Perspective

Some outstanding questions in handling of cohesionless particles

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Abstract

This article describes the author’s perspective on important challenges in physical understanding and mathematical description of (a) the rheology of cohesionless granular materials and (b) phenomena observed in two important applications, namely discharge from bins and dense-phase pneumatic conveying. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

The focus of this article is on gaps in our understanding of mechanics of particulate systems, its translation into mathematical models and the solution of such models. The scope of particle technology is simply too broad to reduce the range of issues to a small number of key questions. Therefore, we limit our attention to a few problems involving cohesionless granular materials where, in spite of the large body of work that has already been done, we do not have satisfactory answers. In particular, we will touch upon some outstanding issues in modeling discharge from hoppers and bins, and dense-phase pneumatic conveying.

In these problems, we are interested in scaling up the performance from small units to large ones or even more ambitiously, designing large units from first principles. This invariably calls for continuum models (a biased view of the author), so the problem reduces to determination of accurate rheological models that recognize practical constraints on solving the continuum model equations. We will first discuss in Section 2 general issues on the rheology of granular materials. Sections 3 and 4 are devoted to the two applications mentioned above, namely bin discharge and pneumatic conveying, respectively.

2. Some general issues in granular flows

It is well known that granular materials manifest vastly different rheological behavior under rapid shear flow conditions and slow, quasi-static deformation. In rapid shear flow, the particles interact primarily through binary inelastic collisions. Here, both the shear and normal stresses manifest a strong typically second-order dependence on shear rate [4]. Constitutive relations for the rheological behavior in this regime have been obtained by adapting the kinetic theory of dense gases to spherical and slightly inelastic particles [29] and binary mixtures of particles [18].

Most industrial applications involve handling of particles having a distribution of size and shape. The effects of shape and size distribution on the rheological characteristics remain largely unexplored. Recent simulations by Hrenya et al. [14] suggest that simple methodologies to account for the effects of smooth size distribution on rheology should be possible. In applications such as gas- and liquid-fluidized beds, the interstitial fluid can and does have a significant impact on the fluctuating motion of the particles [23], and we are only beginning to tackle this problem.

Nonuniform distribution of particles that take the form of clusters and streamers occurs readily in dilute systems as a result of inelastic collisions (e.g., see Refs. [13,44]) and/or interaction with the interstitial fluid [43,49]. These structures are dynamic in nature, and they interact frequently with each other, producing a distribution of sizes

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and shapes ranging from a few to tens of hundreds of particle diameters. Even for the simple case of identical spherical particles with negligible effect of interstitial fluid, effective rheological models that represent an appropriate average over such nonuniform structures remain elusive. When interstitial fluid plays a major role, as in fluidized beds, the problem is further complicated [43].

At the other end of the spectrum in rheology, namely under quasi-static flow conditions where the particles in a dense assembly make enduring contact with their neighbors, the shear stress is essentially independent of shear rate [1]. A ubiquitous feature of flow in the quasi-static regime is the formation of slip plane(s) or shear layer(s) where much of the deformation occurs. On either side of a slip plane, the solid assembly can support a shear stress without undergoing any deformation. Bead pack calculations with frictional spheres [10] suggest that Lade–Duncan [25] form of yield surfaces captures the general features of granular plasticity. However, predictive models that connect particle-level information to continuum rheological description are needed.

In quasi-static flows, stresses are transmitted through formation of multiple force chains, which are constantly being formed and broken when flow occurs. Even in static systems, the load is not borne uniformly by these force chains and their strengths are also not uniform [5,26]. Although effective constitutive models represent averages over many realizations (i.e., snapshots of particle configurations), variability from one realization to another can be large. Thus, for example, the actual location of a slip plane where failure occurs may manifest appreciable variability among seemingly identical experiments.

When a transient network of slip planes is present (e.g., in slow downhill flow of a dense assembly of particles in a vertical duct), constitutive models should represent an average over the effects of interacting slip planes [41], and this still remains a challenge.

In the intermediate regime, a frictional-collisional rheological model that treats the rapid and quasi-static flow contributions additively has been used to analyze plane shear flow [19], chute flow [20], pneumatic transport [37] and hopper discharge [21]. However, this approach does not have a sound physical basis, and a unified theory that bridges the two extremes is needed.

