Reputation and Accountability in Repeated Elections*

Rainer Schwabe†

Princeton University

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Abstract

This paper studies a model of infinitely repeated elections in which voters try to select competent politicians and provide them with incentives to exert costly effort. Markov perfect equilibria are proven to be incapable of incentivising effort. However, a class of equilibria satisfying a weaker version of Markov perfection, as well as weak renegotiation-proofness, is

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†PhD candidate - Department of Economics, Princeton University. Email: rschwabe@princeton.edu.

proposed and its existence proven. Voters use reputation-dependent performance cutoffs to make reelection decisions. Politicians' effort is decreasing in reputation, and expected effort is decreasing in tenure.

1. Introduction

The role of elections in democratic societies has been a subject of much debate among scholars of politics. The Downsian (Downs 1957) idea that the primary purpose of elections is to select a position in policy or ideological space dominated the early Political Economics literature. In the classic formulation, politicians are able to commit to campaign promises. More recently, the concept of elections as a way of disciplining politicians has gained considerable traction[1]. The main contribution of this strand of the literature was to conceptualize the relationship between the electorate and the incumbent politician as one between principal and agent in which the electorate uses the threat of not reelecting the incumbent to incentivize him to produce outcomes desireable to voters. Thus, these models abstract from ideological differences among voters and argue that valence issues are at the heart of the role of elections.

A later series of papers, perhaps most famously Rogoff and Sibert's (1988) study of the political budget cycle, maintain their focus on valence issues but argue that voters use elections as opportunities to select a "good type" of politician. Typically, this means competent politicians who are most able to produce the

[1] The seminal works in this tradition, Barro (1973) and Ferejohn (1986), have been cited 515 and 724 times respectively according to Google Scholar.

results desired by the voters. Indeed, voters in the United States claim in opinion polls that they understand elections more as an opportunities to select "good types" than as sanctioning devices (Fearon 1999). Thus, voters will use an incumbent's performance to learn about his competence and condition his reelection on his reputation.

These two perspectives on the role of elections are by no means incompatible. If competent politicians are desireable because they are more likely to deliver good results, then good performance should lead to high reputation and reelection while poor performance should lead to low reputation and the election of a challenger. It is clear, then, that if voters condition reelection on reputation they are sanctioning poor performance and rewarding good performance. Therefore, by using elections to select for competent politicians, voters are providing incentives for politicians to work in their interest.

In this paper we study a simple, infinite-horizon model of repeated elections with no term limits. There are two types of politician in our model: H (high) and L (low). H-types are competent in the sense that they may choose to exert effort in order to improve the expected utility of voters. L-types, on the other hand, are incompetent in that they do not have the ability to improve outcomes, or it is too costly for them to do so.

Because of the repeated nature of the elections, the set of equilibria is large and complex. In fact, any reelection strategy may be supported as part of a sequential equilibrium (see Proposition 1). Therefore, a main focus of this paper is on equilibrium selection. The incumbent's reputation is a payoff-relevant state variable in this model so that Markov Perfect equilibria are a natural place to start. In the first of our main results we establish that Markovian strategies are not complex enough to allow the voter to incentivize politicians to provide effort.

Our second main result establishes existence of a class of equilibria in which H-types are incentivized to exert positive effort. Voters condition their reelection strategy only on the two most recent periods' reputation, so that a weaker version of Markov Perfection holds. Incumbents are reelected only if their observed performance exceeds a performance cutoff which varies with initial reputation. We refer to this class of equilibria as ***equilibria in reputation-dependent performance cuto/s*** (RDC).

As in the seminal model of Ferejohn 1986, in RDC equilibria voters are left indifferent between reelecting an incumbent and electing an inexperienced challenger. However, in this paper the indifference arises endogenously from equilibrium strategy profiles rather than stemming from homogoneity among politicians. Additionally, we find that voter indifference has an important theoretical justification (see Claim 1 and Proposition 3): not only is any equilibrium implementing voter indifference weakly renegotiation-proof, but any equilibrium which is weakly renegotiation-proof, and in which the value of electing a new politician is history independent, must use voter indifference on a set of reputations of positive measure.

Several of the predictions RDC equilibria make about the career dynamics of politicians mirror those derived in models with term limits. For instance, politicians of high reputation exert lower effort. Also, in expectation, reputation is positively related to tenure so that, for a given politician, tenure is negatively related to performance (see Claim 2). In any given period, an incumbent will be reelected only if his reputation is above a given cutoff. In prior work, which we mention in the following subsection, these predictions were derived from a last period effect where high types were assumed to produce better results in spite of the lack of incentives. Here, they are consequences of a fully dynamic equilibrium strategy specification.

1.1. Related Literature

While the selection and incentivizing roles of elections have mostly been studied separately, there is a growing number of papers which take a unified approach.

Much of this work builds on work by Holmström (1999) on career concerns, with the relationship most directly apparent in Persson and Tabellini (2000, ch. 4.5). Notable contributions include Reed (1994), Banks and Sundaram (1998), Fearon (1999), Berganza (2000), Ashworth (2005), and Besley (2006, ch. 3.3). Each of these papers studies a model in which voters consider both the selection and incentivizing roles of elections and politicians face term limits. Additionally, several papers have applied models like these to the study of subjects such as constituency service (Ashworth and Bueno de Mesquita 2008), the incumbency advantage (Ashworth and Bueno de Mesquita 2006), and transfers to special interest groups (Coate and Morris 1995 and Lohmann 1998) to name a few.

In these models with term limits, voters are assumed to benefit from having a 'high' type in office even if the prospect of reelection is not available to the voter as an incentivising tool. This reflects the standard view that: "If honesty and competence are at stake, we should expect politician quality to be what political scientists call a valence issue - every citizen wants more of it" (Besley 2005). While this may be a natural assumption if 'high' types are assumed to be more honest than 'low' types (as in Coate and Morris 1995), it may be less appealing if politicians are thought to differ in their ability (as in Ashworth 2005, Banks and Sundaram 1998, and others). Indeed, one could argue that talented individuals could more effectively fleece the electorate. In this paper we take the view that, in the absence of electoral incentives, high types and low types will perform equally on average.

Banks and Sundaram (1993) study this issue in a fully dynamic framework with no term limits. However, they focus on stationary strategies in which voters do not condition on a politician's reputation. Therefore, there is no place in their analysis for career dynamics, and equilibrium behavior is quite similar to that predicted by models focusing only on accountability. Additionally, the equilibrium they propose is not weakly renegotiation-proof; it requires that players follow continuation strategies whose associated payoffs are strictly Pareto dominated by other continuation strategies played in equilibrium.

It is also worth noting the relation to work on repeated elections by Duggan (2000) and Banks and Duggan (2006). They model politicians as differing in their spatial policy preferences. Voters use the incentive of reelection to induce politicians to temper their policy choices while in office. However, because there is no uncertainty in the execution of policy and strategies are stationary, there is no evolution of beliefs about the incumbent's preferences beyond their first period in office. The model in this paper can be interpreted in the context of spatial competition by thinking of incompetent politicians as preference outliers and competent politicians as having preferences which differ from the voters' but are close enough to them that they may be willing to temper policy (rather than exert effort). While the two-type assumption seems somewhat contrived in the spatial setting, our results provide an interesting contrast to those of Banks and Duggan.

Finally, this paper is related to the literature on dynamic principal-agent interactions outside of the political sphere. The approach taken here differs from that taken in much that literature in two main dimensions. First, this paper focuses on the use of a retention rule rather than a compensation contract as an incentivizing mechanism. Second, in most of the literature on principal-agent relationships the principal is assumed to be a Stackelberg first-mover, leaving the agent only his reservation utility. In this paper, we look at Nash equilibria which admit the possibility that the gains from interaction may be shared. Indeed, in the RDC equilibria which we focus on, the agent reaps all of the benefits from increases in his reputation and enjoys utility strictly greater than his reservation value. For a more complete discussion of these issues and related papers we refer the reader to Banks and Sundaram 1998.

