# Band-Preference Dynamic Spectrum Management in a DSL Environment

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*Abstract*— This paper introduces an algorithm for spectrum management for digital subscriber line (DSL) systems based on band preference. The proposed method influences the usage of spectrum through band preference factors that subtly modify the loading algorithm of DSL modems. Ad-hoc algorithms for computing such band preference factors are discussed. Simulation results in a practical ADSL environment show that the performance of the proposed method is better than that of Iterative Water-filling (IWF) [1] and is close to that of Optimal Spectrum Balancing (OSB) [2], even with a small number of control parameters.

#### I. INTRODUCTION

While DSL systems are widespread in today's data access networks, there still exist several barriers to achieving higher data rates. Chief among these barriers is Far-End Crosstalk (FEXT), which is the electro-magnetic interference from other same-direction users in the binder. In order to mitigate FEXT, current ADSL systems rely on a Static Spectrum Management (SSM) scheme to set power spectral density masks (PSDMASKs) for all the modems [3]. PSDMASKs limit each modem's transmitted power so that its FEXT into other users can be guaranteed to be lower than an acceptable level. However, this form of static spectrum management must be designed conservatively, and thus its overall performance is much lower than what can be achieved by Dynamic Spectrum Management (DSM).

Techniques for DSM may be stratified into levels of coordination [4]. In Level 0 DSM such as Iterative Water-filling (IWF), each user views other users' signals as noise and seeks to maximize its data rate in a fully distributed manner. In Level 1 DSM, a Spectrum Management Center (SMC) is able to send limited control commands to each modem such as rate or power back-off. In Level 2 DSM, an SMC coordinates the spectra of all modems centrally. In Level 3 DSM, complete coordination, or 'vectoring' occurs as all modems terminate at the same multiplexor, resulting in a MIMO channel [5].

This paper considers Level 2 DSM, for which much work has been undertaken. The "optimal spectrum balancing" (OSB) algorithm attempts to maximize the weighted sum rate of all users [2]. Several methods for reducing OSB's exponential complexity have been reported in [6], [7], and [8]. However, these methods require central controllers with significant control overheads that may be limiting when system parameters change rapidly. Band Preference Spectrum Management (BPSM) avoids these problems by instead relying on the inherent adaptive capability of each of the DSL modems. A central controller infrequently communicates to each modem which frequency bands are preferable (and conversely undesirable) for loading. Cognizant of these "band preferences", each DSL modem then autonomously adapts to any subsequent channel variations. Thus, BPSM significantly reduces control overhead while allowing a largely distributed implementation.

Other techniques for mitigating the control and overhead problem have been studied in [9]. [10] discusses a different form of BPSM based on setting PSDMASKs.

The novel approach of the proposed BPSM algorithm is to employ power scaling factors instead of a PSDMASK [10] or "reference line" [9]. These scaling factors are, heuristically speaking, penalties that are given to tones. During the bitloading process, a modem usually finds the tone that requires minimum energy to load a new bit. Under the proposed algorithm, the modem instead finds the tone that requires minimum *penalized* energy. If a central controller determines that it is desirable for some bands to load a smaller number of bits (*e.g.* to protect other users from FEXT), large scaling factors may be given to those bands. In this way, spectrum can be managed without direct control of each modem.

The remainder of the paper is organized as follows: Section II introduces the system model of multi-user DSL systems and formulates the problem. Section III details the proposed band-preference algorithm and provides the simulation results. Section IV concludes the paper.

