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Sub-grid drag models for horizontal cylinder arrays immersed in 
gas-particle multiphase flows

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Abstract

Immersed cylindrical tube arrays often are used as heat exchangers in gas-particle fluidized beds. In 
multiphase computational fluid dynamics (CFD) simulations of large fluidized beds, explicit resolution 
of small cylinders is computationally infeasible. Instead, the cylinder array may be viewed as an effective 
porous medium in coarse-grid simulations. The cylinders’ influence on the suspension as a whole, manifest-
ished as an effective drag force, and on the relative motion between gas and particles, manifested as a 
correction to the gas-particle drag, must be modeled via suitable sub-grid constitutive relationships. In 
this work, highly resolved unit-cell simulations of flow around an array of horizontal cylinders, arranged 
in a staggered configuration, are filtered to construct sub-grid, or ‘filtered’, drag models, which can be 
implemented in coarse-grid simulations. The force on the suspension exerted by the cylinders is com-
prised of, as expected, a buoyancy contribution, and a kinetic component analogous to fluid drag on a 
single cylinder. Furthermore, the introduction of tubes also is found to enhance segregation at the scale
of the cylinder size, which, in turn, leads to a reduction in the filtered gas-particle drag.

**Keywords:** Computational fluid dynamics (CFD); Cylinders; Filtered models; Fluidization; Multiphase flow; Multiscale

1 Introduction

Cylindrical tube arrays often are used in industrial fluidized bed systems as internal heat exchangers. The presence of these tube bundles strongly affects the dynamics of particle clusters and gas bubbles, which, in turn, affects the macroscopic flow (Yurong et al., 2004; Asegehegn et al., 2011; Schreiber et al., 2011, to name a few studies). To obtain good quantitative predictions, the grid used in computational fluid dynamics (CFD) simulations needs to be fine enough to resolve flow phenomenon at the smallest length-scales (Agrawal et al., 2001). To resolve the flow around cylinders a few centimeters in diameter in devices that may be tens of meters tall, the required number of cells would be \(O(10^6)\) in two-dimensional (2D) models and up to \(O(10^9)\) in three-dimensional (3D) CFD models. The prohibitive computational cost of such fine-grid simulations necessitates the development of multiscale methods where the cylinders do not have to be explicitly resolved.

One approach is to use coarse-grid simulations, where the cylinders are replaced by an effective uniform, stationary porous medium, and the effects of the unresolved cylinders on the gas-particle flow are captured using sub-grid models. The objective of this work is to construct these sub-grid models by filtering the results from periodic cell CFD simulations with immersed horizontal cylinder arrays. The influence of the immersed cylinders on the gas-solid suspension is mainly felt via two ways. First, the cylinders can exert a drag force directly on the suspension. Second, the cylinders may change the clustering behavior of particles, which indirectly affects the filtered gas-particle drag. Both mechanisms are investigated, and the corresponding filtered models for cylinder-suspension drag and gas-solid drag are presented.

2 Background

A characteristic feature of gas-particle flows in fluidized beds is the rapid formation and dissociation of inhomogeneous particle clusters. Macroscopic flow properties, such as solid fraction profile and particle mass inventory, are strongly influenced by the gas flow around these microscopic particle clusters (Igci et al., 2008;
Parmentier et al., 2012). Particle clusters are known to have a characteristic length scale of $O(10d_p)$, where $d_p$ is the particle diameter (Agrawal et al., 2001). Hence, for applications where particle diameters typically range from 50 µm to 500 µm, cell sizes on the order of a few millimeters are required—small enough to resolve fine particle clusters. For large-scale industrial applications, where the devices can be up to 10 m in size, $O(10^3)$ cells are needed along each direction, i.e., millions of cells for 2D and billions of cells for 3D simulations. Simulations with $O(10^6) - O(10^9)$ cells are numerically very expensive and generally require access to powerful computing resources.

Often, arrays of heat transfer tubes are immersed in fluidized beds. To accurately resolve the flow around individual tubes, the cell size must also be sufficiently smaller than the cylinder diameter and spacing, in addition to being small enough to resolve fine particle clusters. These high-resolution simulations provide detailed information on the microscopic flow of clusters and bubbles. However, for commercial-scale devices, the macroscopic quantities, such as bed height, solids holdup, and axial pressure variation, usually are of greater interest.

A number of authors have proposed using sub-grid, or ‘filtered’, constitutive models in coarse-grid CFD simulations, similar to the large-eddy simulation technique used in single-phase turbulence. These coarse-grid simulations cannot explicitly resolve small-scale features such as clusters, but their influences are incorporated as sub-grid constitutive relationships (Agrawal et al., 2001). In a series of papers from Princeton University (Andrews IV et al., 2005; Igci et al., 2008; Igci and Sundaresan, 2011a,b; Igci et al., 2011; Milioli et al., 2013), sub-grid models for filtered gas-solid drag, particle- and gas-phase viscosities, and particle-phase pressure are developed by filtering the results from highly-resolved simulations of gas-particle flows in smaller periodic domains. Parmentier et al. (2012) further show, through a budget analysis, that the filtered sub-grid drag is the most important correction. Implementations of their sub-grid drag model in coarse-grid simulations, without any corrections for filtered stresses, produce reasonably good results for macroscopic flow predictions. Other authors have proposed alternate sub-grid gas-particle drag corrections, a discussion of the various formulations is presented by Schneiderbauer et al. (2013).

Although most research has focused on gas-solid flows away from boundaries, corrections for vertical walls are introduced in Igci and Sundaresan (2011b). The filtered gas-solid drag coefficient is found to be
significantly smaller close to the walls, which strongly suggests that immersed boundaries—such as an array of cooling tubes—can significantly affect the filtered relationships.

No attempts have been made to develop sub-grid models for immersed cylinder arrays. There may be hundreds of cooling tubes in commercial devices, which makes it computationally infeasible to resolve the cylinders explicitly in CFD simulations. However, it may be possible to replace the exact representation of the cylinder array by an effective uniform, stationary porous medium. The objective of this work is to develop filtered relationships that model the effects of the cylinder array on the gas-solid suspension. The approach followed in this study is based on the work of the Princeton University group, particularly the methods used by Igci et al. (2008) and Igci and Sundaresan (2011a). Highly-resolved simulations of a periodic cell with immersed cylinders are analyzed to construct filtered constitutive relationships for cylinder-suspension drag and gas-solid drag. Sub-grid models for stresses presently are ignored as Parmentier et al. (2012) have shown the drag corrections to be adequate.

3 Filtered two-fluid model equations for cylinder drag

Kinetic-theory-based microscopic two-fluid model equations (Ding and Gidaspow, 1990; Gidaspow, 1994) have been used to simulate multiphase flow in a number of fluidized bed systems, and have been the starting point of several recent studies that have developed filtered models (Andrews IV et al., 2005; Igci et al., 2008; Igci and Sundaresan, 2011a,b; Parmentier et al., 2012; Holloway and Sundaresan, 2012; Milioli et al., 2013; Agrawal et al., 2013). In this work, the microscopic two-fluid model also is used to construct filtered drag models with an immersed cylinder array. The flow is assumed to be isothermal and non-reactive. A summary of the microscopic two-fluid model conservation equations is presented in Table 1, which are solved using the open-source code MFIX (Syamlal et al., 1993; Syamlal, 1998).

Instead of the more detailed partial differential equation formulation, a simpler algebraic approximation of the granular energy equation is used (Eq. 5 in Table 1, where $\theta_s$ is the solids granular temperature). This is a necessary choice so that the cut-cell capability in MFIX (Dietiker, 2012) can be used to model curved cylinder boundaries. Li et al. (2011) also have used the cut-cell feature and algebraic approximation for granular energy to model a tube bundle in a bubbling bed. Their simulation results show reasonably good
agreement with experimental measurements of bubble dynamics, suggesting that the algebraic approximation is an acceptable simplification.

Table 1: Summary of microscopic two-fluid model equations.

<table>
<thead>
<tr>
<th>Mass conservation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial (\rho_s \phi_s)}{\partial t} + \nabla \cdot (\rho_s \phi_s \mathbf{v}_s) = 0 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Momentum conservation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial (\rho_s \phi_s \mathbf{v}_s)}{\partial t} + \nabla \cdot (\rho_s \phi_s \mathbf{v}_s \mathbf{v}_s) = -\nabla \cdot \mathbf{\sigma}_s - \phi_s \nabla \cdot \mathbf{\sigma}<em>s + \mathbf{f}</em>{gs} + \rho_s \phi_s \mathbf{g} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Granular energy conservation (algebraic formulation adapted from Syamlal (1987))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_s = \left[ -K_1 \phi_s \text{tr}(D_s) + \sqrt{K_1 \text{tr}(D_s) \phi_s}^2 + 4K_2 \phi_s \left[ K_2 \text{tr}^2(D_s) + 2K_3(D_s : D_s) \right] \right]^{2} )</td>
</tr>
</tbody>
</table>

where \( D_s = \frac{1}{2} \left[ \nabla \mathbf{v}_s + (\nabla \mathbf{v}_s)^T \right] \),

\( K_1 = 2(1 + e_{pp})\rho_s \phi_s \),

\( K_2 = \frac{4d_p \rho_s (1 + e_{pp})\phi_s g_o}{3\sqrt{\pi}} - \frac{2}{3} K_3 \),

\( K_3 = \frac{d_p \rho_s}{2} \left\{ \frac{\sqrt{\pi}}{3} \left[ 0.5 (1 + 3e_{pp}) + 0.4(1 + e_{pp})(3e_{pp} - 1)\phi_s g_o \right] + \frac{8\phi_s g_o (1 + e_{pp})}{5\sqrt{\pi}} \right\} \),

\( K_4 = \frac{12(1 - e_{pp}^2)\rho_s \phi_s \phi_s g_o}{d_p \sqrt{\pi}} \).

