Optimal Resource Allocation via Geometric Programming for OFDM Broadcast and Multiple Access Channels

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***Abstract*— For multi-user orthogonal frequency division multiplexing (OFDM) systems, efficient optimal rate and power allocation algorithms are presented via geometric programming (GP), a special form of convex optimization problem for which very efficient interior point methods exist. Both multiple access channel (MAC) and broadcast channel (BC) are considered and the following two resource allocation problems are of main interest: weighted sum-rate maximization (WSRmax) and weighted sum-power minimization (WSPmin). Utilizing degradedness of BC on each tone, WSRmax and WSPmin in the BC can be all formulated as GP. By using the duality relation between MAC and BC, it is shown that the above resource allocation problems in the MAC can be converted into GP problems as well. This GP perspective of multi-user OFDM resource allocation problems provides numerical efficiency as well as strong scalability for any additional constraints of GP form.**

I. Introduction

The architecture of many communication networks falls into one of two categories: multiple access channel (MAC) or broadcast channel (BC) [1]. Examples of the MAC and BC are the uplink and downlink of a wireless LAN network, respectively. In the uplink, a number of mobile terminals send independent information to the access point (AP), and in the downlink, the AP broadcasts messages, which are often independent, to each mobile terminal (MT). With dramatically increasing demand in high data rate services, orthogonal frequency division multiplexing (OFDM) has drawn much attention as a promising technique for the next generation wireless communication systems. With perfect channel side information (CSI) at both base station (BS) and MTs, as the number of tones goes to infinity, OFDM is shown to achieve the capacity of Gaussian BC and MAC with inter-symbol interference (ISI), or with frequency-selective fading.

To achieve the channel capacity, superposition coding and successive decoding at the BS can be utilized in downlink and uplink OFDM systems, respectively [1]. By using such techniques, OFDM systems can dynamically allocate communication resources like power and rate on each tone in order to satisfy various targets such as maximization of system throughput or minimization of total transmit power. With each user's target data rate fixed, power minimization reduces inter-cell interference levels in both uplink and downlink as well as extends the battery life of each MT in the uplink.

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Over the last decade, much progress has been made on resource allocation for scalar Gaussian MAC and BC with ISI, where each MT and the BS are equipped with a single antenna. In [2], Cheng and Verdu characterized the capacity region of Gaussian MAC with ISI, and showed that the optimal input power spectral densities can be viewed as a generalization of the single-user water-filling spectrum. However, the lack of efficient numerical algorithms triggered much research to solve resource allocation problems efficiently by utilizing the inherent structure of the Gaussian MAC. A breakthrough was made by Tse and Hanly [3], where polymatroid structure was used to characterize the capacity region of fading MAC, and marginal utility functions were introduced to develop algorithms that have strong greedy flavors. These results can be directly extended to Gaussian MAC and BC with ISI [4].

Recently, [5] proposed an efficient algorithm applicable to sum-rate maximization in Gaussian OFDM MAC by utilizing iterative water-filling (IWF) technique, which was first introduced for power control in interference channels [6]. The application of IWF has been further extended to sum-power minimization problem in Gaussian OFDM MAC by [7]. However, for general weighted sum-rate maximization or weighted sum-power minimization problems in Gaussian OFDM MAC and BC, finding numerical algorithms with lower complexity still remains non-trivial. Also, because of the increasing demand in multi-media services such as video and audio streaming, real-time and non real-time traffic often coexist in the network. Thus, the constraints of resource allocation problems become more complicated, which requires developing new algorithms.

This paper introduces yet another powerful tool, geometric programming (GP), into the family of numerical algorithms for various resource allocation problems in OFDM MAC and BC. GP is a special case of convex optimization for which very efficient interior point methods have been developed [8]. GP has a variety of applications in communication systems [9], which include the cross-layer resource allocation [10]. This paper primarily focuses on the following two resource allocation problems in OFDM MAC and BC: weighted sum-rate maximization (WSRmax) and weighted sum-power minimization (WSPmin). By using the "degradedness" of the BC on each tone, as well as duality relation between MAC and BC [11], this paper shows that all these resource allocation problems in the OFDM MAC and BC can be formulated by GP. This GP perspective of multi-user OFDM resource allocation problems provides numerical efficiency as well as strong scalability for any additional constraints of GP form.