The combined effects of frictional and collisional interactions are probably most significant in shear layers and, hence, a validated model for the structure of the shear layers is an important frontier. From a practical viewpoint, shear layers occur in many situations such as landslides, avalanches in sand dunes, rotary drums and rupture layers in bins. They also occur in cylindrical Couette cells, which are often used for rheological characterization of granular materials, so a theory which can predict the structure of shear layers would help in the interpretation of test results.

In addition to a rheological model, continuum equations for granular flow (or statics) require appropriate boundary conditions. A substantial amount of work has been done on boundary conditions describing collisional interaction of spherical particles with frictional or bumpy walls [16,17,19,27,38]. At the other extreme, where an assembly of granular material slides gently on a solid surface, the classical Mohr–Coulomb frictional relation between shear and normal stresses with a constant angle of wall friction is commonly employed. However, measurements of wall friction angles in bins show large fluctuations [3].

In the intermediate regime, some particles interact with the bounding walls via collisions, some roll on the surface, and the remainder slide along the wall. We do not have validated boundary conditions for this regime, which is important in applications such as chute flow [20,28].

In sum, some of the challenges on the rheological behavior of granular materials and fluid-solid mixtures are as follows.

1. Develop rheological models in the rapid flow regime that account for the effects of particle size distribution.

2. Develop reliable constitutive models for quasi-static granular flow, which connect grain-level variables to locally averaged quantities. For example, how does surface roughness (as quantified by varying level of friction coefficient) or small departure from spherical shape or particle size distribution affect the packing density of particles, the consolidation behavior and the deformation characteristics? It should be possible to develop via computational experiments [10] constitutive models that allow us to go from grain-level details to averaged quantities in well-defined, idealized scenarios.

3. Develop and validate a model for shear layers.

4. Develop a unified theory for rheology that applies across all regimes.

5. Develop and validate wall boundary conditions, particularly in the intermediate regime, for use with continuum models.

6. Develop coarsened constitutive models (for use in simulation of large-scale processes) that represent averages over clusters in the dilute regime and transient network of shear planes in the dense regime.

3. Storage and discharge of granular materials in hoppers and bins

The seemingly simple problems of predicting (a) stresses that develop upon storing granular material in a hopper or a bin, and (b) the stresses and discharge rate when this material is allowed to flow through a hole at the bottom, continue to remain a challenge [32,33]. Depending on the hopper material and angle, and the properties of the granular material, sustained discharge can take the form of a mass flow or a funnel flow. In bins equipped with a conical hopper attached to the bottom of a cylindrical vessel, mass-flow, mixed flow where the stagnant zones do not extend all the way till the free surface or pipe flow where the stagnant zone extends up to the free surface and...
the material discharges through a narrow central pipe, may be obtained \[33,47\]. In the case of flat-bottomed bins, mass flow is not possible.

Being able to predict the flow pattern that is likely to occur during discharge, given the geometry and the material properties, is essential for successful design of hoppers and bins. In the context of bins, where large switch stresses can have a destructive influence, one would like to be able to predict the maximum and minimum stresses at any location during filling and discharge \[3,30,33,39,47,48\]. These, however, remain elusive (e.g., see Refs. \[33,39\]).

It is known that in some systems the method used to fill the bin affects the discharge behavior. For example, distributed filling of barley in flat bottomed bins gave pipe flow while eccentric stream filling resulted in mixed flow \[34\]. In this instance, the distributed filling produced a more homogeneous and denser packing. Askegaard et al. \[2\] observed that a small change in humidity from 15% to 12% changed the flow pattern during discharge of barley grains from pipe flow to mixed flow. As remarked by Nielsen \[33\], one of the basic questions that remains unanswered is the physics controlling the shift between mixed and pipe flow. Consequently, it may be important to analyze the evolution of stresses and the packing density during the filling process in order to obtain the correct initial conditions for the discharge process. Furthermore, we must also be able to characterize how seemingly secondary variables (such as a small change in the humidity level) affect the constitutive relations governing the deformation of granular assembly. This remains a challenge \[33,39\].