<table>
<thead>
<tr>
<th>Gas phase stress</th>
</tr>
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<tbody>
<tr>
<td>( \mathbf{\sigma}_g = p_g \mathbf{I} - \mu_g \left[ \nabla \mathbf{v}_g + (\nabla \mathbf{v}_g)^T - \frac{2}{3}(\nabla \cdot \mathbf{v}_g) \mathbf{I} \right] )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solid phase stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathbf{\sigma}_s = [\rho_s \phi_s (1 + 4\eta \phi_s g_o)\theta_s - \eta \mu_s (\nabla \cdot \mathbf{v}_s)] \mathbf{I} - 2\mu_s \mathbf{S} )</td>
</tr>
</tbody>
</table>

where \( \mathbf{S} = \frac{1}{2} \left[ \nabla \mathbf{v}_s + (\nabla \mathbf{v}_s)^T \right] - \frac{1}{3}(\nabla \cdot \mathbf{v}_s) \mathbf{I} \).
\[
\mu_s = \left(\frac{2 + \alpha}{3}\right) \left[ \frac{\mu^*}{g_o \eta (2 - \eta)} \left( 1 + \frac{8}{5} \phi_s \eta \eta g_o \right) \left( 1 + \frac{8}{5} \eta (3 \eta - 2) \phi_s g_o \right) + \frac{3}{5} \eta \mu_b \right],
\]

\[
\mu_b = \frac{256 \mu \phi_s^2 g_o}{5 \pi}, \quad \mu^* = \frac{\mu}{1 + \frac{2 \beta_g \mu}{(\rho_s \phi_s)^2 g_o \theta_s}}, \quad \mu = \frac{5 \rho_s d_p \sqrt{\pi \theta_s}}{96},
\]

\[
g_o = \frac{1}{1 - \left( \frac{\phi_s}{\phi_{s,\text{max}}} \right)^{1/3}}, \quad \eta = \frac{1 + e_{pp}}{2}, \quad \alpha = 1.6, \quad \phi_{s,\text{max}} = 0.64.
\]

Gas-solid drag (Wen and Yu, 1966)

\[
f_{g,s} = \beta_{g,s,\text{micro}} (v_g - v_s), \quad (8)
\]

where

\[
\beta_{g,s,\text{micro}} = 3 \left(1 - \phi_s\right) \rho_s \phi_s |v_g - v_s| \mu_g (1 - \phi_s)^{2.65},
\]

\[
C_D = \begin{cases} 
\frac{24}{Re_p} (1 + 0.15 Re_p^{0.687}) & Re_p < 1000 \\
0.44 & Re_p \geq 1000
\end{cases}
\]

\[
Re_p = \frac{(1 - \phi_s) \rho_s d_p |v_g - v_s|}{\mu_g}.
\]

The objective of this work is to develop filtered drag relationships that model the presence of a cylinder array. In this study, a staggered arrangement of horizontally oriented cylinders is used (Fig. 1) because it is a common configuration for cooling tube bundles (Kim et al., 2003; Li et al., 2011). Sometimes, arrays of vertical cooling tubes are also employed, but to develop filtered models for vertical cylinder arrays, computationally expensive 3D simulations must be necessarily performed. Deriving filtered corrections for horizontal cylinder arrays is more tractable as it is easier to construct idealizations of horizontal tubes using 2D simulations. In the future, filtered coarse-grain models for vertical tubes (or other configurations of interest) can be developed by performing additional 3D simulations and following the method described in this work.

The derivation of the filtered sub-grid conservation equations is based on the approach described in Igci.
et al. (2008). The simulation domain is comprised of a square unit cell with an immersed cylinder array (Fig. 1). The edges of the unit cell are periodic boundaries. A vertical pressure drop $\Delta P_{\text{per}}$ is prescribed to drive the flow along the direction of gravity, which acts vertically downwards. It also is possible to generate a mean flow along the horizontal direction—perpendicular to gravity—by imposing a horizontal pressure gradient. To investigate drag anisotropy due to cylinders, a number of simulations with horizontal flow are performed (see Sec. 7). However, in a majority of the simulations presented, the predominant flow is vertical.

The control volume (CV) is defined as the region occupied by the gas and solids, not including the cylinders, shaded gray in Fig. 1. A uniform ‘box’ filter $G(x, y, z)$ is defined over the entire shaded volume $V_{\text{CV}}$ such that:

$$\int\int\int_{V_{\text{CV}}} G(x, y, z) \, dx \, dy \, dz = 1,$$

which yields a constant value of $G(x, y, z) = 1/V_{\text{CV}}$. The filtered solid fraction $\bar{\phi}_s$ and filtered solids velocity $\bar{v}_s$ are defined as:

$$\bar{\phi}_s = \int\int\int_{V_{\text{CV}}} G(x, y, z) \phi_s \, dx \, dy \, dz = \frac{1}{V_{\text{CV}}} \int\int\int_{V_{\text{CV}}} \phi_s \, dx \, dy \, dz,$$

$$\bar{\phi}_s \bar{v}_s = \int\int\int_{V_{\text{CV}}} G(x, y, z) \phi_s v_s \, dx \, dy \, dz = \frac{1}{V_{\text{CV}}} \int\int\int_{V_{\text{CV}}} \phi_s v_s \, dx \, dy \, dz,$$

respectively, where $\phi_s$ and $v_s$ are the unfiltered variables from highly-resolved simulations. The gas phase filtered volume fraction $\bar{\phi}_g$ and filtered velocity $\bar{v}_g$ are defined in a similar manner. Note that the filtered variables are differentiated from the unfiltered microscopic quantities by adding accents over the symbols: straight overbars denote volume-averaged variables whereas tilde accents are used for Favre-averaged velocities.

Applying the filter $G(x, y, z)$ to the mass conservation equations (Eqs. 1 and 2 in Table 1) yields:

$$\frac{\partial (\rho_s \bar{\phi}_s)}{\partial t} + \nabla \cdot (\rho_s \bar{\phi}_s \bar{v}_s) = 0,$$

$$\frac{\partial (\rho_g \bar{\phi}_g)}{\partial t} + \nabla \cdot (\rho_g \bar{\phi}_g \bar{v}_g) = 0,$$

where $\rho_s$ and $\rho_g$ are the solid- and gas-phase densities, respectively. From the simulations performed in this work, the gas density variation across the simulated domain is found to be minuscule. Therefore, compressibility effects are negligible.
The conservation equation for the net suspension momentum, obtained by adding Eqs. 3 and 4 from Table 1, is the starting point for deriving the cylinder-suspension drag model:

\[
\frac{\partial}{\partial t} (\rho_s \phi_s \mathbf{v}_s + \rho_g \phi_g \mathbf{v}_g) + \nabla \cdot (\rho_s \phi_s \mathbf{v}_s \mathbf{v}_s + \rho_g \phi_g \mathbf{v}_g \mathbf{v}_g) = -\nabla \cdot (\sigma_s + \sigma_g) + (\rho_s \phi_s + \rho_g \phi_g) \mathbf{g},
\]

(14)

where \(\sigma_s\) and \(\sigma_g\) are the solid and gas phase stress tensors, and \(\mathbf{g}\) is the specific body force. Applying the filter over the CV yields:

\[
\frac{1}{V_{CV}} \iiint_{CV} \left[ \frac{\partial}{\partial t} (\rho_s \phi_s \mathbf{v}_s + \rho_g \phi_g \mathbf{v}_g) \right] \, dx \, dy \, dz \\
+ \frac{1}{V_{CV}} \iiint_{CV} \left[ \nabla \cdot (\rho_s \phi_s \mathbf{v}_s \mathbf{v}_s + \rho_g \phi_g \mathbf{v}_g \mathbf{v}_g) \right] \, dx \, dy \, dz \\
= \frac{1}{V_{CV}} \iiint_{CV} \left[ -\nabla \cdot (\sigma_s + \sigma_g) \right] \, dx \, dy \, dz \\
+ \frac{1}{V_{CV}} \iiint_{CV} \left[ (\rho_s \phi_s + \rho_g \phi_g) \mathbf{g} \right] \, dx \, dy \, dz
\]

(15)

In Eq. 15, the first term on the left-hand side represents the filtered temporal term. Despite having a relatively small value close to zero (during statistical steady state), this term is retained in the analysis. The second term on the left represents the net advective momentum flux for the CV. For a periodic cell, this term is always zero and need not be considered. The last term on the right-hand side of Eq. 15 is the filtered body force, which can be easily evaluated, especially when \(\mathbf{g}\) is constant.

The expression for filtered stress gradients, specifically, the first term on the right, can be converted to a surface integral using the divergence theorem. Limits of the surface integral are defined by the surface area of the periodic boundaries, \(S_{per}\), and the surface area of the immersed cylinders, \(S_{cyl}\):

\[
\frac{1}{V_{CV}} \iiint_{CV} \left[ -\nabla \cdot (\sigma_s + \sigma_g) \right] \, dx \, dy \, dz =
\]

\[
\frac{1}{V_{CV}} \int_{S_{per}} \left[ -\mathbf{(\sigma_s + \sigma_g)} \cdot \mathbf{n} \right] \, dS + \frac{1}{V_{CV}} \int_{S_{cyl}} \left[ -\mathbf{(\sigma_s + \sigma_g)} \cdot \mathbf{n} \right] \, dS.
\]

(16)

The unit vector \(\mathbf{n}\) is the outward normal of the periodic and cylinder boundaries, and \(dS\) is a differential surface element. The terms on the right-hand side of Eq. 16 represent the surface traction on the periodic boundaries and the cylinder array, acting per unit volume of the suspension. The traction due to the solid phase integrated over periodic boundaries always equals zero (solid stresses are periodic). However, because
there is a pressure drop $\Delta P_{\text{per}}$ specified across the periodic boundaries, the integral of the traction due to the gas phase is not zero. Hence, the gas-solid suspension traction integrated over the periodic surfaces, i.e., the first term on the right-hand side of Eq. 16, simplifies to the average gas phase pressure gradient imposed across the periodic boundaries.