Notation: Vectors are bold-faced. $\mathbb{R}^n$ denotes the set of real n-vectors and $\mathbb{R}^n_+$ denotes the set of nonnegative real n-vectors. The symbol $\succeq$ (and its strict form $\succ$) is used to denote the componentwise inequality between vectors: $\mathbf{x} \succeq \mathbf{y}$ means $x_i \geq y_i$, $i = 1, 2, \cdots, n$.

II. System Model and Problem Formulation

In this section, downlink and uplink OFDM system models are described as well as the WSRmax and WSPmin problems are mathematically formulated. This formulation considers a transmission system with K users and N tones where the BS and each user are equipped with a single antenna. It is assumed that the ISI is completely removed by exploiting OFDM techniques, i.e. the frequency response is flat within each tone. In the downlink case, total transmit power is constrained to P_{tot}, and in the uplink case, each user has individual power constraint P_i where i is the user index.

On user k's tone n, the channel gain is denoted by $H_k(n)$, and a zero-mean independent and identically distributed (i.i.d.) Gaussian noise with variance $\sigma_k^2(n)$ is added at the receiver part. For the uplink case, $\sigma_k(n)$ is replaced with $\sigma(n)$ since BS is the only receiver. The channel signal-to-noise ratio (SNR) for user k's tone n is defined as $g_k(n) = |H_k(n)|^2/\sigma_k^2(n)$, and let $r_k(n)$ and $p_k(n)$ denote rate and power allocation on user k's tone n. This paper assumes perfect CSI at both BS and each user, which enables BS to dynamically allocate power and rate on each tone according to channel conditions. Multiple users are allowed to share each tone, and the BS performs superposition coding in the downlink and successive decoding in the uplink. Fig. 1 summarizes OFDM BC and MAC models. Formulations of each resource allocation problem in OFDM BC and MAC are presented in the next two subsections.

A. Resource Allocation Problems for OFDM BC

In the downlink, the BS encodes multi-user messages using superposition coding with a proper encoding order. Also, each receiver performs successive decoding with a decoding order identical to the encoding order. It can be assumed that the ordering is the same on every tone, which is shown to be sufficient for achieving the overall capacity region [12]. Let $\pi(\cdot)$ denote the message encoding order at the BS where $\pi(i) < \pi(j)$ means that user i's message is encoded earlier than user j's message. With superposition coding, one user can remove the interference caused by other users' messages encoded earlier. Therefore, the rate for user k's tone n is represented as

$$r_k(n) = \frac{1}{2}\log_2\left(1 + \frac{p_k(n)g_k(n)}{1 + g_k(n)\sum_{i:\pi(i)>\pi(k)} p_i(n)}\right). \quad (1)$$

First, the WSRmax problem can be formulated as follows.

$$\begin{aligned} \text{maximize} \quad & \sum_{k=1}^{K} \mu_k \sum_{n=1}^{N} r_k(n) \\ \text{subject to} \quad & \sum_{k=1}^{K}\sum_{n=1}^{N} p_k(n) \leq P_{tot} \\ & p_k(n) \geq 0 \quad \forall k \;\text{ and }\; \forall n, \end{aligned} \quad (2)$$