#### **II. SYSTEM MODEL AND PROBLEM DEFINITION**

This paper considers a multi-user Discrete Multi-Tone based (DMT) DSL system of L users, which models a copper-wire binder group. For each tone, the channel can be expressed as a linear system as follows:

$$y_{u,n} = \sum_{i=1}^{L} H_n^{(u,i)} x_{i,n} + n_{u,n} \quad (u = 1, \cdots L, \ n = 1, \cdots, N),$$
(1)

where  $H_n^{(u,i)}$  is the  $(u,i)^{\text{th}}$  entry of the channel matrix that represents crosstalk from the transmitter *i* to the receiver *u*,  $y_{u,n}$  is the output of user *u*,  $x_{i,n}$  is the input of user *i*,  $n_{u,n}$ is the noise of user *u* at tone *n*, and *N* is the total number of used tones. In this model, no signal coordination is assumed between lines and the signals from other users are treated as noise; such a multi-user channel is often called an "interference channel". Under this assumption, the rate of user u is proportional to:

$$\mathbf{b}_{n}^{(u)} = \log_{2} \left( 1 + \frac{1}{\Gamma_{n}} \frac{|H_{n}^{(u,u)}|^{2} \mathbf{p}_{n}^{(u)}}{\sigma_{n}^{2} + \sum_{j \neq u} |H_{n}^{(u,j)}|^{2} \mathbf{p}_{n}^{(j)}} \right)$$
  
=  $\log_{2} \left( 1 + \frac{1}{\Gamma_{n}} \cdot \mathbf{g}_{n}^{(u)} \cdot \mathbf{p}_{n}^{(u)} \right)$  (bit/dim)  
 $R_{u} = \sum_{n=1}^{N} \mathbf{b}_{n}^{(u)},$  (2)

where  $\mathbf{g}_n^{(u)} = |H_n^{(u,u)}|^2 / \left(\sigma_n^2 + \sum_{j \neq u} |H_n^{(u,j)}|^2 \mathbf{p}_n^{(j)}\right)$  is the normalized channel gain,  $\mathbf{p}_n^{(u)}$  is the transmit power user u at tone n, and  $\Gamma_n$  is the implementation gap on tone n.

## **III. BAND-PREFERENCE ALGORITHM**

## A. Power Scaling Factors, $\alpha_n$

The case of a single DSL modem is first considered. A good DMT modem, in the absence of PSD masks, loads bits to approximate the following "water-filling" condition in a tone set  $E = \{1, ..., N\}$ 

$$\mathbf{p}_{n}^{WF} = \left(K_{1} - \frac{\Gamma_{n}}{\mathbf{g}_{n}}\right)^{+}, \quad n \in E,$$
$$\sum_{n=1}^{N} \mathbf{p}_{n}^{WF} = P, \tag{3}$$

where  $K_1 \in \mathbb{R}$  is a nonnegative constant, P is a total power constraint, and  $(x)^+ \triangleq \max(x, 0)$ . We will say that waterfilling is the process by which  $\mathbf{p}_n^{WF}$  and  $K_1$  satisfying (3) are found (for given  $\Gamma$ ,  $\mathbf{g}_n$  and P). Scaling factors that modify (3) are introduced as follows.

*Definition 1:* The *scaled water-filling condition* is said to hold when the following conditions are satisfied

$$\mathbf{p}_{n} = \left(\frac{K_{2}}{\alpha_{n}} - \frac{\Gamma_{n}}{\mathbf{g}_{n}}\right)^{+}, \quad n \in E,$$
$$\sum_{n=1}^{N} \mathbf{p}_{n} = P, \ K_{2} \ge 0$$
(4)

Accordingly, the process of finding  $\mathbf{p}_n^{WF}$  and  $K_2$  that satisfy (4) (for given  $\Gamma, \mathbf{g}, \alpha$  and P) is termed scaled water-filling. Note that for  $\alpha = \mathbf{1}$ , scaled water-filling is equivalent to water-filling.

In (4), the factors  $\alpha$  may be interpreted as a tone-dependant penalty that is useful for controlling the modem's power on that tone. For all n such that  $\alpha_n = \infty$ , observe that one must have  $\mathbf{p}_n = 0$  in order that the scaled-water-filling condition hold. An intuitive interpretation is that setting the penalty on tone n (namely  $\alpha_n$ ) to  $\infty$  has the effect of disabling tone n.