The second term on the right-hand side of Eq. 16 represents the filtered drag force (per unit suspension volume) exerted on the gas-solid suspension by the immersed cylinders. As the objective of this work is to express the filtered drag force in terms of other filtered variables, this term is of primary interest. Combining Eqs. 15 and 16, the filtered cylinder-suspension drag $\bar{f}_{c,gs}$ can be written as:

$$\bar{f}_{c,gs} = \frac{1}{V_{CV}} \int_{SA_{cyl}} \int \left[ -(\sigma_s + \sigma_g) \cdot \hat{n} \right] dS$$

$$= \frac{\partial}{\partial t} (\rho_s \bar{\phi}_s \bar{v}_s + \rho_g \bar{\phi}_g \bar{v}_g) + \frac{1}{V_{SA_{per}}} \int_{SA_{per}} (p_g \mathbf{I}) \cdot \hat{n} dS - (\rho_s \bar{\phi}_s + \rho_g \bar{\phi}_g) \mathbf{g}.$$  

A closure for $\bar{f}_{c,gs}$ needs to be constructed in terms of the filtered gas and particle volume fractions, velocities, and geometric parameters.

The derivations of filtered momentum conservation equations for gas and solid phases separately are described in Igci et al. (2008) and will not be repeated here. The final filtered momentum conservation equations are given by:

$$\frac{\partial (\rho_s \bar{\phi}_s \bar{v}_s)}{\partial t} + \nabla \cdot (\rho_s \bar{\phi}_s \bar{v}_s \bar{v}_s) = -\nabla \cdot \bar{\sigma}_s - \bar{\phi}_s \nabla \cdot \bar{\sigma}_g + \bar{f}_{g,s} + \bar{f}_{c,gs} + \rho_s \bar{\phi}_s \mathbf{g},$$  

$$\frac{\partial (\rho_g \bar{\phi}_g \bar{v}_g)}{\partial t} + \nabla \cdot (\rho_g \bar{\phi}_g \bar{v}_g \bar{v}_g) = -\bar{\phi}_g \nabla \cdot \bar{\sigma}_g - \bar{f}_{g,s} + \rho_g \bar{\phi}_g \mathbf{g}.$$  

These same assumptions and simplifications, discussed in Igci et al. (2008) and Agrawal et al. (2001), also are applicable to Eqs. 19 and 20. However, a notable difference between the filtered momentum equations developed in this work and those presented in Igci et al. (2008) and Igci and Sundaresan (2011a) is the introduction of a new term—$\bar{f}_{c,gs}$—representing the cylinder-suspension drag. For reasons explained in Sec. 7, the cylinder-suspension drag is included in the solids momentum equation and not partitioned between the two phases.

The filtered gas-solid drag $\bar{f}_{g,s}$ also appears in Eqs. 19 and 20. The filtered gas-solid drag is given by:

$$\bar{f}_{g,s} = \frac{1}{V_{CV}} \int_{CV} \int \int \beta_{g,s,\text{micro}} (\mathbf{v}_g - \mathbf{v}_s) dx dy dz.$$  

(21)
Strictly speaking, the filtered gas-solid drag should also include the contribution from the filtered correlated fluctuations of microscopic solids fraction and gas phase stress gradient \((\vec{\alpha}_s \nabla \cdot \sigma'_g)\). However, this contribution is known to be much smaller compared to the quantity given by Eq. 21 (Agrawal et al., 2001; Igci et al., 2008) and, therefore, is not included in the present analysis. It will be shown that the existing sub-grid model for \(f_{g,s}\) (Igci et al., 2008) must be modified when cylinders are immersed in the flow. Closure for the filtered cylinder-suspension drag \(f_{c,g,s}\) is presented in Sec. 7, and the modifications to the filtered gas-solid drag \(f_{g,s}\) are discussed in Sec. 8.

4 Highly-resolved two-fluid model simulations

To construct closures for the filtered cylinder-suspension drag \(f_{c,g,s}\) and filtered gas-solid drag \(f_{g,s}\), highly resolved 2D simulations of gas-particle flow in a periodic unit cell (Fig. 1) are performed using the open-source code MFIX. The periodic unit cell contains eight immersed cylinders arranged in a staggered pattern, as shown in Fig. 1. Initial attempts using a non-staggered arrangement revealed that successive rows of cylinders should be offset to avoid undesirable flow behavior, namely, formation of gas flow channels between the tubes and particles settling on top of the cylinders. Choosing a staggered arrangement creates a more tortuous path and prevents channeling of the gas flow. The periodicity of the cylinder arrangement dictates that the edge length \(L_{per}\) must be an integer multiple of the cylinder spacing \(a_c\). The effect of varying \(L_{per}\) (also the filter length \(\Delta_{filter}\)) is discussed in Sec. 5.

A schematic of the computational domain showing the grid and cut-cell treatment of cylinder boundaries is presented in Fig. 2. The cut-cell approach approximates each cylinder as a polygon with a large number of sides whose vertices are the intersections of grid lines and cylinder’s circular edge. The cells, whole or truncated, falling inside the cylinder volume are excluded from computations. This cut-cell approach has been used to simulate complex boundaries in single-phase flows (for example, Causon et al., 2000), as well as multiphase gas-particle flows (Li et al., 2011). The implementation of the cut-cell feature in MFIX is documented in Dietiker (2012). In this work, the no-slip boundary condition is applied for both gas and solid phases at the cylinder walls, representing the case where the immersed tubes have the greatest impact on suspension flow. The second-order Superbee discretization scheme is used for convective terms appearing
in the conservation equations. For numerical stability, an explicit time marching scheme that allows under-
relaxation of the changes in the field variables is used (refer to Syamlal, 1998).

All quantities reported in this paper are made dimensionless using the particle terminal velocity \( v_t \),
particle density \( \rho_s \), and characteristic length scale \( v_t^2/g \), following the scaling used in Igci et al. (2008) and
Igci and Sundaresan (2011a,b). Although an alternative length scale of interest is provided by the particle
diameter \( d_p \), Agrawal et al. (2001) demonstrate that the effect of particle size is mainly felt through \( v_t \), which,
in turn, depends on \( d_p \). Unless otherwise noted, \( v_t^2/g \) is used as the characteristic length scale in this work.
Table 2 summarizes the particle properties and the characteristic quantities used for nondimensionalization.
Henceforth, variables that have been made dimensionless are marked by a superscript asterisk.

The gas phase density (\( \rho_g \)) and viscosity (\( \mu_g \)) are computed assuming a mixture of nitrogen (76%), carbon
dioxide (18%), and water vapor (6%). This particular composition is of interest as the motivation for this
problem is the need to develop multiscale models for carbon-capture devices, described in Sarkar et al. (2013).
The gas properties are provided in Table 2, but it should be noted that the absolute values of \( \rho_g \) and \( \mu_g \) are
unimportant as they appear implicitly in the terminal velocity (\( v_t \)), which is used to nondimensionalize all results.

Parametric studies of periodic cell simulations are performed for different (dimensionless) combinations
of cylinder diameters (\( D_{cyl}^* \)) and spacings (\( a_{cyl}^* \)). For every cylinder configuration, the average (filtered) solid
fraction \( \bar{\phi}_s \) in the domain is varied between 0.01 to 0.60. Figure 3 depicts snapshots of the simulations with
varying \( \bar{\phi}_s \) for a representative cylinder configuration. Furthermore, for each \( \bar{\phi}_s \) value, the periodic pressure
drop \( \Delta P_{per} \) is systematically varied to obtain filtered drag measurements at different gas and solids velocities.
Increasing \( \Delta P_{per} \) results in larger gas and solids velocities and vice versa, but the quantitative dependence is
not known in advance. The pressure drop \( \Delta P_{per} \), expressed in terms of the hydrostatic suspension weight, is
varied between 0.65 to 1.35 times \( (\bar{\phi}_s \rho_s + \bar{\phi}_g \rho_g)gL_{per} \) to obtain the velocity range of interest. Thus, hundreds
of highly-resolved periodic slice simulations are performed for different combinations of cylinder diameter,
cylinder spacing, solid fraction, and gas/solids velocities (prescribed via \( \Delta P_{per} \)).

Flow in the periodic cell simulations is chaotic: rapid formation and dissociation of clusters are observed
at smaller \( \bar{\phi}_s \) values, while rising gas bubbles are seen at larger \( \bar{\phi}_s \). Figure 4 shows a typical temporal profile
Table 2: Summary of invariant simulation parameters.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceleration due to gravity</td>
<td>$g$</td>
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</tr>
<tr>
<td>Particle diameter</td>
<td>$d_p$</td>
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<tr>
<td>Particle density</td>
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<td>441 kg/m³</td>
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<tr>
<td>Particle terminal velocity</td>
<td>$v_t$</td>
<td>0.2697 m/s</td>
</tr>
<tr>
<td>Gas density</td>
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<td>1.14 kg/m³</td>
</tr>
<tr>
<td>Gas viscosity</td>
<td>$\mu_g$</td>
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</tr>
<tr>
<td>Characteristic length scale</td>
<td>$v_t^2/g$</td>
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</tr>
<tr>
<td>Characteristic time</td>
<td>$v_t/g$</td>
<td>0.0275 s</td>
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<tr>
<td>Characteristic volumetric force</td>
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</tr>
<tr>
<td>Particle-particle restitution coefficient</td>
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</tr>
<tr>
<td>Particle-wall restitution coefficient</td>
<td>$e_{pw}$</td>
<td>0.9</td>
</tr>
</tbody>
</table>

of the filtered gas and solids vertical velocities. Starting from a zero value, $\tilde{v}_g^*$ and $\tilde{v}_s^*$ pass through an initial transient period before reaching a statistical steady state. Thereafter, the velocity values (and all other measurements) randomly fluctuate about the mean value; these fluctuations are due to rapid changes in the local clustering behavior. All measurements reported in this work are computed after statistical steady state is achieved.