2. The Model

We study a discrete-time, infinite horizon model of a democratic society. In order to focus on the problems of selecting competent politicians and providing them with incentives to perform well, we abstract from ideological differences in the electorate. Instead, we model citizens as a single, infinitely-lived representative voter.

2.1. Preferences, Timing, and Information in the Stage Game

Each period (indexed by $t \in \{1, 2, ...\}$), the voter must select a politician to carry out a task. There is an infinite set P of potential politicians from which the voter may choose. Each politician is infinitely-lived and may serve for as many periods, or terms, in office as the voter asks him to.

After the voter elects a politician, the politician exerts effort $a \in \mathbb{R}_+ = [0, \infty)$. This effort impacts, but does not perfectly determine, results $r \in \mathbb{R}$ which we interpret as the voter's stage-game utility.

In order to consider differences in competence, we assume that politicians are one of two types: H or L. For H-types, effort is related to results via a conditional distribution function $F(r, a)$ with density $f(r, a)$. For ease of notation, we normalize units of effort so that effort exerted equals expected results:

$\bar{r}(a) = \int_{-\infty}^{\infty} r f(r,a) dr = a$.

H-types receive per-period utility $u(a)$ when in office, and 0 otherwise. Effort is costly so that u is weakly decreasing in a with[2] $u'(0) = 0$ and $u'(a) < 0 \; \forall a > 0$. We also assume that $u(a) > 0$ for all $a \in [0, \bar{a})$.

L-type politicians, on the other hand, are unable[3] to affect the distribution of r so that it is always $F(r, 0)$ when an L-type is in office. They receive a payoff $u_L > 0$ when in office and 0 otherwise, so that they are always willing to serve if elected. Because L-types are always willing to hold office but cannot make choices which influence payoffs in this game, we will focus on the behavior of H-types.

We make the following assumptions on the distribution $F(r, a)$.

$$\text{Full support: } f(r,a) > 0 \text{ for all } r \text{ and } a. \tag{A1.}$$

This assumption is standard in games with imperfect monitoring (Abreu, Pearce, and Stachetti 1989). It guarantees that effort levels can never be perfectly inferred by observing results.

[2] Alternatively, we could assume the derivative at zero to be negative but small without changing the results.

[3] Alternatively, effort is too costly for L-types for it to be worthwhile exerting.

$$f(r,a) \text{ is twice continuously differentiable in both arguments.} \tag{A2.}$$

Monotone Likelihood Ratio Property (MLRP): $\dfrac{f(x,a)}{f(x,a')} > \dfrac{f(y,a)}{f(y,a')}$ whenever $x > y$ and $a > a\prime$.

(A3.)

This assumption ensures that high results are more likely when high effort has been exerted (Milgrom 1981).

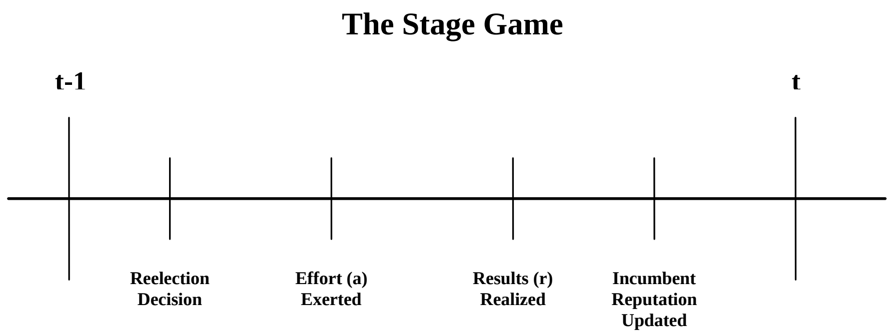
$$\text{Immutability: } f(r,a) = f(r+k, a+k) \text{ for any } k \in \mathbb{R}. \tag{A4.}$$

We make this assumption for analytical convenience. It guarantees that the shape of the distribution f is the same for all effort levels a, so that increases in effort simply shift the distribution to the right. It also implies that outcomes can be written as the sum of the effort choice and a zero-mean stochastic component (ε), a common modeling choice: $r = a + \varepsilon$.

A politician's type is the private information of the politician[4]. The voter

[4] All of the results presented in this paper would extend to a model in which information is

Figure 2.1: Timing of the Stage Game



will assign a probability μ_j of being an H-type to each politician j. We call μ_j politician j's ***reputation***. For ease of notation, when referring to the incumbent's reputation we drop the subscript j. Note that the expected stage-game payoff to the voter when an H-type incumbent exerts effort a is μa, so that reputation is payoff relevant.

The proportion of H-types among the set of potential politicians P is μ_0. Because new politicians are selected randomly from this set, μ_0 will also be the reputation of any politician at the beginning of his first term.

symmetric and politicians learn about their own type along with the voter.

2.2. Histories, Strategies, and the Repeated Game

At time t, the voter will have information about who has been in office and what rewards the voter has received in all previous periods, $1, 2, ..., t$. We call this information a t-history and label it h_t. Let H denote the set of all possible t-histories.

A ***reelection strategy*** is a measurable function $\sigma : H \rightarrow [0,1]$ denoting the probability with which the voter will reelect the incumbent, conditional on all currently available information.

Similarly, an ***e/ort strategy*** is a measurable function $a : H \rightarrow \mathbb{R}_+$ denoting the effort which a given politician will exert conditional on being in office and on all currently available information.

A belief function $\hat{\mu} : H \rightarrow [0,1]^\infty$ is a measurable function specifying, for each politician in P, the probability with which the voter believes that politician to be an H-type. For any politician j who has never held office, $\hat{\mu}_j = \mu_0$ regardless of the t-history h_t. For ease of notation, we use $\hat{\mu}$ to denote the incumbent's reputation and summarize the relevant information as the previous period's result and reputation (r, μ). In equilibrium, beliefs about a politician's type evolve according to Bayes rule:

$$\hat{\mu}(r,\mu) = \frac{\mu f(r,a)}{\mu f(r,a) + (1-\mu) f(r,0)}$$

Because our distribution f satisfies the MLRP, $\hat{\mu}(r,\mu)$ is strictly increasing in r.

It is important to note that different histories can lead to the same incumbent reputation. We can group these together to define a coarser partition of the set of all histories as follows: if $\hat{\mu}(h^1) = \hat{\mu}(h^2)$ for $h^1, h^2 \in H$ then $h^1, h^2 \subset \hat{h} \in \hat{H}$. We will refer to this as a Markovian partition of histories and we will use this definition of $\hat{H}$ in the following section to define Markovian strategies and other concepts.

A natural way to make a Markovian partition finer is to allow it to distinguish histories not only by current reputation but also by the previous period's. We will call the following partition Markov-2: if $\hat{\mu}(h^1_t) = \hat{\mu}(h^2_{t'})$ and $\hat{\mu}(h^1_{t-1}) = \hat{\mu}(h^2_{t'-1})$ then $h^1, h^2 \subset \mathring{h} \in \mathring{H}$.

Given a strategy profile (σ, a) and beliefs $\hat{\mu}$, the voter can compute his expected future payoffs at h_t. We denote this by $V(\sigma, a, \hat{\mu}; h_t)$. Letting $h_{t+1}(r)$ denote the t+1-history reached from h_t after a result r is observed, it may be defined recursively:

$$V(\sigma, a, \hat{\mu}; h_t) = \hat{\mu}(h_t)a(h_t) + \delta \int_{-\infty}^{\infty} f(r, a)V(\sigma, a, \hat{\mu}; h_{t+1}(r))dr$$

Where $\delta \in (0, 1)$ is a discount factor common to the voter and all politicians.