The water-filling and scaled water-filling conditions may be generalized to the setting where the modem has a PSDMASK. Note that this is a strict generalization of (3) and (4) because PSD masks are redundant if larger than the total power constraint  $(\mathbf{C} \succ \mathbf{1} \cdot P)$ . This generalized setting will be considered in the remainder of the paper. The water-filling condition (3) generalizes to

$$K_{2} \geq 0, \nu \geq 0,$$
  

$$0 \geq \left(K_{1} - \frac{\Gamma_{n}}{\mathbf{g}_{n}}\right), \nu_{n} = 0, \quad \text{if } \mathbf{p}_{n} = 0$$
  

$$\mathbf{p}_{n} = \left(K_{1} - \frac{\Gamma_{n}}{\mathbf{g}_{n}}\right), \nu_{n} = 0, \quad \text{if } 0 < \mathbf{p}_{n} < \mathbf{C}_{n},$$
  

$$\mathbf{C}_{n} = \left(\frac{1}{1/K_{1} + \nu_{n}} - \frac{\Gamma_{n}}{\mathbf{g}_{n}}\right), \nu_{n} \geq 0, \quad \text{if } \mathbf{p}_{n} = \mathbf{C}_{n}, \quad (5)$$

for each n. The (generalized) scaled water-filling condition (4) is therefore defined<sup>1</sup> as

Definition 2: For fixed scaling factors  $\alpha \in \overline{\mathbf{R}}^N$ ,  $\alpha \succeq \mathbf{1}$ , the (generalized) scaled water-filling condition is defined as

$$K_{2} \geq 0, \ \nu \geq 0,$$
  

$$0 \geq \left(\frac{K_{2}}{\alpha_{n}} - \frac{\Gamma_{n}}{\mathbf{g}_{n}}\right), \nu_{n} = 0, \quad \text{if } \mathbf{p}_{n} = 0$$
  

$$\mathbf{p}_{n} = \left(\frac{K_{2}}{\alpha_{n}} - \frac{\Gamma_{n}}{\mathbf{g}_{n}}\right), \nu_{n} = 0, \quad \text{if } 0 < \mathbf{p}_{n} < \mathbf{C}_{n},$$
  

$$\mathbf{C}_{n} = \left(\frac{1}{1/K_{2} + \nu_{n}} \cdot \frac{1}{\alpha_{n}} - \frac{\Gamma}{\mathbf{g}_{n}}\right), \nu_{n} \geq 0, \quad \text{if } \mathbf{p}_{n} = \mathbf{C}_{n},$$
(6)

for each *n*. Note again that under the scaled water-filling condition (6),  $\alpha_n = \infty$  implies that  $\mathbf{p}_n = 0$ , and that (6) with  $\alpha = \mathbf{1}$  is equivalent to (5).

The following two theorems show how spectral allocation may be controlled using the scaling factors. We first show that for any channel and any full-power PSD, there exist scaling constants such that the scaled water-filling condition holds.

Theorem 1: For any fixed PSD  $\mathbf{p} \in \mathbb{R}^N_+$  where  $\sum_n \mathbf{p}_n = P$ , channel gains  $\mathbf{g}_n \in \mathbb{R}^N_{++}$ , and gaps  $\Gamma_n \in \mathbb{R}^N_{++}$ , there exist  $\alpha \in \overline{\mathbb{R}}^N_+$ ,  $\alpha \succeq \mathbf{1}$  and  $K_2 \in \mathbf{R}_+$  such that the scaled water-filling condition (6) holds.