5 Effect of filter length

In deriving sub-grid models for gas-particle flows without cylinders, a number of works (Igci et al., 2008; Igci and Sundaresan, 2011a; Parmentier et al., 2012) find the filtered corrections to be dependent on the filter length. In this section, the influence of varying filter length is examined with immersed cylinders. Three filter sizes are simulated: $\Delta_{\text{filter}}^* = 26.98$, $\Delta_{\text{filter}}^* = 40.46$, and $\Delta_{\text{filter}}^* = 53.95$ (Fig. 5). These simulations
are performed using a grid resolution of $\Delta_{\text{grid}}/d_p = 8.33$, average solid fraction $\bar{\phi}_s = 0.30$, and vertical pressure drop $\Delta P_{\text{per}} = 1.10(\bar{\phi}_s \rho_s + \bar{\phi}_g \rho_g)gL_{\text{per}}$. The quantities of interest are the vertical components of filtered cylinder-suspension drag force ($\bar{f}_{cgs,y}$), filtered gas-solid drag force ($\bar{f}_{ggs,y}$), filtered gas velocity ($\bar{\tilde{v}}_g$), and filtered solids velocity ($\tilde{v}_s^*$). The filtered drag force measurements are made dimensionless by the characteristic force $\rho_s g$, while the filtered velocities are made dimensionless by the terminal velocity $v_t$. A comparison of the filtered velocities and drag forces (Fig. 6) shows insignificant changes with varying filter size. Therefore, the filtered drag coefficients—which are calculated from the filtered velocity and drag force measurements—also are unaffected by varying filter size.

It has been shown that the impact of changing filter size is more significant at smaller filter lengths (Igci et al., 2008; Igci and Sundaresan, 2011a; Parmentier et al., 2012). As the filter size increases, the filtered drag coefficient approaches an asymptotic value (for given $\bar{\phi}_s$). For the filter lengths considered in this work ($\Delta_{\text{filter}}^* \geq 26.98$), it appears the filtered drag coefficients are near the asymptotic limit. Indeed, the filter size correction factor given by Igci and Sundaresan (2011a) is already 0.98 at $\Delta_{\text{filter}}^* = 26.95$, which almost equals the asymptotic value of 1 for $\Delta_{\text{filter}}^* \to \infty$. Therefore, the effect of filter size may be safely excluded from this work as the simulated filter lengths are sufficiently large. For the remainder of this work, a filter length of $\Delta_{\text{filter}}^* = 26.98$ is employed, and the generated results are considered applicable to the large filter limit ($\Delta_{\text{filter}}^* \to \infty$).

A further examination of simulation snapshots (Fig. 5) shows no visible differences in particle clustering behavior as the filter length increases. The cylinders appear to act as barriers that geometrically limit the size of the largest clusters (also see Fig. 13). This observation is further supported by Asegehegn et al. (2011) and Schreiber et al. (2011), who have performed experiments and simulations of bubbling beds with horizontal tubes. They have determined that the bubble diameters do not grow with bed height in the presence of tubes, whereas they do so in beds without tubes. Formation of large bubbles and clusters is prevented as the cylinders tend to cleave larger flow structures. As the highly-resolved simulations can reproduce flow patterns seen experimentally, there is reason to believe the two-fluid model can capture reality. Hence, the two-fluid model equations are a justifiable basis for constructing filtered corrections.
6 Effect of grid size

The influence of varying grid resolution also is investigated in this study. Agrawal et al. (2001) have demonstrated that the grid size to particle diameter ratio ($\Delta_{grid}/d_p$) should be $O(10)$ to resolve fine particle clusters. Igci et al. (2008) have found that filtered measurements are grid-independent using $\Delta_{grid}/d_p$ ratios smaller than 16.67. In this work, the variation of the quantities of interest, namely, filtered velocities and filtered drag force measurements, are investigated for varying $\Delta_{grid}/d_p$ ratios. Figure 7(a) shows that the filtered solids velocity ($\tilde{v}_s^*$) is unaffected by changing grid size. The value $\tilde{v}_g^*$ is found to increase slightly with decreasing grid size, but the difference is small and well within one standard deviation of $\tilde{v}_g^*$ fluctuations (indicated by scatter bars in Fig. 7(a)). Decreasing the grid size does not change the filtered cylinder-suspension drag $\bar{f}_{c,gs,y}^*$, but results in a minor reduction in the filtered gas-solid drag $\bar{f}_{g,s,y}^*$ (Fig. 7(b)).

Fine clusters in the flow are resolved slightly better using smaller grid sizes. The gas is able to bypass these now-resolved fine clusters, which explains the small increase in gas velocity $\tilde{v}_g^*$ and corresponding small decrease in the gas-particle drag $\bar{f}_{g,s,y}^*$. However, the marginal improvement obtained using a smaller grid size of $\Delta_{grid}/d_p = 4.76$ over the $\Delta_{grid}/d_p = 8.33$ case does not justify the considerable increase in computational cost, especially when hundreds of highly-resolved simulations are needed to construct the sub-grid models. Moreover, the filtered solids velocity $\tilde{v}_s^*$ and cylinder-suspension drag $\bar{f}_{c,gs,y}^*$ are practically grid-independent. For the purposes of this work, a resolution of $\Delta_{grid}/d_p = 8.33$ is considered sufficiently accurate. Note that this resolution already is finer than that adopted by Igci et al. (2008), who have employed a $\Delta_{grid}/d_p$ ratio of 16.67.

7 Drag on suspension exerted by cylinders

Explicit resolution of small cylinders in a large-scale simulation is computationally infeasible. Therefore, the cylinder array needs to be modeled in an alternate fashion. One approach is to replace the cylinder array with a stationary porous medium that excludes the volume occupied by the actual cylinders. Implementing such a stationary porous medium in multiphase CFD simulations is fairly straightforward: an additional solid phase is introduced but the conservation equations for this phase are not solved. The effective drag force experienced by the gas-solid suspension due to the cylinders appears as the filtered cylinder-suspension
drag force $\bar{f}_{c, gs}$ in the filtered momentum equations, expressed in terms of the filtered variables ($\bar{\phi}_y, \bar{\phi}_s, \bar{v}_g^*, \bar{v}_s^*$) and geometric parameters ($D_{cyl}^*, a_{cyl}^*$).

The filtered vertical drag $\bar{f}_{c, gs,y}$ is obtained by considering the Y-component of Eq. 18. The drag on the gas and solid phases are not evaluated separately but considered together as the cylinder-suspension drag (it will be reasoned later that the cylinder-gas drag is much smaller compared to the cylinder-solids drag).

To determine the relationship between filtered drag force and filtered velocities, Fig. 8 plots $\bar{f}_{c, gs,y}$ against $\bar{v}_g^*$ and $\bar{v}_s^*$, shown for three representative solid fractions. These results are for the case with $D_{cyl}^* = 4.15$ and $a_{cyl}^* = 13.49$. Other solid fractions and cylinder configurations also are examined but are not presented as figures. It is known that the drag force on a single cylinder immersed in a single-phase medium is proportional to the square of flow velocity. The drag force experienced by a cylinder immersed in a gas is given by $C_{gas} D_{cyl}^* \rho_g v_g^2$, and on a cylinder in a granular flow is $C_{solid} D_{cyl}^* \rho_s \phi_s v_s^2 (D_{cyl} + d_p)$, where $C_{gas}$ and $C_{solid}$ are the drag coefficients (Wassgren et al., 2003; Chehata et al., 2003). The data presented in Fig. 8 strongly suggest that the filtered drag force due to an array of cylinders also is proportional to the square of the filtered flow velocity.

For typical gas-particle flows, the cylinder-solids drag force, which is $O(\rho_s v_s^2)$, is much larger than the cylinder-gas drag, which is $O(\rho_g v_g^2)$, since $\rho_s \gg \rho_g$. Therefore, the gas phase contribution to the filtered cylinder-suspension drag force may be ignored, because the drag force is mainly due to the forces exerted by particles colliding with the tubes. Moreover, the filtered gas velocity ($\bar{v}_g^*$) is strongly correlated to the filtered solids velocity ($\bar{v}_s^*$); in Figs. 8(b) and 8(c), the two curves corresponding to $\bar{v}_g^*$ and $\bar{v}_s^*$ are almost parallel. Hence, it is more appropriate to express the filtered drag force as a function of the solids velocity $\bar{v}_s^*$, given by:

$$
\bar{f}_{c, gs,y} = -\text{sign}(\bar{v}_s^*) \bar{\beta}_{c, gs,y}^* (\bar{v}_s^*)^2 - \bar{\gamma}_{c, gs,y}^*
$$

where $\bar{\beta}_{c, gs}^*$ and $\bar{\gamma}_{c, gs}^*$ are the (dimensionless) model parameters, which are functions of the filtered solid fraction, cylinder diameter, and cylinder spacing. The first term of Eq. 22, $\bar{\beta}_{c, gs,y}^* (\bar{v}_s^* |\bar{v}_s^*|)$, is the kinetic component of the cylinder-suspension drag and follows the familiar quadratic form reported by Wassgren et al. (2003) and Chehata et al. (2003). The sign function, or, alternatively, the absolute value of $\bar{v}_s^*$, is used.

$$
\bar{f}_{c, gs,y} = -\text{sign}(\bar{v}_s^*) \bar{\beta}_{c, gs,y}^* (\bar{v}_s^*)^2 - \bar{\gamma}_{c, gs,y}^* \bar{v}_s^* |\bar{v}_s^*| - \bar{\gamma}_{c, gs,y}^* \bar{v}_s^* |\bar{v}_s^*|)
$$

(22)
to ensure the drag force always acts opposite to the direction of flow.

The model parameters $\bar{\beta}^{*}_{c,gs,y}$ and $\bar{\gamma}^{*}_{c,gs,y}$ are obtained by least-squares fits of the $f^{*}_{c,gs,y}$ and $v^{*}_{s}$ measurements using Eq. 22. An examination of the fits show good agreement with simulation measurements, especially for $\bar{\phi}_s > 0.10$ (representative fits shown by the solid lines in Fig. 8). At smaller solid fractions, most of the particles tend to aggregate into a few clusters (refer to Fig. 3). The chaotic flow of these few individual clusters results in large variances in flow measurements. Therefore, the fitted model parameters are less certain for $\bar{\phi}_s \leq 0.10$. Fortunately, considerably better fits are observed in the solid fraction range $0.20 \leq \bar{\phi}_s \leq 0.50$, where most fluidized beds operate.