Similarly, we denote the value function of an incumbent H-type politician $Q(\sigma, a, \hat{\mu}; h_t)$. It may be defined recursively as:

$$Q(\sigma, a, \hat{\mu}; h_t) = u(a(h_t)) + \delta \int_{-\infty}^{\infty} f(r, a)\sigma(h_{t+1}(r))Q(\sigma, a, \hat{\mu}; h_{t+1}(r))dr$$

Definition 1. *A sequential equilibrium (Kreps and Wilson 1982) is a strategy profile* (σ^*, a^*) *and a belief function* $\hat{\mu}$ *such that:*

1. $V(\sigma^*, a^*, \hat{\mu}; h_t) \geq V(\sigma', a^*, \hat{\mu}; h_t)$ for all σ' and h_t.

2. $Q(\sigma^*, a^*, \hat{\mu}; h_t) \geq Q(\sigma^*, a', \hat{\mu}; h_t)$ for all a' and h_t.

3. $\hat{\mu}$ evolves according to Bayes rule[5] using the strategies (σ^*, a^*).

[5] The full support assumption A1 ensures that Bayes rule is always applicable since all histories are reached with positive probability.

Figure 2.2: Summary of important notation.

r	Voter's stage-game utility.
a	Politician's effort.
$u(a)$	Politician's stage game utility.
$f(r,a)$	pdf of r given a.
V	Voter's value function.
Q	Politician's value function.
σ	Voter's reelection strategy.
μ	Incumbent's reputation.

3. Equilibrium Selection

As in any infintely repeated game, we expect there to be a large set of sequential equilibria. In this section, we discuss the problem of the multiplicity of equilibria and some possibilities for narrowing our focus to those equilibria which are most appealing. We begin with the following result which starkly outlines the problem of multiplicity. Then, we proceed by describing several classes of equilibria of this model and using them to highlight several equilibrium selection criteria.

Proposition 1. *Any reelection strategy σ can be supported as part of a sequential equilibrium.*

To see that this is true, we first identify the equilibrium with the lowest payoffs for all players in the equilibrium set, which we call an equilibrium in ***grim***

strategies. Suppose H-type politicians always choose $a = 0$. Then, the voter is left indifferent among all politicians and may choose any reelection rule. In particular, it is a best response for him never to reelect a politician, regardless of his performance. This reelection strategy makes $a = 0$ also a best response.

Next, we note that this equilibrium may be used as part of other equilibria as a credible punishment to the voter for not following a prescribed reelection strategy. Because the voter's expected payoff can never be worse than 0, the following is an equilibrium for any reelection strategy σ: the voter plays σ on the equilibrium path while politicians play a best response to σ. If the voter ever deviates from σ, equilibrium play switches to grim strategies.

One may object to the equilibria above by arguing that it is implausible that all politicians in P will coordinate on playing grim strategies in the continuation game. Since the physical environment is identical each time a politician is elected to his first term, it seems natural to focus on equilibria in which strategies are the same every time the voter begins a fresh relationship with a politician. This, of course, implies that the value of the outside option for the politician is constant through all histories. In a sense, this is a stationarity condition which we will call ***challenger-stationarity***. Because it is sufficient for our purposes and a weaker condition, we define challenger-stationarity in terms of the value of electing an inexperienced politician.

Definition 2. *An equilibrium satisfies* ***challenger-stationarity*** *if the value of electing an inexperienced politician is history-independent.*

In a closely related paper, Banks and Sundaram (1993) describe an equilibrium of the repeated elections game which satisfies challenger-stationarity (following Banks and Sundaram, we call these ***simple equilibria***). All politicians are held to a single performance standard. When this performance standard is not met, the politician is not reelected. This is the case for politicians of any reputation, even though the expected rewards to the voter are increasing in the incumbent's reputation. This is enforced through the following trigger strategy: after a politician has missed his performance target once, he never expects to be reelected again and will therefore never again exert effort. However, these strategies require that the voter keep the personal history of a politician in mind when making his reelection decision. One might argue for strategies which allow for one politician of given reputation to stand in for another, i.e. an ***anonymity*** cirterion.

Definition 3. *An equilibrium satisfies* ***anonimity*** *if reelection strategies do not depend on candidate identity.*

A more serious criticism of simple equilibria, in our view, is that after a politician with high reputation misses a performance target, both the voter and the politician would benefit from agreeing to keep the politician in office and continue play as if the incumbent had not violated the voter's performance standard. Therefore, the punishment prescribed by the equilibrium is not credible. More formally, the equilibria are not ***Weakly Renegotiation-Proof*** (Farrell and Maskin 1989)[6]. There is a continuation equilibrium with associated payoffs which strictly Pareto dominate those specified as following the history in question.

Farrell and Maskin's definition of WRP equilibrium is as follows: an equilibrium strategy σ is WRP if there do not exist continuation equilibria σ^1 and σ^2 of σ such that σ^1 strictly Pareto dominates σ^2 (i.e. payoffs under σ^1 are strictly greater for both players than under σ^2). To adapt the definition of WRP to the current game, we must take into account that the politician's reputation is payoff relevant, so that continuation payoffs when the politician's reputation is μ may not be feasible when his reputation is $\mu' \neq \mu$. The following definition formalizes

[6] Weak renegotiation-proofness is a condition of ***internal consistency*** in that it makes comparisons between the continuation payoffs of a given equilibrium strategy profile. Competing notions of renegotiation-proofness, such as that advocated by Pearce (1987), call for ***external consistency*** so that comparisons are made across equilibria. In particular, Pearce argues that comparisons should be made among the the lowest continuation payoffs of equilibria. Because not reelecting politicians (giving them continuation payoff of zero) is the voter's only effective tool for providing incentives, this approach is unlikely to narrow the set of equilibria in this game.

this notion.

Definition 4. *A sequential equilibrium is Weakly Renegotiation-Proof (WRP) if, for any two histories $h^1, h^2 \in H$ leading to a reputation μ, i.e. $h^1, h^2 \subset \hat{h} \in \hat{H}$, $V(h^1) > V(h^2)$ implies $Q(h^1) \leq Q(h^2)$ and $Q(h^1) > Q(h^2)$ implies $V(h^1) \leq V(h^2)$.*

In particular, this equilibrium refinement rules out simple equilibria. In fact, the WRP condition is closely related to a voter indifference condition analogous to that exploited by Ferejohn (1986) and which plays a central role in our equilibria with reputation-dependent cutoffs. We describe this relationship in greater detail in Section 4. Trivial examples of weakly renegotiation-proof equilibria are easy to identify: equilibrium in grim strategies is WRP. The question now becomes: "Are there weakly renegotiation-proof equilibria in which the voter has positive expected payoffs?" Although Proposition 2 below makes for a discouraging start to our search, in Section 4 we answer in the affirmative.

Given that reputation is the only payoff relevant state variable, it is natural to look for Markov equilibria in which strategies depend only on reputation. In addition to the standard arguments for Markov Perfect equilibrium (Maskin and Tirole 2001), it is also important for us to address the possibility of Makovian reelection strategies because previous work on related models has tended to predict that voters will use a simple reputation cutoff as a reelection rule[7]. Additionally, related work on repeated elections by Duggan (2000) and Banks and Duggan (2006) has focused of Markovian equilibria.

Definition 5. *A sequential equilibrium is Markov Perfect if:*

- *Reelection strategies are a function only of reputation:* $\sigma : \hat{H} \to [0, 1]$.
- *Effort strategies are a function only of reputation:* $a : \hat{H} \to \mathbb{R}_+$.