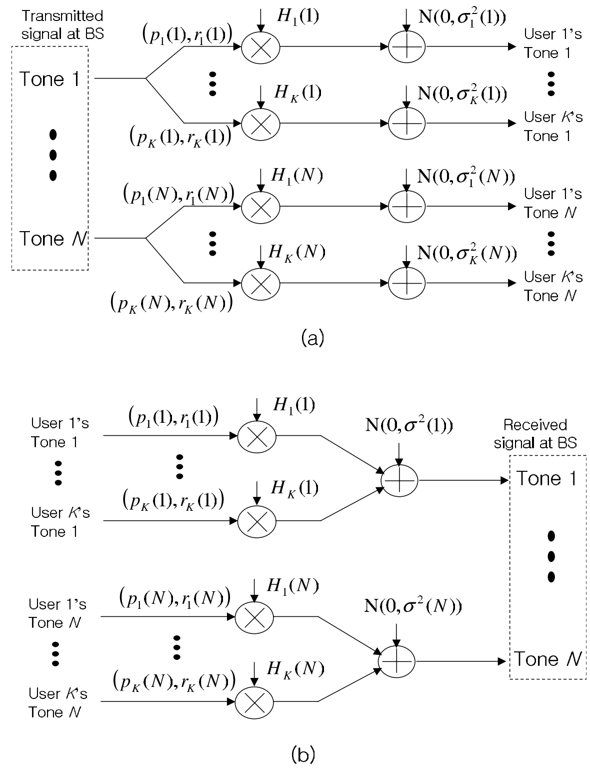

Fig. 1. (a) OFDM BC model. (b) OFDM MAC model.

where $\mu_k \geq 0$ is the weight on rate assigned to user k. Under the total power constraint, this problem's solution is the optimal power and rate allocation that maximizes the weighted sum-rate. The boundary surface of achievable rate region in BC or MAC can be traced by solving WSRmax for all possible weight vectors.

A dual version of WSRmax is WSPmin, which finds the rate and power allocation that minimizes the weighted sum-power with minimum rate constraints on each user. In the downlink, transmit power comes from a single source at the BS. Thus, sum-power minimization (SPmin) problem is of particular interest in BC, which is formulated as

$$\begin{aligned} \text{minimize} \quad & \sum_{k=1}^{K}\sum_{n=1}^{N} p_k(n) \\ \text{subject to} \quad & \sum_{n=1}^{N} r_k(n) \geq R_k \quad \forall k \\ & p_k(n) \geq 0 \quad \forall k \;\text{ and }\; \forall n, \end{aligned} \quad (3)$$

where R_k is user k's minimum rate constraint.

B. Resource Allocation Problems for OFDM MAC

In the uplink case, the BS performs successive decoding with interference cancellation, in which each user's message is successively decoded and subtracted from the received signal. As in the downlink, the same ordering can be assumed over the tones without losing achievable rates. Let $\pi(\cdot)$ denote the decoding order at the BS where $\pi(i) < \pi(j)$ means that user i's message is decoded earlier than user j's message. Then, the rate for user k's tone n is given by

$$r_k(n) = \frac{1}{2}\log_2\left(1 + \frac{p_k(n)g_k(n)}{1+\sum_{i:\pi(i)>\pi(k)} p_i(n)g_i(n)}\right). \quad (4)$$

Using this definition of $r_k(n)$, formulation of WSRmax in the MAC is the same as in the BC except power constraint. Total power constraint is considered in the BC, but each user has an individual power constraint in the MAC. Thus, total power constraint, $\sum_{k=1}^{K}\sum_{n=1}^{N} p_k(n) \le P_{tot}$ is replaced with individual power constraints, $\sum_{n=1}^{N} p_k(n) \le P_k$ for all k in WSRmax for the MAC.

Compared with SPmin in the BC, WSPmin in the MAC includes the weight on each user's power in the objective. Therefore, $\sum_{k=1}^{K}\sum_{n=1}^{N} p_k(n)$ in (3) is replaced with $\sum_{k=1}^{K}\lambda_k\sum_{n=1}^{N} p_k(n)$ where $\lambda_k \ge 0$ is the weight on power assigned to user k. Other than this change in the objective, all the constraints are identical in both cases.

III. Optimal resource allocation via Geometric Programming

In this section, WSRmax and WSPmin problems for downlink and uplink OFDM systems are formulated as geometric programming (GP), a convex optimization problem with efficient algorithms to obtain the globally optimal solution. GP uses monomial and posynomial functions. A monomial function has the form of $h(\mathbf{x}) = cx_1^{a_1}x_2^{a_2}\cdots x_n^{a_n}$, where $\mathbf{x} \succ 0$, $c \ge 0$ and $a_i \in \mathbb{R}$. A posynomial is a sum of monomials $f(\mathbf{x}) = \sum_k c_k x_1^{a_{1k}}x_2^{a_{2k}}\cdots x_n^{a_{nk}}$. Then, GP takes the following form,