Values of $\bar{\beta}^{*}_{c,gs,y}$ obtained from fits are plotted for different cylinder diameters and spacings in Fig. 9(a). The results indicate that increasing the cylinder diameter yields a larger cylinder-suspension drag coefficient. The effect of decreasing the cylinder spacing is qualitatively similar to increasing the cylinder diameter: the cylinder-suspension drag coefficients decrease in both cases. However, for a given solid fraction, the cylinder-suspension drag does not vary linearly with cylinder diameter, which is a departure from the established drag expressions for flow past a single cylinder (i.e., $C_{gas}^{D} \frac{1}{2} \rho_g v^2_{gas} D_{cyl}$ or $C_{solid}^{D} \frac{1}{2} \rho_s \bar{\phi}_s v^2_{s} (D_{cyl} + d_p)$). This observation suggests that the flow fields around neighboring tubes interact, especially as the cylinders become larger or get closer. Figures 3 and 5 clearly show particle clusters often span adjacent cylinders, indicating strong interactions of the flow fields around neighboring tubes.

To implement the cylinder-suspension drag as a sub-grid model in CFD simulations, algebraic fits are preferred. The following fit is proposed for the $\bar{\beta}^{*}_{c,gs,y}$ data presented in Fig. 9(a):

$$\bar{\beta}^{*}_{c,gs,y} = \frac{B_1 \bar{\phi}_s^2}{1 + B_2 \bar{\phi}_s^2}$$

where $B_1$ and $B_2$ are fitted by polynomial functions of $D^{*}_{cyl}$ and $a^{*}_{cyl}$, given in Table 3. Fits for $\bar{\beta}^{*}_{c,gs,y}$, shown as solid black lines in Fig. 9(a), are found to match the simulation measurements reasonably well.

The variation of the second model parameter in Eq. 22, $\bar{\gamma}^{*}_{c,gs,y}$, with $\bar{\phi}_s$ is shown in Fig. 9(b) for different cylinder configurations. By increasing the ratio of total cylinder volume to net suspension volume in the domain, larger $\bar{\gamma}^{*}_{c,gs,y}$ values are obtained. The $\bar{\gamma}^{*}_{c,gs,y}$ curves collapse for those combinations of $D^{*}_{cyl}$ and $a^{*}_{cyl}$ values that have equal $D^{*}_{cyl}/a^{*}_{cyl}$ ratios. From this scaling, it is immediately recognized that $\bar{\gamma}^{*}_{c,gs,y}$ is the contribution due to buoyancy. The direction of the force resulting from the $\bar{\gamma}^{*}_{c,gs,y}$ term also is consistent with
buoyancy. The (dimensionless) theoretical buoyancy per unit volume, assuming a perfectly homogeneous suspension and hydrostatic pressure drop, is given by:

\[ f_{\text{buoy, theo}}^* = \frac{f_{\text{buoy, theo}}}{\rho_s g} = \left( \frac{\pi}{4} D_{\text{cyl}}^2 - \frac{\pi}{4} D_{\text{cyl}}^2 \right) \frac{\rho_s \phi_s + \rho_g \phi_g}{\rho_s}. \]  

(24)

The theoretical buoyancy calculated using Eq. 24, shown by dotted lines in Fig. 9(b), are quite similar in magnitude to the measured \( \bar{\gamma}_{c,gs,y}^* \) values. However, the variation with solid fraction \( \bar{\phi}_s \) is not linear. This non-linearity with \( \bar{\phi}_s \) is most likely due to clustering inhomogeneities, which violate the homogeneity assumptions associated with the linear hydrostatic pressure profile (Eq. 24). Therefore, \( \bar{\gamma}_{c,gs,y}^* \) represents the buoyancy contribution, now non-linear, to the filtered cylinder-suspension drag.

A good fit for \( \bar{\gamma}_{c,gs,y}^* \) is given by:

\[ \bar{\gamma}_{c,gs,y}^* = \left( \frac{\pi}{4} D_{\text{cyl}}^2 - \frac{\pi}{4} D_{\text{cyl}}^2 \right) G_1 \bar{\phi}_s + G_2 \bar{\phi}_s, \]  

(25)

where the values of the fitting constants are determined to be \( G_1 = 1.743 \) and \( G_2 = 2.077 \). The fits are shown using solid black lines in Fig. 9(b).

Thus far, the simulations discussed in this section have a macroscopic flow along the vertical (Y) direction, which only enables calculation of the filtered vertical drag coefficients. To determine the filtered drag coefficients along the horizontal direction, additional simulations are performed with a superimposed horizontal flow. These horizontal flow simulations are conducted for one cylinder configuration (\( D_{\text{cyl}}^* = 4.15 \) and \( a_{\text{cyl}}^* = 13.49 \)), but the horizontal drag model developed will be generalized for other cylinder diameters and spacings.

A systematically varying horizontal pressure drop is prescribed to control the horizontal solids (\( \bar{u}_s^* \)) and gas (\( \bar{u}_g^* \)) velocities, while maintaining a constant vertical pressure drop of 1.00 (\( \bar{\phi}_s \rho_s + \bar{\phi}_g \rho_g \)) per. Analysis of the filtered horizontal cylinder-suspension drag force (\( \bar{f}_{c,gs,x}^* \)) and filtered horizontal solids velocity (\( \bar{u}_s^* \)) measurements reveals the following relationship:

\[ \bar{f}_{c,gs,x}^* = -\text{sign}(\bar{u}_s^*) \bar{\beta}_{c,gs,x}^* (\bar{u}_s^*)^2 = -\bar{\beta}_{c,gs,x}^* (\bar{u}_s^*)^2, \]  

(26)

where \( \bar{\beta}_{c,gs,x}^* \) is the filtered horizontal cylinder-suspension drag coefficient. The horizontal buoyancy contribution is negligible because pressure gradients in the horizontal direction are significantly smaller than the
gravity-induced vertical pressure gradients.

The calculated horizontal cylinder-suspension drag coefficients $\bar{\beta}_{c,gs,x}$ are presented in Fig. 10 for the configuration $D_{cyl}^* = 4.15$ and $a_{cyl}^* = 13.49$. The $\bar{\beta}_{c,gs,x}$ values are fit using a function similar to Eq. 23, given by:

$$\bar{\beta}_{c,gs,x} = \frac{B_3 \phi_s^2}{1 + B_4 \phi_s^2}$$ \hspace{1cm} (27)

As horizontal flow measurements for other cylinder diameters and spacings are unavailable, the dependence of the fitting parameters $B_3$ and $B_4$ on $D_{cyl}^*$ and $a_{cyl}^*$ could not be established. Presently, the fitting parameters are expressed as constants: $B_3 = 0.4543$ and $B_4 = 6.427$. The fit using Eq. 27 is shown as a solid black line in Fig. 10.

Anisotropy in granular flows and fluidized beds is well known (for example, see Natarajan et al., 1995; Koch and Sangani, 1999). A comparison of the drag coefficients (for $D_{cyl}^* = 4.15$ and $a_{cyl}^* = 13.49$) from Figs. 9(a) and 10 shows that $\bar{\beta}_{c,gs,y}$ generally is larger than $\bar{\beta}_{c,gs,x}$. Therefore, the presence of anisotropy in the drag coefficients $\bar{\beta}_{c,gs,y}$ and $\bar{\beta}_{c,gs,x}$ is revealed. The anisotropy is quantified by the ratio of drag coefficients $\bar{\beta}_{c,gs,y}/\bar{\beta}_{c,gs,x}$ (Fig. 11). The ratios calculated directly from the simulation measurements are shown by symbols, and the ratio of fits, i.e., Eq. 23 divided by Eq. 25, is represented by the line. The ratio of fits (line) captures the measured anisotropy (symbols) for the solid fraction range of interest ($0.20 \leq \phi_s \leq 0.50$). For the $\phi_s \leq 0.10$ cases, $\bar{\beta}_{c,gs,y}/\bar{\beta}_{c,gs,x}$ measurements are less certain as greater error is associated with dividing $\bar{\beta}_{c,gs,y}$ by $\bar{\beta}_{c,gs,x}$, both of which have near-zero values. For a sub-grid model, it is proposed that the ratio of fits be used to compute $\bar{\beta}_{c,gs,x}$:

$$\bar{\beta}_{c,gs,x} = \bar{\beta}_{c,gs,y} \left( \frac{B_3 \phi_s^2}{1 + B_4 \phi_s^2} \right) \left( \frac{1.042 \phi_s^2}{1 + 16.02 \phi_s^2} \right)$$ \hspace{1cm} (28)

The constants 1.042 and 16.02 that appear in Eq. 28 are the values of $B_1$ and $B_2$, respectively, for $D_{cyl}^* = 4.15$ and $a_{cyl}^* = 13.49$. Although the expression for $\bar{\beta}_{c,gs,x}$ (Eq. 28) is derived by extrapolating from one cylinder configuration, this result may be used for other cylinder diameters and spacings as a first approximation.

It is expected that the drag coefficient along the cylinder axis $\bar{\beta}_{c,gs,z}$ (i.e., perpendicular to the plane shown in Figs. 1–3) also will exhibit anisotropy, consistent with the anisotropic cylinder configuration. Presumably, the $\bar{\beta}_{c,gs,z}$ values are even smaller than $\bar{\beta}_{c,gs,x}$ values, but this hypothesis can only be ascertained by performing expensive 3D simulations. Note that for typical fluidized beds, accurate modeling of the
vertical drag coefficient $\bar{\beta}_{c,gs,y}^*$ is most important as the flow is predominantly vertical. The vertical velocities ($\tilde{v}_{gs}^*$ and $\tilde{v}_{g}^*$), and consequently, the vertical drag forces, are much larger than the corresponding transverse components. Therefore, anisotropy in the drag coefficient $\bar{\beta}_{c,gs}^*$ is less significant in fluidized bed simulations because the transverse velocities and drag forces usually are quite small.

The additional simulations performed with superimposed horizontal flow were not used to derive the model for filtered vertical cylinder-suspension drag $\bar{f}_{c,gs,y}$ (Eqs. 22, 23, and 25). A preliminary evaluation of the proposed model for $\bar{f}_{c,gs,y}$ can be performed using these horizontal flow simulations as a test set. The filtered drag forces $\bar{f}_{c,gs,y}$ predicted using Eq. 22 are compared against the test set of actual horizontal-flow simulation measurements in Fig. 12. Scatter bars represent one standard deviation of the drag observations. Reasonably good agreement is obtained between the predicted and measured $\bar{f}_{c,gs,y}$ values, and the predictions generally are within one standard deviation of the measurements. This finding also hints that the coupling between the horizontal and vertical flows is not significant, i.e., the horizontal flow does not noticeably affect the vertical drag coefficients and vice versa.