Indeed, Markov equilibrium takes the idea that history can matter only through the state variable even further than WRP[8]. Once again, equilibrium in grim strategies provides a trivial example of a Markov Perfect equilibrium. However, the following result makes clear that the Markovian criterion is too strict to allow for the voter to effectively incentivise H-type politicians.

Proposition 2. *There is no Markov Perfect equlibrium with positive value for the voter* ($V > 0$)*.*

[7] See Reed (1994), Banks and Sundaram (1998), Fearon (1999), Berganza (2000), and Ashworth (2005). In these models with term limits, it is assumed that high types perform better than low types in the last term in which no incentives for effort can be provided. Therefore, incumbents are reelected if their expected type is higher than that of a replacement.

[8] See Farrell and Maskin 1989 for further discussion of the relation between WRP and Markov Equilibria.

A full proof is provided in the Appendix (Section 7.2). To develop some of the intuition behind the proof, suppose that politicians of all reputations provide effort of at least $\alpha > 0$ in equilibrium. If reelection strategies depend only on reputation, the politician's value function Q must be increasing in reputation in order to provide incentives for effort. As a politician's reputation nears 1, the change in his reputation for a fixed but wide set of outcomes r approaches 0. Therefore, the politician's value function Q must increase at least a fixed amount (itself dependent on α) in each of an infinity of ever smaller intervals. However, we know that Q is bounded above by the value of holding office forever while exerting 0 effort: $\frac{u(0)}{1-\delta}$. Therefore, providing incentives for effort at least α for all reputations is infeasible. Conversely, if politicians of reputation at least μ do not provide effort, it is not worthwhile for the voter to reelect them. This in turn, means that politicians should avoid ending up with a reputation higher than μ, and they can only do this by providing lower effort, leading to an unraveling of incentives for incumbents of all reputations.

If we persist in looking for equilibria in which positive effort is exerted while insisting that strategies depend on history in the simplest way possible, the natural next step is to allow for strategies to depend on both the politician's current reputation and his reputation the previous period. Such strategies are Markovian if we take (μ_{t-1}, μ_t) rather than μ_t as our state variable. We refer to these strategies as ***Markov-2***. In Section 4 we provide a thorough description of these equilibria and a proof of their existence. These equilibria are also WRP.

Definition 6. *A sequential equilibrium is Markov-2 Perfect if:*

- *Reelection strategies are a function only of the current and last period's reputation:* $\sigma : \mathring{H} \to [0, 1]$.
- *Effort strategies are a function only of the current and last period's reputation:* $a : \mathring{H} \to \mathbb{R}$.

The following table summarizes the discussion above by listing the equilibrium properties discussed above and which types of equilibria satisfy each of them. We include, in the first column, the class of equilibria featured in this paper and discussed in the following Section 4.

4. Equilibria in Reputation-Dependent Performance Cutoffs (RDC)

Because the strategies we will consider in this section depend only on reputation at the beginning of the term and current performance, we drop the notation noting

Figure 3.1: Classes of equilibria and their properties.

	RDC	Simple (Banks & Sundaram '93)	Grim Trigger	Markov
$V > 0$	**Y**	Y	Y	N
Weak Renegotiation-Proofness	**Y**	N	N	Y
Markov	**N**	N	N	Y
Markov-2	**Y**	N	N	Y
Challenger-Stationarity	**Y**	Y	N	Y
Anonymity	**Y**	N	Y	Y

the dependance of the voter's and the politicians' value functions V and Q on the entire history and strategy profile, and instead emphasize their dependence on reputation: $V(\mu)$ and $Q(\mu)$.

In order to find equilibria in which the voter provides incentives for H-types to provide positive effort but that are WRP and depend on history in the simplest way possible (in this case, are Markov-2), we look to the structure of the equilibria in the baseline models of political agency. In our view, this has the added virtue of providing some continuity in the modeling and understanding of electoral incentives. The seminal work of Ferejohn 1986 makes two important observations:

- Performance cutoffs are effective means of providing incentives to politicians.
- Voter indifference over incumbents and replacements can be exploited to

sustain equilibria with performance cutoffs.

In addition to the connecton to earlier models of political accountability, the condition of voter indifference between reelecting the incumbent or electing a challenger is connected in the current model to the concept weak renegotiation-proofness (WRP, Definition 4). Clearly, voter indifference implies WRP since continuation payoffs are the same for the voter after any history of play, ruling out Pareto improvements.

Claim 1. *Any equilibrium in which* $V(\mu) = V(\mu_0)$ *for all* μ *is weakly renegotiation-proof (WRP).*

The following Proposition goes some way toward establishing the reverse implication; i.e. that WRP implies voter indifference. Specifically, the indifference condition will hold for a set of reputations of positive measure, and that strategies outside of this set will be "uninteresting". In order to do so, we assume that the effort strategies of newly elected politicians do not depend on prior history (i.e. equilibria are challenger-stationary, see Definition 2). This seems natural in the current context where each time a politician is elected for the first time, the continuation game looks identical to the start of the game at time 0.

Proposition 3. *Any equilibrium satisfying weak renegotiation-proofness (WRP) and challenger-stationarity and providing positive value for the voter ($V(\mu_0) > 0$):*

- *There is a subset of reputation space of strictly positive measure $S \subset [0, 1]$ such that, for any $\mu \in S$, if $\hat{\mu}(h) = \mu$ then $V(h) = V(\mu_0)$.*
- *For any $\mu \in S^C = [0, 1]\backslash S$, if $h^1, h^2 \subset \hat{h}(\mu)$ then $\sigma(h^1) = \sigma(h^1) = 1$ or 0. That is, strategies in the complement of S are Markovian and degenerate.*

Proof. Consider any reputation μ such that we can find histories h^1 and h^2 satisfying $\hat{\mu}(h^1) = \hat{\mu}(h^2) = \mu$, $\sigma(h^1) = 1$ and $\sigma(h^2) = 0$ (or strategies are mixed but may lead to reelection after h^1 and dismissal after h^2). Then WRP implies that, because $Q(h^1) > Q(h^2)$, $V(h^1) \leq V(h^2)$. Also, because it is a best response to reelect after h^1, $V(h^1) \geq V(\mu_0)$. Because it is a best response not to reelect after h^2, $V(h^2) \leq V(\mu_0)$. From this we conclude that $V(h^1) = V(h^2) = V(\mu_0)$.

This leaves reputation levels at which incumbents are always reelected or always thrown out of office. However, any reelection strategy leading to this sort of behavior over almost all reputations is essentially identical to a Markov-Perfect reelection strategy. By Proposition 2 this contradicts the premise that the equilibrium in question provides positive value for the voter. ■

In order to extend the logic of voter indifference to the current setting, we must recognize the importance of reputation. If we are to preserve voter indifference over incumbents and replacements, we must use performance cutoffs which adjust to the incumbent's reputation. Otherwise, expected results will be increasing in reputation as in Banks and Sundaram 1989's simple equilibria.

In order to keep the voter indifferent between incumbents and replacements, it must be that $V(\mu) = \mu a(\mu) + \delta \int_{-\infty}^{\infty} f(r, a(\mu)) \left[\sigma(\hat{\mu}(r,\mu))V(\mu) + (1 - \sigma(\hat{\mu}(r,\mu)))V(\mu_0)\right] dr = \mu a(\mu) + \delta V$

so that $a(\mu) = \frac{V(1-\delta)}{\mu}$. Denoting $v = V(1-\delta)$ we write the identity for effort levels which keep the voter indifferent among politicians as:

$$a(\mu) = \frac{v}{\mu}$$

We refer to v as the ***value to the voter*** of an effort profile $a(\mu)$. Note that $a'(\mu) = -\frac{v}{\mu^2}$. Clearly, any equilibrium with positive value to the voter ($v > 0$) will involve a lowest reputation politician which will ever be elected, since $a(\mu) \to \infty$ as $\mu \to 0$.