$$\begin{array}{lrcl} \text{minimize} & f_0(\mathbf{x}) & & \\ \text{subject to} & f_i(\mathbf{x}) & \le & 1 \\ & h_j(\mathbf{x}) & = & 1, \end{array} \quad (5)$$

where f_0 and f_i are posynomials and h_j are monomials. Although this is not a convex optimization problem, with a change of variables: $y_i = \log x_i$ and $b_{ik} = \log c_{ik}$, we can convert it into a convex form as the following:

$$\begin{array}{ll} \text{minimize} & p_0(\mathbf{y}) = \log\sum_k \exp(\mathbf{a}_{0k}^T\mathbf{y} + b_{0k}) \\ \text{subject to} & p_i(\mathbf{y}) = \log\sum_k \exp(\mathbf{a}_{ik}^T\mathbf{y} + b_{ik}) \le 0 \\ & q_j(\mathbf{y}) = \mathbf{a}_j^T\mathbf{y} + b_j = 0 \end{array} \quad (6)$$

A variety of efficient interior point methods have been developed to quickly find the optimal solution of (6) [8].

GP formulation of OFDM resource allocation problems is closely related with the message encoding and decoding order. According to [12], the optimal ordering for various OFDM resource allocation problems is identical over all the tones, which implies that $K!$ possible orderings exist regardless of the number of tones. In the downlink, the channel at each tone forms a degraded broadcast channel where the largest rate region is achieved by encoding the user with higher channel SNR later [1]. The next subsection shows that after determining this tone-dependent ordering on every tone, WSRmax and SPmin in the BC can be converted into GP. Clearly, this ordering cannot perform worse than any other orderings in the above problems. Thus, the optimal rate and power allocation obtained by solving GP must conform to the optimal ordering that is one of $K!$ tone-independent orderings. By using the duality relation between BC and its dual MAC, WSRmax and WSPmin in the MAC can be also solved via GP, which will be shown in the following subsections.

A. GP Formulations for OFDM BC

In the downlink OFDM systems, the achievable rate region of tone n can be represented as

$$C_{BC}\left(\mathbf{m}(n), \sum_{k=1}^{K} p_k(n)\right) = \{r_{\pi_n(i)}(n) : r_{\pi_n(i)}(n) \le \frac{1}{2}\log\left(1 + \frac{p_{\pi_n(i)}(n)}{m_{\pi_n(i)}(n) + \sum_{j<i} p_{\pi_n(j)}(n)}\right), i = 1,\cdots,K\}, \quad (7)$$

where the effective noise variance of user k's tone n, $m_k(n) = 1/g_k(n)$, $\mathbf{m}(n) = [m_1(n),\cdots,m_K(n)]^T$, and $\pi_n(\cdot)$ is the permutation at tone n such that $m_{\pi_n(1)}(n) < m_{\pi_n(2)}(n) < \cdots < m_{\pi_n(K)}(n)$. That is, $\pi_n(\cdot)$ is in order of decreasing channel SNRs on tone n, which is reverse to the encoding order providing the largest rate region. When $\mathbf{r}(n) = [r_1(n),\cdots,r_K(n)]^T$ is on the boundary of the capacity region, solving $p_{\pi_n(i)}(n)$'s in terms of the rate vector $\mathbf{r}(n)$ yields the following equations.

$$\sum_{i=1}^{l} p_{\pi_n(i)}(n) = \sum_{i=1}^{l}\left(m_{\pi_n(i)}(n) - m_{\pi_n(i-1)}(n)\right) \times\exp\left(2\ln 2\sum_{j=i}^{l} r_{\pi_n(j)}(n)\right) - m_{\pi_n(l)}(n),\ l = 1,\cdots,K \quad (8)$$

where $m_{\pi_n(0)}(n) \equiv 0$. As shown in [4], (7) equals

$$C_{BC}\left(\mathbf{m}(n), \sum_{k=1}^{K} p_k(n)\right) = \{r_{\pi_n(i)}(n) : \sum_{i=1}^{K}\left(m_{\pi_n(i)}(n) - m_{\pi_n(i-1)}(n)\right)\exp\left(2\ln 2\sum_{j=i}^{K} r_{\pi_n(j)}(n)\right) \le \sum_{k=1}^{K} p_k(n) + m_{\pi_n(K)}(n),\ r_i(n) \ge 0,\ i = 1,\cdots,K\}. \quad (9)$$