8 Influence of cylinders on gas-solid drag

Apart from a direct drag force exerted on the gas-solid suspension, the cylinders also may affect particle clustering behavior. Figure 13 presents snapshots of cluster formation for $\bar{\phi}_s$ values of 0.30 and 0.50, domains without and with cylinders ($D_{cyl}^* = 2.76$ and $5.53$) are shown. The clusters appear unchanged qualitatively for $\bar{\phi}_s = 0.30$. However, at a larger solid fraction of $\bar{\phi}_s = 0.50$, the gas-solid suspension is more heterogeneous with immersed cylinders, evidenced by the gas bubbles seen in Fig. 13. Without cylinders, the suspension is more homogeneous at $\bar{\phi}_s = 0.50$—almost no gas bubbles are visible. The presence of cylinders, which tends to enhance gas-solid segregation at larger solid fractions, is likely to affect the gas-particle drag $\bar{f}_{g,s}$. Hence, a modified sub-grid model for $\bar{f}_{g,s}$ must be developed.

First, the effect of cylinders on the dimensionless filtered gas-solid slip velocity ($\bar{\tilde{v}}_g^* - \bar{\tilde{v}}_s^*$) is examined. Figure 14 shows the filtered slip velocities for different cylinder diameters and spacings. The slip velocities without immersed cylinders are also included (filled triangular symbols). The introduction of cylinders appears to have a negligible influence on the slip velocity, particularly for $\bar{\phi}_s \geq 0.10$. The uncertainty in
velocity measurements at smaller solid fractions explains the larger variations of slip velocity with cylinder configuration for $\tilde{\phi}_s < 0.10$ (refer to Sec. 7). The slip velocity measurements are fit by the expression:

$$\tilde{v}_g^* - \tilde{v}_s^* = 6.697 e^{-8.140 \tilde{\phi}_s} \left( \tilde{\phi}_s^2 + 2.670 \tilde{\phi}_s + 0.4489 \right),$$

shown as a black line in Fig. 14.

Next, modifications to the model for filtered gas-solid drag coefficient must be developed. The filtered drag coefficient $\tilde{\beta}_{g,s,y}^*$ is related to the filtered drag force and slip velocity as:

$$\tilde{f}_{g,s,y}^* = \tilde{\beta}_{g,s,y}^* (\tilde{v}_g^* - \tilde{v}_s^*).$$

Traditionally, the gas-solid drag coefficient is obtained from a microscopic drag model, such as, Wen and Yu (1966) (Eq. 8 in Table 1), shown by the lightly dotted line ($\tilde{\beta}_{g,s,y}^{*\text{micro}}$) in Fig. 15(a). The filtered slip velocity $\tilde{v}_g^* - \tilde{v}_s^*$, from Eq. 29, and the filtered solid fraction $\tilde{\phi}_s$ are used to calculate $\tilde{\beta}_{g,s,y}^{*\text{micro}}$ (hence, the overbar accent). The microscopic drag model incorrectly overpredicts the gas-solid drag coefficient in coarse grid simulations because the influences of unresolved clusters are ignored (Agrawal et al., 2001). Therefore, the effects of these unresolved clusters must be included via sub-grid drag corrections. The filtered drag model developed by Igci and Sundaresan (2011a) for flows without immersed cylinders is included in Fig. 15(a) for reference, indicated by the heavier dotted line.

Figure 13 clearly illustrates that cylinders immersed in the flow can affect clustering behavior. The filtered gas-solid drag coefficients $\tilde{\beta}_{g,s,y}^*$ for simulations with immersed cylinder arrays are shown in Fig. 15, where the different symbols represent various configurations of cylinder diameter and spacing. For $0 \leq \tilde{\phi}_s \leq 0.30$, the filtered gas-solid drag coefficient with immersed cylinders matches the Igci and Sundaresan (2011a) model for $\tilde{\beta}_{g,s,y}^*$ without cylinders. The particle clustering behavior is unchanged for smaller solid fractions (refer to Fig. 13) and, as a result, the $\tilde{\beta}_{g,s,y}^*$ values remain unaffected.

At larger filtered solid fractions, introduction of cylinders enhances gas-solid segregation. The critical solid fraction value above which inhomogeneities start to affect the filtered gas-solid drag coefficient is $\tilde{\phi}_s = 0.30$. In Fig. 15(a), this is the point where the curves for $\tilde{\beta}_{g,s,y}^*$ without cylinders (heavily dotted line) and with immersed cylinders (solid black line) start to deviate. For $\tilde{\phi}_s \geq 0.30$, the gas-solid drag coefficients with cylinders are smaller than the corresponding values without cylinders. Dense particle clusters generally
form in the vicinity of the cylinders, often on top, which allows the gas to channel through the domain with greater ease, resulting in a decrease in $\bar{\beta}_{\text{g.s.y}}^*$ with immersed cylinders. At the random dense-packing limit of $\phi_{s,\text{max}} = 0.64$, inhomogeneities disappear as there is no additional volume available to form bubbles or clusters. Thus, the microscopic drag coefficient $\bar{\beta}_{\text{g.s.micro}}^*$ and the filtered drag coefficient $\bar{\beta}_{\text{g.s.y}}^*$ (with or without cylinders) converge at $\phi_{s,\text{max}}$.

Although the inhomogeneities visually appear to increase for larger $D_{\text{cyl}}^*$ and smaller $a_{\text{cyl}}^*$ values, no consistent quantitative trend with either quantity could be identified. Figure 15(b) magnifies the data points in the range $0.50 \leq \tilde{\phi}_s \leq 0.60$, with scatter bars representing plus/minus one standard deviation of the $\bar{\beta}_{\text{g.s.y}}^*$ measurements from several simulations (with different pressure gradients). To improve the clarity of scatter bars, data points are shown with a slight horizontal offset. The standard deviations are too large to confidently establish the dependence of $\bar{\beta}_{\text{g.s.y}}^*$ on cylinder diameter and spacing. Moreover, any noteworthy variations in $\bar{\beta}_{\text{g.s.y}}^*$ are only seen for $\tilde{\phi}_s \geq 0.60$—a very high solid fraction rarely encountered during typical fluidized bed operations. Therefore, the influence of cylinder diameter and spacing is not incorporated in the current model. Large scatter bars notwithstanding, the reduction in filtered gas-solid drag due to cylinders still is captured.

The fitted value for $\bar{\beta}_{\text{g.s.y}}^*$ is expressed as:

$$\bar{\beta}_{\text{g.s.y}}^* = \bar{\beta}_{\text{g.s.micro}}^*(1 - h_{2D}), \quad (31)$$

where $\bar{\beta}_{\text{g.s.micro}}^*$ is the previously-defined microscopic drag coefficient. The function $h_{2D}$ in Eq. 31 represents the filtered correction to the microscopic drag model for 2D simulations. As the cylinders do not affect $\bar{\beta}_{\text{g.s.y}}^*$ at smaller solid fractions, $h_{2D}$ for $\tilde{\phi}_s < 0.30$ is taken from Igci and Sundaresan (2011a) without any changes (dotted line in Fig. 16). For $0.30 \leq \tilde{\phi}_s \leq 0.64$, simulation data with cylinders is used to fit $h_{2D}$. The
complete fitted function for $h_{2D}$ is given by:

$$h_{2D} = 1 - \frac{\tilde{\beta}_{g,s,y}}{\hat{\beta}_{s,\text{micro}}}$$

$$= \begin{cases} 
2.7 \phi_s^{0.234} & \phi_s < 0.0012 \\
-0.019 \phi_s^{-0.455} + 0.963 & 0.0012 \leq \phi_s < 0.014 \\
0.868 e^{-0.38 \phi_s} - 0.176 e^{-119.2 \phi_s} & 0.014 \leq \phi_s < 0.25 \\
-4.59 \times 10^{-5} e^{19.75 \phi_s} + 0.852 e^{-0.268 \phi_s} & 0.25 \leq \phi_s < 0.30 \\
(-0.4341 \phi_s + 0.8998) \left[1 - e^{42.68(\phi_s - \phi_{s,\text{max}})}\right] & 0.30 \leq \phi_s \leq 0.64 
\end{cases}$$

(32)

where the modification for $\phi_s \geq 0.30$ is shown by the solid black line in Fig. 16. The value of $h_{2D}$ at $\phi_{s,\text{max}} = 0.64$ goes to zero because clustering inhomogeneities disappear at the packing limit. As detailed in Sec. 5, the effect of filter length does not need to be included in the model, and the filtered gas-solid drag correction is considered valid at the large filter limit ($\Delta_\text{filter} \to \infty$).

The gas-solid slip velocities in the horizontal direction usually are very small, even for cases with superimposed macroscopic horizontal flows. Typically, $\tilde{u}_g^* - \tilde{u}_s^*$ values are an order of magnitude smaller than $\tilde{v}_g^* - \tilde{v}_s^*$. Larger slip velocities arise due to gravity, which is absent along the horizontal direction. Consequently, the horizontal gas-particle drag force is not significant compared to the vertical drag force. Milioli et al. (2013) report that the horizontal measurements are more noisy, and they resort to computing the vertical and horizontal drag coefficients (without cylinders) from vertical flow data. Following their approach, it is suggested that an isotropic gas-solid drag coefficient be used, i.e.:

$$\tilde{\beta}_{g,s,x}^* = \tilde{\beta}_{g,s,y}^*$$

(33)

where $\tilde{\beta}_{g,s,x}^*$ is the filtered horizontal gas-solid drag coefficient with immersed cylinder arrays. The filtered horizontal gas-solid drag is then computed by $\tilde{f}_{g,s,x}^* = \tilde{\beta}_{g,s,x}^* (\tilde{u}_g^* - \tilde{u}_s^*)$.

9 Summary

Sub-grid drag models are developed for cylinder arrays immersed in a gas-particle fluidized bed. Hundreds of periodic 2D simulation measurements are analyzed to construct these filtered models, expressed as functions
of the cylinder diameter, cylinder spacing, solid fraction, and gas/solids velocities. All of the results are made dimensionless using preexisting scaling relationships developed by Igci et al. (2008). These sub-grid models are to be implemented in coarse-grid simulations where, in addition to the gas and solid phases, the cylinder array is represented by an effective stationary porous medium. A set of filtered conservation equations and constitutive relationships to be used in coarse-grid three-phase CFD simulations is provided in the Appendix.