Because effort strategies $a(\mu)$ keep the voter indifferent among reelection strategies, if there exists a performance cutoff function $r(\mu) : [0,1] \to \mathbb{R}$ which makes $a(\mu)$ a best response, this will be a sequential equilibrium.

Definition 7. *An equilibrium in reputation-dependent performance cutoffs (RDC) with value v is a sequential equilibrium in which:*

- *Politicians follow an effort strategy* $a(\mu_t) = \frac{v}{\mu_t}$.
- *The Voter follows a reelection strategy* $\sigma(\mu_{t-1}, \mu_t) = \begin{cases} 1 \text{ if } \mu_t \geq \hat{\mu}(r(\mu_t), \mu_{t-1}) \\ 0 \text{ otherwise} \end{cases}$

The following Theorem states the existence of equilibria in reputation-dependent cutoff strategies. In order to establish existence we make an additional assumption on the distribution of r, specifically that it is log-concave. This property is satisfied by many commonly used distributions, including the Normal distribution (Bagnoli and Bergstrom 2005).

$$f(r, a) \text{ is log-concave.} \tag{A5.}$$

Theorem 1. *There exists a class of equilibria in reputation-dependent performance cutoffs (RDC) in which voters use a reputation-dependent performance cutoff, are indifferent among politicians of all reputations, and receive strictly positive expected utility.*

The proof (in Section 7.1) proceeds as follows: let $Q(\mu)$ be any bounded and well-behaved candidate for the politician's value function. If we have chosen v

carefully, it will be obtainable under $Q(\mu)$ in an RDC equilibrium since $Q(\mu)$ is bounded below by $u(0)$. We then define an operator $T(Q)(\mu) = u(a(\mu)) + \delta \int_{r_Q(\mu)}^{\infty} f(r, a(\mu))Q(\hat{\mu}(r, \mu))dr$ where $r_Q(\mu)$ is a reputation-dependent performance cutoff implementing v. A fixed point of T will be a value function Q with associated cutoff function $r_Q(\mu)$ implementing an effort strategy $a(\mu_t) = \frac{v}{\mu_t}$. Because this effort strategy leaves the voter indifferent between reelecting the incumbent or not, the cutoff function describes a reelection strategy which is a best response. Therefore, once we check sufficient conditions for a fixed point of T, we have found an RDC equilibrium.

RDC equilibria satisfy several desireable properties. They satisfy anonymity and challenger-stationarity. Strategies are also independent of time and symmetric in that all politicians of a given reputation follow the same strategy. Among the class of equilibria in which H-types exert positive effort, RDC equilibria depend on the history of play in the simplest possible way, i.e. they are Markov-2. Finally, RDC equilibria are weakly renegotiation-proof.

5. Career Dynamics and Comparative Statics

A straight-forward implication of the RDC equilibria of Theorem 4 is that effort decreases with reputation. Additionally, politicians stay in office only if their reputation improves, or in the case of ARDC expected reputation rises with tenure. This implies a negative relationship between expected performance and tenure for a given politician (though not across all politicians). This is a prediction which has been emphasized by others, including Banks and Sundaram (1998) and Ashworth (2005), though their derivation relies on last-period effects. As Ashworth (2005) points out, the prediction fits well with the negative correlation between tenure and personal constituent services examined in Bruce, Ferejohn and Fiorina (1990).

Recent work by Galazzo, et. al. (2009) finds a negative relationship between tenure and attendance in the Italian legislature. Attendance may be interpreted as an observation of performance in this context if we reinterpret our model to fit Italy's parliamentary system. In this case, politicians are directly accountable to their party and not to the voters. We might imagine that parties face a similar retention problem to that face by voters in democracies with direct representation, and thus may use RDC strategies.

Claim 2. *For a given politician, expected performance is negatively related to tenure.*

Because of the importance of the voter's outside option in RDC equilibria, it should not be surprising that the best payoff achievable for voters in an RDC equilibrium is increasing in the average reputation of new politicians.

Claim 3. *The highest expected payoff to the voter in an RDC equilibrium is weakly increasig in the proportion of high types in* P (μ_0).

Proof. Because of voter indifference, we may use the same cutoffs and strategies associated with μ_0, leading to the same value function, with any $\mu_0' > \mu_0$. ■

One undesireable attribute of the equilibria used to prove Theorem 2 is that they use performance cutoffs which are above the expected performance of high types (i.e. $r(\mu) - a(\mu) > 0$), and therefore politicians are always reelected with probability strictly less than $\frac{1}{2}$. Additionally, because of the relatively low reelection probability, the politician's value function is lower than it would be in an equilibrium with higher reelection rates, and therefore the highest level of implementable effort would likely be higher in this alternative scenario.

Generally, we would expect similar equilibria using cutoffs below expected performance to exist and guarantee reelection rates strictly higher than $\frac{1}{2}$. However, moving performance cutoffs below expected performance allows for the possibility that you may be reelected when your reputation has decreased, and thus that a politician will be reelected even if it is infeasible for him to be incentivised to provide the required effort to keep indifference. Whether this takes place will depend on the slope of $r(\mu)$, which in turn depends on the shape of $Q(\mu)$, which is an endogenous object. This is the focus of current work by the author.

Conjecture 1. *There is an RDC equilibrium in which politicians are always re-elected with probability strictly greater than* $\frac{1}{2}$.

6. Conclusions

The aim of this paper has been to improve the general understanding of the dual role of elections: selecting competent politicians and incentivizing them to exert costly effort to the benefit of the electorate. In particular, we have focused on the potential interaction between a politician's reputation, the voter's willingness to replace him with a less experienced candidate, and the politician's performance. We have done so in the context of a simple model of repeated elections without term limits which does not assume that competence is desireable to the voter even in the absence of incentivizing mechanisms.

As in many infinitely repeated games, the problem of equilibrium selection has taken center-stage. However, attention paid to this issue has paid off in unexpected ways. We have shed light on the question of whether voters can effectively incentivize politicians by simply conditioning reelection on reputation. The answer is no (Proposition 2), at least in the benchmark model we study. We have uncovered an interesting relationship between weak renegotiation-proofness and the condition that the voter be left indifferent between reelecting the incumbent and electing an inexperienced replacement (Claim 1 and Proposition 3). This has given us fresh perspective on the seminal work in political agency (Ferejohn 1986). Finally, we have considered some of the virtues and limitations of the large set of equilibria in trigger strategies.

Our exploration of the equilibrium set and its refinements led us to generalize the equilibria of Ferejohn 1986 to a model with non-homogeneous politicians (RDC equilibria, Section 4). The use of voter indifference to support performance cutoffs which, in turn, allow the voter to incentivize effort from politicians is consistent with several intuitively appealing equilibrium refinements. Additionally, after establishing existence (Theorem 1), we go on to explore the predictions of the model for political careers. The results presented in Section 5 largely replicate those derived from simpler related models. Our hope is that these predictions now stand on firmer theoretical ground. The author is currently working to expand the set of testable predictions for career dynamics, and to use the current framework to study more specific topics.

7. Appendix

7.1. Existence of RDC Equilibria - proof of Theorem 1

The first derivative with respect to μ of the updating function is:

$$\frac{\partial \hat{\mu}(r,\mu)}{\partial \mu} = \hat{\mu}_2(r,\mu) = \frac{f(r,a(\mu))f(r,0) + \mu(1-\mu)a'(\mu)f_2(r,a(\mu))f(r,0)}{(\mu f(r,a(\mu)) + (1-\mu)f(r,0))^2}$$

It is useful to note that $\hat{\mu}_2(r,\mu) \to 1$ as $a(\mu) \to 0$. The second term in the numerator converges to zero since $f_2(r,a(\mu))f(r,0)$ is uniformly bounded above.