From above relations, WSRmax problem given in (2) can be converted into the following GP.

$$\text{minimize} \qquad \log\exp\left(-\sum_{k=1}^{K}\mu_k\sum_{n=1}^{N} r_k(n)\right)$$

$$\text{subject to} \quad \log\exp\left(-r_k(n)\right) \le 0, \quad \forall\ k, n$$
$$\log \sum_{n=1}^{N}\sum_{k=1}^{K}\left(\frac{m_{\pi_n(k)}(n) - m_{\pi_n(k-1)}(n)}{P_{tot} + \sum_{l=1}^{N} m_{\pi_l(K)}(l)}\right) \times \exp\left(2\ln 2 \sum_{i=k}^{K} r_{\pi_n(i)}(n)\right) \le 0, \tag{10}$$

where the optimization variables are $r_k(n)$'s. Given the optimal rates, the optimal power allocation is derived from (8).

For SPmin in (3), the optimal ordering on each tone is also the one providing the largest rate region, which enables the following GP formulation of SPmin.

$$\begin{aligned} \text{minimize} \quad & \log \sum_{n=1}^{N}\sum_{k=1}^{K}\left(m_{\pi_n(k)}(n) - m_{\pi_n(k-1)}(n)\right) \\ & \times \exp\left(2\ln 2 \sum_{i=k}^{K} r_{\pi_n(i)}(n)\right) \\ \text{subject to} \quad & \log\exp\left(-r_k(n)\right) \le 0, \quad \forall\ k, n \\ & \log\left(\exp(R_k)\exp(-\sum_{n=1}^{N} r_k(n))\right) \le 0 \quad \forall\ k \end{aligned} \tag{11}$$

B. GP Formulations for OFDM MAC

By using duality relation between BC and MAC, the results obtained for the downlink can be extended for GP formulations of WSRmax and WSPmin in the uplink. Given a BC, its dual MAC has the channel SNRs and a total power constraint that are the same as in the original BC. [11] showed that any points in the BC capacity region can be also achieved in its dual MAC if the decoding order in the dual MAC is reverse to the encoding order in the BC. Since the total power for both channels is identical, the rate allocation minimizing sum-power in the MAC can be solved via GP in its dual BC by using (11). Note that from the above argument on ordering, the decoding order on tone n in the MAC is equal to the permutation $\pi_n(\cdot)$ defined in the previous subsection. Once the optimal rate allocation for SPmin is obtained by solving GP, the corresponding power allocation in the MAC can be determined from the following equation.

$$p_{\pi_n(k)}(n) = \frac{(2^{2r_{\pi_n(k)}(n)} - 1)\cdot 2^{2\sum_{i=k+1}^{K} r_{\pi_n(i)}(n)}}{g_{\pi_n(k)}(n)}, \ \forall\ k, n \tag{12}$$

where $r_{\pi_n(K+1)}(n) \equiv 0$ for all n. This equation is derived by applying the tone-dependent optimal ordering to (4).

In the uplink case, each user has different power source so that WSPmin problem is more useful than SPmin. With general non-equal weights, the tone-dependent optimal ordering can be different from that defined in SPmin. However, by utilizing channel scaling method, the optimal ordering for WSPmin in the MAC can be easily determined, and this problem becomes solvable via GP as well. Define the scaled power $p'_k(n) = \lambda_k p_k(n)$ where λ_k is the weight on user k's power. Then, close observation of (4) reveals that if the channel SNR is also scaled such that $g'_k(n) = g_k(n)/\lambda_k$, the mutual information in terms of scaled powers and channel SNRs remains the same as that before scaling [2]. Therefore, we can convert WSPmin in the MAC into SPmin in terms of $p'_k(n)$ and $g'_k(n)$, which is solved via GP.