*Proof:* A constructive proof is given. Choose  $K_2 = \max_{n \in E}(\mathbf{p}_n + \Gamma_n/\mathbf{g}_n)$ , which satisfies  $0 \leq K_2 < \infty$ . This choice of  $K_2$  implies that  $0 \leq \mathbf{p}_n \leq K_2 - \frac{\Gamma_n}{\mathbf{g}_n}$  for all  $n \in E$ . For each  $n \in E$ , either  $\mathbf{p}_n = 0$ , or  $\mathbf{p}_n > 0$ . For  $n \in E$  such that  $\mathbf{p}_n = 0$ , choose  $\alpha_n = \infty$  and  $\nu_n = 0$  to satisfy (6). For  $n \in E$  such that  $\mathbf{p}_n > 0$ , choose  $\alpha_n = K_2/(\mathbf{p}_n + \frac{\Gamma_n}{\mathbf{g}_n}) < \infty$  and  $\nu_n = 0$ . It may be verified by substitution that this choice of  $\alpha_n$  satisfies (6). Because  $\mathbf{p}_n \leq K_2 - \frac{\Gamma}{\mathbf{g}_n}$ , it also holds  $\alpha \succeq 1$ .

Note that in Theorem 1, the dual variables  $\nu_n$  associated with the PSD masks may be chosen to always be 0. This mathematical property may be interpreted as showing that the proper selection of  $\alpha_n$  acts as a "virtual PSD mask" and makes PSD mask constraint in Theorem 2 redundant (in this singleuser setting).

The following theorem shows that for every set of channel parameters and fixed scaling constants, there exists exactly one PSD satisfying the scaled water-filling condition.

<sup>&</sup>lt;sup>1</sup>It may be verified that the condition (6) reduces to (4) when  $\mathbf{C} \succ \mathbf{1} \cdot P$ .

Theorem 2: For any fixed  $\alpha \in \overline{\mathbb{R}}^N_+$  where  $\alpha \succeq \mathbf{1}$ , channel gains  $\mathbf{g}_n \in \mathbb{R}^N_{++}$ , and gaps  $\Gamma \in \mathbb{R}^N_{++}$ , there exists a unique  $\mathbf{p} \in \mathbb{R}^N_+$  satisfying (6). Furthermore,  $\sum_{n \in E} \mathbf{p}_n = P$  unless  $\alpha = \mathbf{1} \cdot \infty$ .

*Proof:* For n such that  $\alpha_n = \infty$ , observe that  $\mathbf{p}_n = 0$  by (6). Define F to be the remaining tone indices, that is,  $F \triangleq E - \{n : \alpha_n = \infty\}$ . Consider the following convex optimization problem

$$\max \sum_{n \in F} \frac{1}{\alpha_n} \log \left( 1 + \frac{\mathbf{g}_n \mathbf{p}_n}{\Gamma_n} \right)$$
  
subject to  $\mathbf{p} \succeq 0, \ n \in F$   
 $\sum_{n \in F} \mathbf{p}_n \le P,$   
 $\mathbf{C} \succeq \mathbf{p}, \ n \in F.$  (7)

Observe that the objective of (7) is strictly convex in **p** because  $\log(1+x)$  is strictly convex on  $x \in \mathbb{R}_+$ . Furthermore, there exists a feasible point to the optimization (7), namely  $\mathbf{p} = \mathbf{0}$ , and the feasible set is closed and bounded. The optimization problem therefore has a unique optimal value, call it  $\mathbf{p}^*$ . Because the objective is strictly increasing in  $\mathbf{p}_n$  for  $n \in F$ , it follows that  $\sum_{n \in E} \mathbf{p}_n = \sum_{n \in F} \mathbf{p}_n = P$  (unless  $F = \emptyset$ ).

It is known that the Karhn-Kush-Tucker (KKT) conditions are necessary and sufficient for optimality of a convex optimization problem satisfying these properties [11]. It can be shown by direct computation that the KKT conditions of the optimization (7) are precisely (6). Therefore, because  $\mathbf{p}^*$  is unique optimal solution to (7),  $\mathbf{p}^*$  is also the unique value satisfying (6).