When facing a reputation-dependent performance cutoff, an H-type politician with reputation μ solves the problem:

$$\max_a \left\{ u(a) + \delta \int_{r(\mu)}^{\infty} f(r,a)Q(\hat{\mu}(r,\mu))dr \right\}$$

To implement performance v (or effort strategy $a(\mu) = \frac{v}{\mu}$) with a reputation-dependent cutoff $r(\mu)$ we must have:

$$u'(a(\mu)) + \delta \int_{r(\mu)}^{\infty} f_2(r,a(\mu))Q(\hat{\mu}(r,\mu))dr = 0$$

i.e., the first order condition of the politician's problem is satisfied at $a(\mu)$.

And this must be true at every reputation point μ so that the derivative of the FOC with respect to μ must be 0:

$u''(a(\mu))a'(\mu)-\delta r'(\mu)f_2(r(\mu),a(\mu))Q(\hat{\mu}(r(\mu),\mu))+\delta\int_{r(\mu)}^{\infty} f_2(r,a(\mu))Q'(\hat{\mu}(r,\mu))\hat{\mu}_2(r,\mu)+ f_{22}(r,a(\mu))a'(\mu)Q(\hat{\mu}(r,\mu))dr = 0$

Solving for $r'(\mu)$:

$$r'(\mu) = \frac{\frac{1}{\delta}u''(a(\mu))a'(\mu) + \int_{r(\mu)}^{\infty} f_2(r,a(\mu))Q'(\hat{\mu}(r,\mu))\hat{\mu}_2(r,\mu) + f_{22}(r,a(\mu))a'(\mu)Q(\hat{\mu}(r,\mu))dr}{f_2(r(\mu),a(\mu))Q(\hat{\mu}(r(\mu),\mu))} \tag{7.1}$$

The Fundamental Theorem of Differential Equations guarantees the existence of a function $r(\mu)$ satisfying the equation above as long as the first order condition is feasible and we can bound $r(\mu)$ away from the point where $f_2(r(\mu),a(\mu)) = 0$ (for symmetric distributions, this point is $a(\mu)$), since the RHS of the expression above is continuous and the domain of $r(\mu)$ is compact.

Before presenting a proof of existence of these equilibria, we select a feasible value for the voter: $v > 0$. For analytical convenience, we focus on cutoffs where $r(\mu) - a(\mu) > 0$ and $f_2(r(\mu),a(\mu)) > 0$.

A lower bound for the value of holding office is $\bar{Q} = u(0)$. Using this lower bound as a hypothetical constant value function, we have:

$$u'(a(\mu,v)) = \delta \int_{r(\mu)}^{\infty} f_2(r,a(\mu,v))\bar{Q}dr = \delta\bar{Q}f(r(\mu),a(\mu,v))$$

Clearly, this equality cannot hold for v large enough. However, as $v \to 0$, $a(\mu,v) \to 0$ and therefore $u'(a(\mu,v)) \to 0$. However, $\bar{Q} > 0$, so that the equation must hold for appropriate $r(\mu)$ for v low enough (but still strictly positive). Indeed, we can guarantee that a strictly positive v may be sustained as above even if we restrict attention to cutoffs satisfying $r(\mu) - a(\mu) > L$ for any given lower bound L.

Lemma 1 (Bounded Derivative). *For any continuously differentiable function Q with absolutely K-bounded first derivative, $\left|\frac{\partial T(Q)}{\partial \mu}\right| < K$ for any $\mu \in [\mu_0, 1]$.*

Proof. $\frac{\partial T(Q)}{\partial \mu} = \frac{\partial\left[u(a(\mu))+\delta\int_{r(\mu)}^{\infty} f(r,a(\mu))Q(\hat{\mu}(r,\mu))dr\right]}{\partial \mu} =$

$u'(a(\mu))a'(\mu)+\delta\int_{r(\mu)}^{\infty} f_2(r,a(\mu))a'(\mu)Q(\hat{\mu}(r,\mu))dr+\delta\int_{r(\mu)}^{\infty} f(r,a(\mu))Q'(\hat{\mu}(r,\mu))\hat{\mu}_2(r,\mu)dr-$
$\delta r'(\mu)f(r(\mu),a(\mu))Q(\hat{\mu}(r(\mu),\mu))$

The first two terms add up to zero by the politician's F.O.C. Substituting equation 7.1 into the fourth term:

$\delta\int_{r(\mu)}^{\infty} f(r,a(\mu))Q'(\hat{\mu}(r,\mu))\hat{\mu}_2(r,\mu)dr$
$-\frac{f(r(\mu),a(\mu))}{f_2(r(\mu),a(\mu))}\left(u''(a(\mu))a'(\mu)+\delta\int_{r(\mu)}^{\infty} f_2(r,a(\mu))Q'(\hat{\mu}(r,\mu))\hat{\mu}_2(r,\mu)+f_{22}(r,a(\mu))a'(\mu)Q(\hat{\mu}(r,\mu))dr\right)$

We first consider the terms which include Q'. Combining them gives:

$$\left|\int_{r(\mu)}^{\infty} \hat{\mu}_2(r,\mu) Q'(\hat{\mu}(r,\mu)) \left(f(r,a(\mu)) - \frac{f(r(\mu),a(\mu))}{f_2(r(\mu),a(\mu))} f_2(r,a(\mu)) \right) dr \right|$$
$$< K \left|\int_{r(\mu)}^{\infty} \hat{\mu}_2(r,\mu) \left(f(r,a(\mu)) - \frac{f(r(\mu),a(\mu))}{f_2(r(\mu),a(\mu))} f_2(r,a(\mu)) \right) dr \right|$$
$$< K \left| \frac{f(r(\mu),a(\mu))}{f_2(r(\mu),a(\mu))} \int_{r(\mu)}^{\infty} \hat{\mu}_2(r,\mu) f_2(r,a(\mu)) dr \right|$$

Where we use assumption A5. to derive both inequalities.

Because $\lim_{a\to 0} \hat{\mu}_2(r,\mu) = 1$ and using assumption A5. again we can say that the last expression is finite for any $r(\mu) > a(\mu)$ and low enough $a(\mu)$. Therefore, $\lim_{r(\mu)\to\infty} \frac{f(r(\mu),a(\mu))}{f_2(r(\mu),a(\mu))} \int_{r(\mu)}^{\infty} \hat{\mu}_2(r,\mu) f_2(r,a(\mu)) dr = 0$.

As argued in the text preceding the Lemma, since Q is bounded below, we may choose $r(\mu)$ as large as we like while still supporting positive effort. In particular, we may choose $r(\mu)$, and thus $a(\mu)$, so that the following inequality holds for all $\mu > \mu_0$:

$$\delta \int_{r(\mu)}^{\infty} f(r,a(\mu)) \hat{\mu}_2(r,\mu) dr - \frac{f(r(\mu),a(\mu))}{f_2(r(\mu),a(\mu))} \delta \int_{r(\mu)}^{\infty} f_2(r,a(\mu)) \hat{\mu}_2(r,\mu) dr < .9$$

Therefore, $\delta \int_{r(\mu)}^{\infty} \left[f(r,a(\mu)) Q'(\hat{\mu}(r,\mu)) - \frac{f(r(\mu),a(\mu))}{f_2(r(\mu),a(\mu))} f_2(r,a(\mu)) Q'(\hat{\mu}(r,\mu)) \right] \hat{\mu}_2(r,\mu) dr < .9K$.