In other instances, the stored material can undergo slow consolidation during storage and develop tensile strength (often quantified through an unconfined yield strength, e.g., see Rhodes, \[52\]) which depends on the local compressive stresses and time of storage, or can relax slowly through rearrangement \[33\]. Then, the initial condition and the constitutive model for deformation for the discharge process are considerably more complex. Such consolidation is an important consideration in the storage of viscoelastic particles in large silos in chemical industries (and transportation in railroad cars, etc.) (e.g., see Ref. \[40\]), as it can change the flow pattern during discharge dramatically.

The three common methods to analyze bin discharge are kinematic models, finite element simulation of continuum models and discrete element simulation of particle assemblies \[39\]. The kinematic models afford reasonable results for the stagnant zone in some cases (for example, see Ref. \[31\]), but unfortunately, they do not predict stresses field. Furthermore, the rate of discharge needs to be specified as input, so that the flow patterns inside the bin can be computed. Hence, this method is of limited value.

The finite element codes that are commonly being used cannot simulate filling process in a realistic manner \[39\] and, hence, the initial state prior to discharge has to be specified. With suitable constitutive models, they can be used to simulate the evolution of tensile strength resulting from consolidation during storage, provided a reasonable guess for the initial state of fill and stress fields can be specified. Challenges in this approach include exit boundary conditions \[22\] and reliable constitutive models.

In the discrete element method, one can, in principle, analyze the filling and consolidation events; however, it is simply impossible to simulate large units without using large, fictitious particles each of which represent thousands of actual particles.

In the case of material such as fly ash or fine powder used in chemical and pharmaceutical industries, the discharge process can be influenced strongly by the interstitial gas \[33,47\]. Therefore, the discharge must be viewed as a two-phase flow problem (and probably as an unsteady flow). While simple 1-D analyses of flow in hoppers have been performed to demonstrate that the gas phase can modify the flow characteristics (e.g., see Ref. \[42\]), multi-dimensional (and unsteady) analysis allowing for the effect of interstitial fluid remains to be performed.

One of the fundamental problems in simulations of discharge from flat-bottomed bins is to predict the shapes of the stagnant zones at the bottom of bins in a mixed flow mode and validate using experimental data. A satisfactory demonstration that simulations using continuum models are capable of capturing the stagnant zones accurately is, to the best of the author’s knowledge, unavailable.

The method of filling and/or small imperfections in the bins can introduce asymmetries in the stress and packing density, so a seemingly symmetrical bin can manifest asymmetric flow patterns during discharge \[33\]. The effect of small imperfections suggests that the imperfections directly or indirectly influence formation of rupture surfaces during discharge and, hence, the stress field. It is also conceivable that symmetric discharge may be unstable and give way to asymmetric discharge patterns, with the imperfection providing the necessary kick to cause the destabilization—this line of inquiry does not appear to have been pursued in the literature before. Such stability analysis calculations may also give useful insights into the flow problem.

It has been reported that the run-to-run variability in the stress field during bin discharge can be fairly large \[12\], suggesting that the manner in which the stresses relax is a stochastic process. Constitutive relations for continuum models usually represent averages of several different realizations of granular microstructures, so computations based on such models can at best give an average solution. Simulations allowing for constitutive relations having stochastic terms should be useful to explore if catastrophic stress excursions can occur.

One of the surprising outcomes of benchmark simulations described by Rotter et al. \[39\] is that finite element simulations of a test problem carried out by different
groups yielded vastly different results. This is due at least in part to differences in initial conditions—which may be interpreted as a manifestation of the effect of the filling method. The benchmark simulations also suggest that the predictions are sensitive to changes in the constitutive relations or parameter values. It is also conceivable that the results are sensitive to grid resolution.

Some of the challenges that go beyond what was described in Section 2) to be met before we can hope to come close to predicting silo behavior via continuum models can be summed up as follows:

1. Demonstrate and validate that the shape of the stationary layer in bin discharge can be captured satisfactorily by continuum analysis.
2. Explain what controls the transition between mixed flow and pipe flow.
3. Develop a method (perhaps, stochastic) to investigate run-to-run variability to analyze the possibility of stress excursions.
4. Investigate when symmetry may be broken through inherent instability.
5. Investigate how interstitial gas influences the dynamics of the discharge process and stress excursions.