The filtered model for cylinder-suspension vertical drag is comprised of a kinetic term, proportional to the square of solids velocity, and a buoyancy-like contribution. Anisotropy of the kinetic drag contribution is also discovered, indicating that horizontal drag coefficients are smaller.

The presence of cylinders also affects particle clustering behavior, which alters the filtered gas-particle drag. At larger solid fractions, cylinders enhance gas-solid segregation at the scale of cylinders’ diameter, lowering the interphase drag. To account for immersed cylinders, a revision to the filtered gas-particle drag model from Igci and Sundaresan (2011a) is presented.

The sub-grid drag models presented are derived from 2D simulations as it is computationally too expensive to run hundreds of 3D simulations. This work identifies the principal sub-grid corrections required and presents a methodology for constructing these corrections, which in future can be followed for extending the filtered drag models to 3D systems. Igci et al. (2008) have shown that the qualitative trends are preserved between filtered models derived from 2D and 3D simulations.

It is also recognized that the use of other boundary conditions such as partial-slip walls, instead of the non-slip walls used in this study, may affect the sub-grid drag corrections. The choice of constitutive relationships, especially for the particle-phase stress, may result in additional modifications to the filtered models. The influence of boundary conditions and constitutive relationships are fruitful topics for further investigation and may be addressed in a future study.

The motivation behind developing filtered models is to eventually replace computationally expensive, highly-resolved simulations of large fluidized beds with faster, coarse-grid CFD models that include these sub-grid corrections. At present, efforts are being directed toward verifying the fidelity of the sub-grid models. Macroscopic flow predictions obtained using filtered models will be compared against highly-resolved simu-
lations of large fluidized beds with hundreds of tubes. Validation of the filtered models against experimental measurements from sufficiently large beds will be addressed in future studies.

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Appendix: Implementation of filtered three-phase model equations

In a coarse-grid CFD simulation, the cylinder array will be viewed as a uniform, stationary porous medium. The conservation equations and constitutive models need to be expressed in terms of three phases: solids, gas, and the new porous medium representing the cylinder array. In this section, the filtered conservation and constitutive relationships are presented in terms of the three-phase model variables, denoted by the corresponding uppercase characters. The volume fraction of the immersed cylinder array phase $\Phi_c$ is defined as:

$$\Phi_c = \frac{V_{cyl}}{V_{tot}} = \frac{\pi D_{cyl}^2}{\frac{\pi}{4} a_{cyl}^2},$$

(34)
where $V_{cyl}$ is the volume occupied by cylinders and $V_{tot}$ is the total volume of all three phases, i.e., the net unit cell volume. Overbar and tilde accents over the symbols represent filtered volume and Favre averages, respectively, computed over the net cell volume. The filtered solids and gas fractions in a coarse-grid cell of the three-phase model are defined as:

$$\bar{\Phi}_s = \frac{1}{V_{tot}} \iiint_{cell} \phi_s \, dx \, dy \, dz = (1 - \bar{\Phi}_c) \bar{\phi}_s,$$  \hspace{1cm} (35)$$

$$\bar{\Phi}_g = \frac{1}{V_{tot}} \iiint_{cell} \phi_g \, dx \, dy \, dz = (1 - \bar{\Phi}_c) \bar{\phi}_g.$$  \hspace{1cm} (36)

The filtered velocities in the three-phase model are defined as:

$$\bar{\Phi}_s \bar{V}_s = \frac{1}{V_{tot}} \iiint_{cell} \phi_s v_s \, dx \, dy \, dz = (1 - \bar{\Phi}_c) \bar{\phi}_s \bar{v}_s,$$  \hspace{1cm} (37)$$

$$\bar{\Phi}_g \bar{V}_g = \frac{1}{V_{tot}} \iiint_{cell} \phi_g v_g \, dx \, dy \, dz = (1 - \bar{\Phi}_c) \bar{\phi}_g \bar{v}_g,$$  \hspace{1cm} (38)

which, when combined with Eqs. 35 and 36, yield $\bar{V}_s = \bar{v}_s$ and $\bar{V}_g = \bar{v}_g$. A coarse-grid simulation implementing the three-phase model will compute solutions for $\bar{\Phi}_s$, $\bar{\Phi}_g$, $\bar{V}_s$, and $\bar{V}_g$. Therefore, the conservation and constitutive equations must be recast in terms of these coarse-grid variables. Table 3 summarizes the filtered models developed in this work expressed as functions of the three-phase model variables.

Although Igci et al. (2008) and Igci and Sundaresan (2011a) have presented sub-grid models for filtered solid phase pressure and viscosity, such corrections are not developed in this study. The effect of the sub-grid stress corrections on macroscopic predictions is not expected to be significant (refer to Sec. 2 for a discussion of the work by Parmentier et al., 2012). The importance (or lack thereof) of filtered stress models remains to be addressed and will be studied in the future. Until then, it is proposed that the model for filtered solids stress presented in Igci and Sundaresan (2011a) be implemented in coarse grid simulations, together with the sub-grid corrections presented in Table 3. It should be noted that the Igci and Sundaresan (2011a) model for the filtered solid-phase stress is an approximation for systems with tubes, as the effects of immersed cylinders are not considered.

### Table 3: Summary of filtered three-fluid model equations.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\Phi}<em>s = \frac{1}{V</em>{tot}} \iiint_{cell} \phi_s , dx , dy , dz = (1 - \bar{\Phi}_c) \bar{\phi}_s$</td>
<td>Mass conservation for solids</td>
</tr>
<tr>
<td>$\bar{\Phi}<em>g = \frac{1}{V</em>{tot}} \iiint_{cell} \phi_g , dx , dy , dz = (1 - \bar{\Phi}_c) \bar{\phi}_g$</td>
<td>Mass conservation for gas</td>
</tr>
<tr>
<td>$\bar{\Phi}<em>s \bar{V}<em>s = \frac{1}{V</em>{tot}} \iiint</em>{cell} \phi_s v_s , dx , dy , dz = (1 - \bar{\Phi}_c) \bar{\phi}_s \bar{v}_s$</td>
<td>Momentum conservation for solids</td>
</tr>
<tr>
<td>$\bar{\Phi}<em>g \bar{V}<em>g = \frac{1}{V</em>{tot}} \iiint</em>{cell} \phi_g v_g , dx , dy , dz = (1 - \bar{\Phi}_c) \bar{\phi}_g \bar{v}_g$</td>
<td>Momentum conservation for gas</td>
</tr>
</tbody>
</table>

25
Table 3 (continued from previous page)

\[
\frac{\partial (\rho_s \Phi_s)}{\partial t} + \nabla \cdot (\rho_s \Phi_s \bar{V}_s) = 0 \quad (39)
\]

\[
\frac{\partial (\rho_g \Phi_g)}{\partial t} + \nabla \cdot (\rho_g \Phi_g \bar{V}_g) = 0 \quad (40)
\]

Momentum conservation

\[
\frac{\partial (\rho_s \Phi_s \bar{V}_s)}{\partial t} + \nabla \cdot (\rho_s \Phi_s \bar{V}_s \bar{V}_s) = -\nabla \cdot \Sigma_s - \Phi_s \nabla \cdot \bar{V}_s + \bar{F}_{g.s} + \bar{F}_{c.gs} + \rho_s \Phi_s \bar{g} \quad (41)
\]

\[
\frac{\partial (\rho_g \Phi_g \bar{V}_g)}{\partial t} + \nabla \cdot (\rho_g \Phi_g \bar{V}_g \bar{V}_g) = -\Phi_g \nabla \cdot \bar{V}_g - \bar{F}_{g.s} + \rho_g \Phi_g \bar{g} \quad (42)
\]

Cylinder-suspension drag

\[
\bar{F}_{c.gs} = (\rho_s \bar{g}) (1 - \Phi_c) \bar{f}_{c.gs}^*, \quad (43)
\]

where \(\bar{f}_{c.gs}^* = \left\{ \begin{array}{l} \bar{f}_{c.gs,x}^* \\
\bar{f}_{c.gs,y}^* \end{array} \right\} = \left\{ \begin{array}{l} -\bar{\beta}_{c.gs,x}^* \left( \frac{\bar{U}_s}{v_1} \right) \\
-\bar{\beta}_{c.gs,y}^* \left( \frac{\bar{V}_s}{v_1} \right) - \bar{\gamma}_{c.gs,y}^* \end{array} \right\}, \quad (44)
\]

\[
\bar{\beta}_{c.gs,x}^* = \frac{B_3 \left( \frac{\Phi_s}{1 - \Phi_s} \right)^2}{1 + B_4 \left( \frac{\Phi_s}{1 - \Phi_s} \right)^2}, \quad (45)
\]

\[
\bar{\beta}_{c.gs,y}^* = \frac{B_1 \left( \frac{\Phi_s}{1 - \Phi_s} \right)^2}{1 + B_2 \left( \frac{\Phi_s}{1 - \Phi_s} \right)^2}, \quad (46)
\]

\[
B_1 = \frac{0.1106 D_{cyl}^4 + 1.047 D_{cyl}^3 - 2.354 D_{cyl}^2 + 1.957 D_{cyl}}{a_{cyl}^2 - 22.74 a_{cyl} + 134.0}, \quad (47)
\]

\[
B_2 = \frac{-6.273 D_{cyl}^3 + 40.86 D_{cyl}^2}{a_{cyl}^2 - 26.86 a_{cyl} + 196.3}, \quad (48)
\]

\[
B_3 = 0.4543, \quad B_4 = 6.427, \quad (49)
\]

\[
\bar{\gamma}_{c.gs,y}^* = \frac{1}{2} \left( \frac{\bar{U}_s^2}{v_1} - \frac{\pi}{4} D_{cyl}^2 \right), \quad (50)
\]

\[
G_1 = 1.743, \quad G_2 = 2.077, \quad (51)
\]