The maximum value Q can take is the value of exerting minimum effort (0) and holding office forever: $\frac{u(0)}{1-\delta}$. Thus, because

$\left| \int_{r(\mu)}^{\infty} f_{22}(r,a(\mu)) dr \right| < B$ for some $B > 0$ we can conclude that

$\left| \int_{r(\mu)}^{\infty} f_{22}(r,a(\mu)) a'(\mu) Q(\hat{\mu}(r,\mu)) dr \right| < B \frac{v}{\mu_0^2} \frac{u(0)}{1-\delta}$ where v is determined by the choice of $r(\mu)$ made above.

Similarly, by assumption $|u''(a(\mu))| < \infty$. We may focus on a closed interval $a \in [0, \frac{v}{\mu_0}]$ so that the second derivative is uniformly bounded above:

$|u''(a(\mu))| < U$ for some $U > 0$.

Using these bounds, we have that the absolute value of the derivative above is bounded by:

$0.9K + \frac{f(r(\mu),a(\mu))}{f_2(r(\mu),a(\mu))} \frac{v}{\mu_0^2} \left(U + B\frac{u(0)}{1-\delta} \right) < K$

The first term is strictly less than K. The second term does not depend on K, so that choosing K high enough makes it strictly less than $0.1K$. ■

Definition 8. *Let $C([0,1])$ be the space of bounded, continuous functions f : $[0,1] \to \mathbb{R}$.*

Let $\hat{C} \subset C([0,1])$ be the restriction of this space to functions with K-bounded first derivative and codomain $[u(0), \frac{u(0)}{(1-\delta)}]$.

It is clear that $\hat{C}$ is nonempty, bounded, closed, and convex.

Definition 9. *The operator $T : \hat{C} \to \hat{C}$ is:*

$T(Q)(\mu) = u(a(\mu)) + \delta \int_{r_Q(\mu)}^{\infty} f(r, a(\mu))Q(\hat{\mu}(r, \mu))dr$

A fixed point of this operator will define a value function for the politician in a reputation-dependent cutoff equilibrium.

We must first verify that T maps $\hat{C}$ to $\hat{C}$.

That $T(Q)$ has a K-bounded derivative is verified in Lemma 1.

That $T(Q)$ is continuously differentiable in μ is immediate from the differentiability of f, Q, $a(\mu)$, and $r(\mu)$.

Next we verify the necessary conditions for the application of Schauder's fixed point theorem (Stokey and Lucas 1989, ch. 17.4):

The class of functions $T(\hat{C})$ is equicontinuous under the sup norm since the first derivative of all functions in the class are uniformly bounded (citation needed).

Lemma 2. *The operator T is continuous.*

1. **Proof.** Let $\{Q_i\}_{i\in\mathbb{N}} \subset \hat{C}$ be a sequence of functions converging to Q in the sup norm.

   Then, for any $\beta > 0$ $\exists j \in \mathbb{N}$ such that $\forall i > j$, $\|Q_i - Q\| < \beta$.

   $(T(Q_i) - T(Q))(\mu) = \delta \int_{r_Q(\mu)}^{\infty} f(r, a(\mu)) \left[Q_i(\hat{\mu}(r,\mu)) - Q(\hat{\mu}(r,\mu))\right] dr$

   $+\delta \int_{r_{Q_i}(\mu)}^{r_Q(\mu)} f(r, a(\mu)) Q_i(\hat{\mu}(r,\mu)) dr$

   if $r_{Q_i}(\mu) > r_Q(\mu)$. For the reverse case, an identical argument may be used.

   The first term converges to zero by definition of Q_i.

   The second term converges to zero because $r_{Q_i}(\mu) \to r_Q(\mu)$. To see this, consider the following equality derived from the politician's F.O.C.:

$\int_{r_Q(\mu)}^{\infty} f_2(r, a(\mu)) \left[Q_i(\hat{\mu}(r, \mu)) - Q(\hat{\mu}(r, \mu))\right] dr = \int_{r_{Q_i}(\mu)}^{r_Q(\mu)} f_2(r, a(\mu)) Q_i(\hat{\mu}(r, \mu)) dr$

Again, the term on the LHS converges to zero by convergence of Q_i. Hence the RHS must also converge to zero. However, because $r_{Q_i}(\mu) > a(\mu)$ and $Q_i(\hat{\mu}(r, \mu)) \geq u(0)$, the terms inside the integral are bounded away from zero. Therefore, it must be that $r_{Q_i}(\mu) \to r_Q(\mu)$.

We have now established that $\|T(Q_i) - T(Q)\| \to 0$ so that T is a continuous operator. ■

We may now apply Schauder's FPT to find a value function and a reputation-dependent cutoff function $r(\mu)$ implementing effort strategy $a(\mu, v)$.

This completes the proof of existence.

7.2. Impossibility of Markov Perfect Equilibria with positive effort - proof of Proposition 2

Consider first the case in which there is a lower bound $b > 0$ on the effort exerted by politicians with reputation in $[x, 1)$. Using the politician's FOC, we know that his value function must must satisfy

$$\delta \int_{-\infty}^{\infty} f_2(r, a(\mu)) \sigma(\hat{\mu}(r, \mu)) Q(\hat{\mu}(r, \mu)) dr \geq -u'(b) = B > 0$$

In what follows, for ease of exposition we write $\hat{Q}(\hat{\mu}(r,\mu))$ for $\sigma(\hat{\mu}(r,\mu))Q(\hat{\mu}(r,\mu))$.

Because $\int_0^\infty f_2(r,a(\mu))dr < \infty$, we can find a value $r^* \in \mathbb{R}_+$ such that

$\delta \int_{-r^*}^{r^*} f_2(r,a(\mu))\hat{Q}(\hat{\mu}(r,\mu))dr \geq \delta \int_{-\infty}^{\infty} f_2(r,a(\mu))\hat{Q}(\hat{\mu}(r,\mu))dr - \varepsilon$

for some fixed $\varepsilon \in (0, \frac{B}{2})$.

Suppose $\hat{Q}$ is weakly monotonic. If $\hat{Q}$ is weakly decreasing, the integrals above will be weakly negative and thus the F.O.C. will not be satisfied. Suppose $\hat{Q}$ is weakly increasing. By the monotone likelihood ratio property (A3.) we know that there is a unique point at which $f_2(\hat{r}(\mu), a(\mu)) = 0$ with the derivative being negative to the left and positive to the right of that point. Denote this point $\hat{r}(\mu)$. Then,

$\delta \int_{-r^*}^{r^*} f_2(r,a(\mu))\hat{Q}(\hat{\mu}(r,\mu))dr$

$\leq \delta \int_{\hat{r}(\mu)}^{r^*} f_2(r,a(\mu))\hat{Q}(\hat{\mu}(r^*,\mu))dr + \delta \int_{-r^*}^{\hat{r}(\mu)} f_2(r,a(\mu))\hat{Q}(\hat{\mu}(-r^*,\mu))dr$

$\leq \delta \left[\hat{Q}(\hat{\mu}(r^*,\mu)) - \hat{Q}(\hat{\mu}(-r^*,\mu)\right] k$

for $k = \int_{\hat{r}(\mu)}^{r^*} f_2(r,a(\mu))dr - \int_{-r^*}^{\hat{r}(\mu)} f_2(r,a(\mu))dr > 0$, for all μ.

Therefore, $\hat{Q}(\hat{\mu}(r^*,\mu)) - \hat{Q}(\hat{\mu}(-r^* + a(\mu),\mu) \geq \frac{B}{2\delta k} > 0$ for all μ.

Given μ and r^*, there is a μ' such that $\mu = \hat{\mu}(-r^*,\mu')$. Therefore, $\hat{Q}$ must increase by at least $\frac{B}{2\delta k}$ over $[\hat{\mu}(-r^*,\mu), \hat{\mu}(r^*,\mu')]$. Because this process can be repeated indefinitely, this implies that $\hat{Q}$ grows without bound, which is a contradiction. Therefore, there can be no Markov reelection strategy leading to a weakly monotonic $\hat{Q}$ over any interval $[x,1]$ which provides positive expected benefits to the voter.