GP formulation of WSRmax for the MAC is not straightforward compared to other problems so far. The optimal tone-independent ordering is automatically determined from the given weight vector, but this ordering doesn't guarantee the feasibility of GP formulations because of the individual power constraints. This paper shows that by employing Lagrange dual decomposition, WSRmax can be solved via iterative GP. First, convert WSRmax in the MAC to the minimization problem by multiplying -1 and taking the exponential on the objective. Lagrangian of this problem is defined over domain $\mathcal{D}$ as

$$\mathcal{L}(\{p_k(n)\},\{r_k(n)\},\boldsymbol{\lambda}) = \exp\left(-\sum_{k=1}^{K}\mu_k\sum_{n=1}^{N} r_k(n)\right) + \sum_{k=1}^{K}\lambda_k\left(\sum_{n=1}^{N} p_k(n) - P_k\right), \tag{13}$$

where $\boldsymbol{\lambda} \succeq 0$ and the domain $\mathcal{D}$ is defined as the set of all non-negative $p_k(n)$'s for all k and n. Then, the Largrange dual function is represented as

$$f(\boldsymbol{\lambda}) = \min_{\{p_k(n)\},\{r_k(n)\}\in\mathcal{D}} \mathcal{L}(\{p_k(n)\},\{r_k(n)\},\boldsymbol{\lambda}). \tag{14}$$

For a fixed $\boldsymbol{\lambda}$, the minimization problem in (14) can be formulated via GP as the following. First, define the scaled power $p'_k(n) = \lambda_k p_k(n)$, and the scaled channel SNR $g'_k(n) = g_k(n)/\lambda_k$. Then, in terms of $p'_k(n)$ and $g'_k(n)$, the minimization of Lagrangian in (13) is equivalent to maximizing the weighted sum-rate and minimizing the sum-power simultaneously. In the dual BC, the optimal encoding order on each tone for WSRmax and SPmin is equal to the order of increasing scaled channel SNR. From this reasoning, (14) can be converted into GP as follows.

$$\begin{aligned} \text{minimize} \quad & \log\Bigg(\exp\left(-\sum_{k=1}^{K}\mu_k\sum_{n=1}^{N} r_k(n)\right) \\ & + \sum_{n=1}^{N}\sum_{k=1}^{K}\left(m'_{\pi'_n(k)}(n) - m'_{\pi'_n(k-1)}(n)\right) \\ & \times \exp\left(2\ln 2\sum_{i=k}^{K} r_{\pi'_n(i)}(n)\right)\Bigg) \\ \text{subject to} \quad & \log\exp\left(-r_k(n)\right) \le 0, \quad \forall\ k, n \end{aligned} \tag{15}$$

where $m'_k(n) = 1/g'_k(n)$ and $\pi'_n(\cdot)$ is the permutation at tone n such that $m'_{\pi'_n(1)}(n) < m'_{\pi'_n(2)}(n) < \cdots < m'_{\pi'_n(K)}(n)$, or $\pi'_n(\cdot)$ is in order of decreasing scaled channel SNRs on tone n. With the optimal rate and power allocation obtained by this GP, $f(\boldsymbol{\lambda})$ can be derived from (13).

Finally, the dual optimal solution is obtained by maximizing $f(\boldsymbol{\lambda})$ over $\boldsymbol{\lambda} \succeq 0$. Since the original WSRmax in the MAC is a convex optimization problem, the duality gap is zero, which means that the dual optimal objective always equals the primal optimal objective [8]. This maximization can be

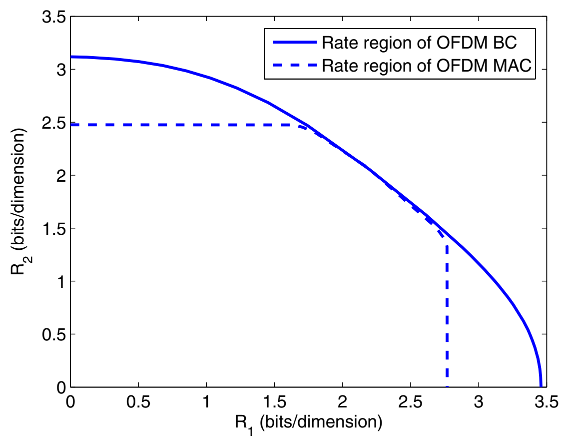

Fig. 2. Rate regions of OFDM BC and MAC ($N = 64$, $K = 2$, $P_{tot} = NK = 128$ in BC, $P_1 = P_2 = \frac{P_{tot}}{2} = 64$ in MAC. Each user's average channel SNR per tone = 10 dB)