It is interesting to mention an intriguing phenomenon known as silo music. The following experiment can be easily performed in a laboratory. Construct a bin by attaching an orifice plate to one end of a glass tube, and clamp the bin to a metal stand in a vertical position. Close the orifice with a piece of cotton and fill the tube with silica gel (50–300 μm). Upon removing the cotton, the silica gel begins to flow and a loud booming sound, referred to as ‘silo music’ by Tejchman and Gudehus [46], can be heard. The sound persists until the head of material (H) in the tube falls below a critical value, \( H_0 \), which depends on the orifice diameter. For \( H < H_0 \), the material continues to flow, but without the booming sound. A plausible mechanism of sound generation is as follows. The vibrations of the ambient air can be induced by vibration of the air column in the tube and/or the tube and supporting stand, which, in turn, may be generated by the axial vibration associated with stick-slip motion of the granular column, leading to the silo music.

It is known that avalanches on the sloping faces of sand dunes in the desert sometimes emit booming sounds. Nori et al. [35] noted that sound-producing sand, although known for centuries, remains one of nature’s more puzzling phenomena. It is not clear whether the same mechanism gives rise to booming dunes and silo music. Experiments with booming sand [11] suggest that the phenomenon is rather difficult to reproduce, and sensitive to the presence of impurities, etc. On the other hand, silo music appears to be a more robust phenomenon [15].

In large scale silos, silo music takes the form of an annoying noise associated with powerful vibrations which may damage the silo [45], and so a study of this phenomenon is of significant practical relevance.

4. Dense-phase pneumatic conveying

Fluidization and pneumatic conveying in vertical, horizontal or inclined ducts are common operations in particle handling. The former is widely encountered in chemical reactors, while the latter is frequently employed to transport particles from one location to another in a plant. In both instances, spatial and temporal nonuniformities readily occur. The details of these nonuniformities depend on the material properties of the particles and the fluid, the geometry of the vessel/duct and throughput. A variety of flow regimes has been observed and documented in the literature for fluidization as well as pneumatic conveying (e.g., see Refs. [6,8]).

Much progress has been made on formulating models for gas–particle flows, which treat the two phases as interpenetrating continua [9]. The general mathematical structure of these equations is the same for most dispersed two-phase flow problems, while the closure relations for the effective stresses and the interphase interaction depend on the type of phases involved. In gas–solid flows, the challenge is to specify in a reliable manner these closure relations in terms of gas and particle properties. This is precisely the same situation we had in the previous sections. However, we now have to be concerned with additional issues, such as the influence of gas phase turbulence on the fluctuating motion of particles (e.g., in dilute phase

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This was true for discrete element simulations as well, but we will focus on the finite element simulation of continuum model equations.
pneumatic transport) and the effect of structures such as clusters and bubbles on effective interphase interaction and stresses. The author has already described elsewhere some of the challenges in modeling the hydrodynamics of fluidized suspensions [43], so only the problem of dense phase pneumatic conveying in horizontal ducts is considered below. While dilute phase pneumatic conveying is more easily achieved, dense phase conveying is attractive from the point of view of minimizing particle damage.

Fine powders that fluidize easily and do not deaerate quickly (typically Geldart Type A powders) are generally easy to transport in dense-phase mode [50]. In contrast, larger particles deaerate easily and this leads to very different performance characteristics in the dense phase transport regime [6,50,51]. In particular, low-velocity slug flow can be achieved as one of the regimes (see Fig. 1), where particle damage can be minimized. Simplified analyses of pressure drop across and velocity of steady, fully developed slugs can be found in the literature [7,24,36]. One of the striking features seen in Fig. 1 is the regime of unstable transport obtained at intermediate conditions. In this regime, flow occurs with very large excursions in gas pressure and the stresses exerted on the duct by the moving slugs. The flow alternates between fast-moving unstable slugs and dilute phase transport with increasing levels of deposition. Which of these two modes dominates depends on the location of the operating condition inside the unstable regime—the unstable slugs dominate close to the dense-phase slugging boundary, while they are less frequent near the dilute-phase boundary. One of the important challenges ahead of us is to predict the two boundaries of the unstable region, for specified gas, particles and duct (i.e., diameter, length, configuration and material of construction). In order to do this, we need a better understanding of how the particles inside the slugs move and transmit stresses to the wall. Only then will we be able to identify the mechanism driving this instability and the stability boundaries.

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