Gas-solid drag

\[
\bar{F}_{g.s} = (\rho_s \bar{g}/v_1) (1 - \Phi_c) \bar{\beta}_{g.s}^* (\bar{V}_g - \bar{V}_s), \quad (44)
\]
where, $\bar{\beta}_{g,s} = \frac{1}{(\rho_s g/v_t)} \bar{\beta}_{g,s,\text{micro}}(1 - h_{2D})$,

$$
\bar{\beta}_{g,s,\text{micro}} = \frac{3}{4} \rho_g \left(\frac{1 - \frac{\Phi_s}{1 - \Phi_c}}{d_p}\right) \left|\hat{V}_g - \hat{V}_s\right| \left(1 - \frac{\Phi_s}{1 - \Phi_c}\right)^{-2.65},
$$

$$
C_D = \begin{cases} 
\frac{24}{Re_p}(1 + 0.15Re_p^{0.687}) & Re_p < 1000 \\
0.44 & Re_p \geq 1000 
\end{cases},
$$

$$
Re_p = \frac{(1 - \frac{\Phi_s}{1 - \Phi_c}) \rho_g d_p |\hat{V}_g - \hat{V}_s|}{\mu_g},
$$

$$
h_{2D} = \begin{cases} 
2.7 \left(\frac{\Phi_s}{1 - \Phi_c}\right)^{0.234} & \frac{\Phi_s}{1 - \Phi_c} < 0.0012 \\
-0.019 \left(\frac{\Phi_s}{1 - \Phi_c}\right)^{-0.455} + 0.963 & 0.0012 \leq \frac{\Phi_s}{1 - \Phi_c} < 0.014 \\
0.868 e^{-0.38\left(\frac{\Phi_s}{1 - \Phi_c}\right)} - 0.176 e^{-119.2\left(\frac{\Phi_s}{1 - \Phi_c}\right)} & 0.014 \leq \frac{\Phi_s}{1 - \Phi_c} < 0.25 \\
-4.59 \times 10^{-5} e^{19.75\left(\frac{\Phi_s}{1 - \Phi_c}\right)} + 0.852 e^{-0.268\left(\frac{\Phi_s}{1 - \Phi_c}\right)} & 0.25 \leq \frac{\Phi_s}{1 - \Phi_c} < 0.30 \\
\left[-0.4341 \left(\frac{\Phi_s}{1 - \Phi_c}\right) + 0.8998\right] \left[1 - e^{-42.68\left(\frac{\Phi_s}{1 - \Phi_c}\right)-0.64}\right] & 0.3 \leq \frac{\Phi_s}{1 - \Phi_c} \leq 0.64
\end{cases}.
$$

References


S. W. Kim, J. Y. Ahn, S. D. Kim, and D. Hyun Lee. Heat transfer and bubble characteristics in a fluidized


Figure 1: Schematic of the unit cell considered in this work. The dotted horizontal and vertical boundaries are periodic, with a superimposed pressure drop along the vertical direction. The control volume (CV) used for filtering is shaded gray.

Figure 2: Cut-cell representation of the cylinder boundaries showing the fluid cells simulated, and the blocked cells excluded from computations.
Figure 3: Simulation snapshots of the particle distributions for varying filtered solid fractions $\bar{\phi}_s$. These images are for $D_{\text{cyl}}^* = 4.15$, $a_{\text{cyl}}^* = 13.49$, and $\Delta P_{\text{per}} = 1.10(\bar{\phi}_s \rho_s + \bar{\phi}_g \rho_g)gL_{\text{per}}$.

Figure 4: Temporal variations of the filtered gas velocity $\bar{v}_g^*$ (top) and filtered solids velocity $\bar{v}_s^*$ (bottom) showing initial transient behavior and statistical steady state. These measurements are for $D_{\text{cyl}}^* = 4.15$, $a_{\text{cyl}}^* = 13.49$, $\bar{\phi}_s = 0.30$, and $\Delta P_{\text{per}} = 1.10(\bar{\phi}_s \rho_s + \bar{\phi}_g \rho_g)gL_{\text{per}}$. 

\[ t^* = \frac{t}{(v_t/g)} \]
Figure 5: Snapshots of solid volume fraction ($\phi_s$) distributions for varying dimensionless filter length $\Delta_{\text{filter}}^*$, which also is the edge-length $L_{\text{per}}$ of the periodic domain.

Figure 6: Effect of varying dimensionless filter size $\Delta_{\text{filter}}^*$ on time-averaged dimensionless values of (a) filtered gas velocity $\tilde{v}_g^*$ and filtered solids velocity $\tilde{v}_s^*$, and (b) filtered gas-solid drag $\tilde{f}_{gs,\text{y}}^*$ and filtered cylinder-suspension drag $\tilde{f}_{c,gs,\text{y}}^*$. Scatter bars represent plus/minus one standard deviation of all recorded observations. The effect of filter length is studied using $D_{\text{cyl}}^* = 4.15$, $a_{\text{cyl}}^* = 13.49$, $\bar{\phi}_s = 0.30$, $\Delta P_{\text{per}} = 1.10(\bar{\phi}_s \rho_s + \bar{\phi}_g \rho_g)gL_{\text{per}}$, and $\Delta_{\text{grid}}/d_p = 8.33$. 
Figure 7: Effect of varying grid size $\Delta_{\text{grid}}$, scaled by particle diameter $d_p$, on time-averaged values of (a) dimensionless filtered gas velocity $\tilde{v}_g^*$ and filtered solids velocity $\tilde{v}_s^*$, and (b) dimensionless filtered gas-solid drag $\tilde{f}_{g,s,y}^*$ and cylinder-suspension drag $\tilde{f}_{c,g,s,y}^*$. Scatter bars represent plus/minus one standard deviation of all recorded observations.

Figure 8: The dimensionless filtered cylinder-suspension drag $\tilde{f}_{c,g,s,y}^*$ plotted against filtered gas and solids velocities for (a) $\phi_s = 0.05$, (b) $\phi_s = 0.30$, and (c) $\phi_s = 0.50$. Solid lines represent the least-squares fit between $\tilde{v}_s^*$ and $\tilde{f}_{c,g,s,y}^*$ using Eq. 22. Horizontal and vertical scatter bars indicate plus/minus one standard deviations of all velocity and drag measurements, respectively. These figures are for the case $D_{\text{cyl}}^* = 4.15$ and $a_{\text{cyl}}^* = 13.49$.
Figure 9: Dimensionless filtered vertical cylinder-suspension drag parameters (a) $\bar{\bar{\beta}}_{c.g.s.,y}^*$ and (b) $\bar{\bar{\gamma}}_{c.g.s.,y}^*$, as functions of filtered solid fraction $\bar{\phi}_s$ for varying cylinder diameter ($D_{*\text{cyl}}$) and cylinder spacing ($a_{*\text{cyl}}$). Solid lines in (a) and (b) represent the best fit using Eqs. 23 and 25, respectively. Dotted lines in (b) represent the theoretical buoyancy given by Eq. 24.

Figure 10: Dimensionless filtered horizontal cylinder-suspension drag coefficient $\bar{\bar{\beta}}_{c.g.s.,x}^*$ as a function of filtered solid fraction $\bar{\phi}_s$. The solid line represents the best fit obtained using Eq. 27.
Figure 11: Anisotropy between the vertical and horizontal drag coefficients, quantified by the ratio $\bar{\beta}_{c,gs,y} / \bar{\beta}_{c,gs,x}$, as a function of $\bar{\phi}_s$. The symbols represent the ratios obtained from simulation measurements, and the solid line represents the ratio of fits (from Eqs. 23 and 27). The dotted segment for smaller $\bar{\phi}_s$ values indicates the region where simulation measurements (square symbols) are less certain.

Figure 12: Comparison of the predicted filtered vertical cylinder-suspension drag $\bar{f}_{c,gs,y}^*$ (obtained using Eq. 22) with the test set values measured from simulations with superimposed horizontal flow. The test set measurements, plotted along the vertical axis, were not used to derive Eq. 22. Scatter bars represent plus/minus one standard deviation of statistical steady-state measurements.
Figure 13: Clustering behavior of solids without cylinders (first column) and with immersed cylinder arrays (second and third columns). The top row shows snapshots for $\bar{\phi}_s = 0.30$, when cylinders have a negligible effect on clustering behavior. The bottom row corresponds to $\bar{\phi}_s = 0.50$, when the presence of cylinders enhances gas-particle segregation. The effects of segregation are visibly manifested as gas bubbles and dense particle clusters around the cylinders.

Figure 14: The dimensionless filtered gas-solid slip velocity ($\tilde{v}^*_g - \tilde{v}^*_s$) as a function of filtered solid fraction $\bar{\phi}_s$ for varying cylinder diameters and spacings. The slip velocity without immersed cylinders also is shown. The line represents the fit given by Eq. 29. The dotted segment for smaller $\bar{\phi}_s$ values indicates the region where simulation measurements (symbols) are less certain.
Figure 15: (a) The dimensionless filtered gas-solid drag coefficient $\bar{\beta}_{g,s,y}^*$ as a function of filtered solid fraction $\bar{\phi}_s$ for varying cylinder diameters and spacings. The microscopic (unfiltered) drag coefficient $\bar{\beta}_{g,s,micro}^*$ (Eq. 8) is shown for reference as the lightly dotted line (top). The heavier dotted line (middle) represents the filtered drag coefficient in a domain without cylinders (from Igci and Sundaresan, 2011a). The solid line (bottom) is the modified fit for $\bar{\beta}_{g,s,y}^*$ with immersed cylinders. (b) A close-up of data points in the range $0.50 \leq \bar{\phi}_s \leq 0.60$. Scatter bars represent plus/minus one standard deviation of $\bar{\beta}_{g,s,y}^*$ measurements.
Figure 16: Variation of $h_{2D}$ as a function of filtered solid fraction $\bar{\phi}_s$ for varying cylinder diameters and spacings. The dotted line represents the $h_{2D}$ function given by Igci and Sundaresan (2011), applicable to systems without cylinders. The solid line is the proposed modification to the $h_{2D}$ function, given by Eq. 32, which accounts for the differences due to immersed cylinders at larger solid fractions ($\bar{\phi}_s \geq 0.30$).