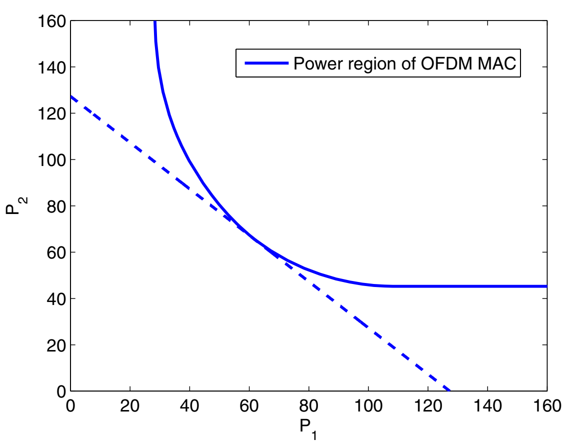

Fig. 3. Power region of OFDM MAC ($N = 64$, $K = 2$, $\mathbf{R} = [2.05 \;\; 2.19]^T$(bits/dim), and the channel SNRs are the same as in Fig. 1)

done by iterating the following steps until each user's power converges to individual power constraint: find $f(\boldsymbol{\lambda})$ via GP for a fixed $\boldsymbol{\lambda}$, and update $\boldsymbol{\lambda}$ to the direction of increasing $f(\boldsymbol{\lambda})$. The update of $\boldsymbol{\lambda}$ can be efficiently done by using the ellipsoid method, which is a type of sub-gradient search methods for non-differentiable functions. The ellipsoid method is shown to converge in $\mathcal{O}(n^2)$ iterations where n is the number of variables [8]. A sub-gradient for $f(\boldsymbol{\lambda})$ required in the ellipsoid method is $d_k = \sum_{n=1}^{N} p_k^*(n) - P_k$ for all k, where $\{p_k^*(n)\}$ optimizes the minimization problem in the definition of $f(\boldsymbol{\lambda})$.

IV. Numerical results and discussion

This section provides some simulation results generated using GP formulations for multi-user OFDM resource allocation problems. Fig. 2 presents two achievable rate regions of OFDM BC and MAC where $N = 64$, $K = 2$, $P_{tot} = NK = 128$ in BC, $P_1 = P_2 = \frac{P_{tot}}{2} = 64$ in MAC. Channel SNRs are assumed to be i.i.d. exponentially distributed with each tone's average SNR of 10 dB. The same set of channel SNRs are used for both BC and MAC. In Fig. 2, boundary points of rate regions are obtained by solving WSRmax via GP for all possible weight vectors. Since $P_1 + P_2 = P_{tot}$ as well as both OFDM BC and MAC have the same channel SNRs, duality relation holds between these two channels. Therefore, both rate regions always share at least one boundary point, which can be observed in Fig. 2.

Fig. 3 illustrates the power region for the same OFDM MAC as in Fig. 2, with the target rate vector of $\mathbf{R} = [2.05 \;\; 2.19]^T$ bits per dimension. Boundary points of power region are characterized by solving WSPmin via GP for all possible weight vectors. The given target rate vector is a boundary point shared by both OFDM BC and MAC in Fig. 2. Thus, as in Fig. 3, the minimum sum-power required to support these target rates is equal to the total power used in Fig. 2

V. Conclusion

In downlink and uplink OFDM systems, various resource allocation problems are formulated as geometric programming (GP), a special form of convex optimization that can be solved very efficiently. This paper presents GP formulations of two major problems: weighted sum-rate maximization and weighted sum-power minimization. Without violating GP structure, a variety of rate constraints can be added, which is essential for satisfying each user's various quality of service (QoS) requirement. In multi-user OFDM systems, GP emerges as a powerful tool that provides high numerical efficiency as well as strong scalability.

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