We are left with the possibility of a $\hat{Q}$ which is non-monotonic over every interval of the form $[x,1]$. Suppose we have found such a $\hat{Q}$. Then,

$\delta \int_{-r^*}^{r^*} f_2(r,a(\mu))\hat{Q}(\hat{\mu}(r,\mu))dr \geq \frac{B}{2}$ for all μ.

By A4. we can rewrite $f_2(r,a(\mu))\hat{Q}(\hat{\mu}(r,\mu))$ as $f_2(r,0))\hat{Q}(\hat{\mu}(r+a(\mu),\mu))$. Choose $t>0$ and define $\hat{Q}^t(\hat{\mu}(r,\mu)) = \hat{Q}(\hat{\mu}(r+a(\hat{\mu}(t,\mu)),\hat{\mu}(t,\mu)))$ a function of μ and r. Then,

$\delta \int_{-r^*}^{r^*} f_2(r,a(\hat{\mu}(t,\mu)))\hat{Q}(\hat{\mu}(r,\hat{\mu}(t,\mu)))dr = \delta \int_{-r^*}^{r^*} f_2(r,0)\hat{Q}^t(\hat{\mu}(r,\mu))dr \geq \frac{B}{2}$

Therefore,

$\delta \int_{-r^*+a(\mu)}^{r^*} f_2(r,0)\frac{1}{\hat{\mu}(r^*,\mu)-\hat{\mu}(-r^*,\mu)} \int \hat{Q}^t(\hat{\mu}(r,\mu))dtdr \geq \frac{B}{2}$

So that the moving average $\frac{1}{\hat{\mu}(r^*,\mu)-\hat{\mu}(-r^*,\mu)} \int \hat{Q}^t(\hat{\mu}(r,\mu))dt$ is a candidate for our value function generating positive effort. If it is weakly monotonic, the first part of the proof leads to a contradiction. If it is not, then we note that the moving average is necessarily flatter than the original $\hat{Q}$, and that the limit of repeating the moving average procedure is a weakly monotonic (indeed linear) $\hat{Q}$. Formally, define $\hat{Q}_i$ to be a new candidate value function after i iterations of the moving average operation and let $\hat{Q}_\infty = \lim_{i\to\infty} \hat{Q}_i$. Then, for any interval $[a,b] \subset [\mu_0,1]$ and $\varepsilon > 0$, there exists an integer I such that for all $i' > I$, $\left\| \hat{Q}_i - \hat{Q}_\infty \right\|_\infty < \varepsilon$

over $[a, b]$. Therefore, $\delta \int_{-r^*}^{r^*} f_2(r, 0)\hat{Q}_{i'}(\hat{\mu}(r, \mu))dr \geq \frac{B}{2} - \eta(\varepsilon)$ for some $\eta(\varepsilon)$ which goes to zero as ε does. We are left with a contradiction by the arguments made above for weakly monotonic value functions.

Now, we consider the case where there is no lower bound on effort exerted. The following Lemmas provide constraints on what can happen in such a hypothetical equilibrium.

Lemma 3. *In any Markov equilibrium with positive value* $V(\mu_0) > 0$, *every interval of the form* $[\mu, 1]$ *must contain reputation points at which politicians are reelected with strictly positive probability.*

Proof. Suppose not. Let $\hat{r}(a)$ denote the outcome which would keep the politician's reputation constant:

$$\hat{r}(a) = \{r | \hat{\mu}(r, \mu) = \mu\}$$

Note that, using Assumption A3., $\hat{r}(a) < a$ (if r is normally distributed $\hat{r}(a) = \frac{a}{2}$).

Then consider the first order condition of a politician with the ***highest reputation which is reelected with positive probability*** μ:

$$u'(a(\mu)) + \delta \int_{-\infty}^{\hat{r}(a)} f_2(r, a(\mu)) Q(\hat{\mu}(r, \mu)) dr < 0 \text{ for any } a(\mu).$$

Because $f_2(r, a(\mu))$ is negative for all values below $a(\mu)$. Therefore, $a(\mu) = 0$ and μ is an absorbing state. Since we assumed $V(\mu_0) > 0$, it is not a best response for the voter to reelect a politician with reputation μ, contradicting the definition of μ. ■

Lemma 4. *Consider a Markov Perfect Equilibrium with positive value for the voter $V(\mu_0) > 0$. In every reputation interval of the form $[\mu, 1]$ there must be a subset of positive measure in which politicians exert effort above some fixed lower bound $b > 0$.*

Proof. Suppose not. Then, choose a lower bound $b < \frac{1}{2}V(\mu_0)$ and let $[\mu, 1]$ be an interval over which effort is bounded above by b almost everywhere. V is bounded above by the constant function $\bar{V} = \frac{\bar{a}}{1-\delta}$ where $u(\bar{a}) = 0$. Let k satisfy $\sum_{i=0}^{k} \delta^i b + \sum_{i=k+1}^{\infty} \delta^i \bar{V} < V(\mu_0)$. By Lemma 3, there must be reputations arbitrarily close to 1 which are reelected with positive probability. Because effort is bounded, we may choose a reputation (call it $\hat{\mu}$) which is reelected with positive probability and from which the probability of transitioning out of $[\mu, 1]$ in k periods or fewer (call it p) is arbitrarily small. In particular, if we choose $p < \frac{V(\mu_0)}{\delta \bar{V}}$, an upper bound on the value to the voter of having a politician with reputation $\hat{\mu}$ in office ($V(\hat{\mu})$) is:

$$V(\hat{\mu}) < (1-p)\left(\sum_{i=0}^{k}\delta^i b + \sum_{i=k+1}^{\infty}\delta^i \bar{V}\right) + p\delta\bar{V} < V(\mu_0)$$

If the politician is reelected in each of his first k terms. Note that the probability of transitioning to a point in $[\mu, 1]$ at which effort higher than b is exerted is zero because this may happens only on a subset of measure 0, and therefore this possibility does not affect the calculation of expected rewards.

If he does not survive k terms, then $V(\hat{\mu})$ is less than:

$$V(\hat{\mu}) < b + \delta V(\mu_0) < V(\mu_0)$$

Therefore, it is not a best response to reelect a politician when his reputation is $\hat{\mu}$, which contradicts the definition of $\hat{\mu}$. ■

Given Lemma4, if we have a weakly monotonic value function we need only to modify the arguments above as follows. Instead of moving to a reputation satisfying $\mu = \hat{\mu}(-r^* + a(\mu'), \mu')$ we move to one satisfying $a(\mu') > b$ and $\mu < \hat{\mu}(-r^* + a(\mu'), \mu')$. Once again, we conclude that $\hat{Q}$ must increase by at least a fixed amount $\frac{B}{2\delta k}$ infinitely many times, contradicting its boundedness.

To deal with non-monotonic candidate value functions $\hat{Q}$ we note that, given Lemma 4, repeated application of the moving average operation ensures that the value of all integrals $\delta \int_{-r^*(\mu)}^{r^*(\mu)} f_2(r, a(\mu))\hat{Q}_i(\hat{\mu}(r, \mu))dr$ will be positive. Because these are defined on a closed set $[\mu', 1]$, there exists a minimum value of these integrals. Now, we may apply the same arguments as above: $\lim_{i\to\infty} \hat{Q}_i$ is weakly monotonic, i.e. repeated application of the moving average operation leads to a $\hat{Q}$ which is approximately weakly monotonic and therefore to a contradiction of the boundedness of $\hat{Q}$.

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