## B. Scaled Bit-Loading

Current bit-loading algorithms [12] require only slight modification to include scaling factors for the proposed BPSM scheme. The algorithm is described as follows:

Initialization:

- 1 Set  $\Delta \mathbf{p}'_n(1) = \alpha_n \cdot (\mathbf{p}_n(1) \mathbf{p}_n(0)), \ \mathbf{b}_n = 0 \ \forall n$ Iteration:
- 2 If  $\sum \mathbf{p}_n \ge P$ , or  $\min_n(\Delta \mathbf{p}'_n(\mathbf{b}_n+1)) = \infty$ , then stop.
- 3 Set  $m = \arg \min_n \Delta \mathbf{p}'_n(\mathbf{b}_n + 1)$ .
- 4 If  $\mathbf{b}_m + 1 \leq b_{max}$ , and  $\mathbf{p}_m(\mathbf{b}_m + 1) \leq \mathbf{C}_m$ , go to 5; else set  $\Delta \mathbf{p}'_m(\mathbf{b}_m + 1) = \infty$ , and then go to 2.
- 5 Set  $\mathbf{b}_m = \mathbf{b}_m + 1$ ,  $\Delta \mathbf{p}'_m(\mathbf{b}_m + 1) = \alpha_m \cdot (\mathbf{p}_m(\mathbf{b}_m + 1) \mathbf{p}_m(\mathbf{b}_m))$ , and then go to 2.

where  $\mathbf{b}_n$  is the number of bits loaded on tone n,  $\mathbf{p}_n(\mathbf{b}_n) \triangleq (2^{\mathbf{b}_n} - 1) \cdot \Gamma_n/\mathbf{g}_n$ , is the power to load  $\mathbf{b}_n$  bits on tone n,  $b_{\text{max}}$  is the maximum bits per tone,  $\mathbf{C}_n$  is the PSDMASK at tone n, and P is the maximum power per user. As seen above, the only modification is to scale the incremental energy tables in DMT modem. The following theorem shows the efficiency of this bit-loading algorithm.

*Theorem 3:* The allocation generated by the scaled integerbit-loading algorithm is an *undominated* or *efficient* solution to (7). Furthermore, the terminating value of  $\sum_{n} \mathbf{b}_{n}/\alpha_{n}$  found by the algorithm is within 1 of the optimal value of (7). *Proof:* Efficiency is defined in [13] and can be shown as in [13,  $\S$ 8]. Suboptimality of less than 1 can be shown as a consequence of [13, Thm. 3].

In step 3 of the loading process, the tone that has the minimum incremental energy is found and a bit is loaded on that tone. The incremental energy can be equivalently expressed as follows.

$$\Delta \mathbf{p}_{n}'(\mathbf{b}_{n}+1) = \alpha_{n} \cdot (\mathbf{p}_{n}(\mathbf{b}_{n}+1) - \mathbf{p}_{n}(\mathbf{b}_{n}))$$

$$= \alpha_{n} \cdot ((2^{\mathbf{b}_{n}+1}-1) - (2^{\mathbf{b}_{n}}-1)) \cdot \Gamma_{n}/\mathbf{g}_{n}$$

$$= \alpha_{n} \cdot 2^{\mathbf{b}_{n}} \cdot \Gamma_{n}/\mathbf{g}_{n}$$

$$= \alpha_{n} \cdot (\mathbf{p}_{n}(\mathbf{b}_{n}) + \Gamma/\mathbf{g}_{n})$$
(8)

By interpreting  $\alpha_n \cdot (\mathbf{p}_n(\mathbf{b}_n) + \Gamma/\mathbf{g}_n)$  as a scaled waterlevel on tone *n* at the moment when  $\mathbf{b}_n$  bits are loaded, the proposed bit-loading process loads a bit on the tone that has the minimum scaled water-level and it is coincident with scaled water-filling condition.

#### C. Multiuser Use of Band Preference

Band preference is designed for deployment in multi-user networks. In this setting, the users' gains g depend on the the power allocations chosen by *other* users. The multiuser convergence properties of the proposed band preference algorithm are beyond our present scope. However, the system architecture of a spectrum management center (SMC) using band preference is of central interest.

In particular the SMC, with knowledge of the channel and noise, may compute band preference coefficients  $\alpha_n^{(k)}$  for each user k and distribute them infrequently to the modems over control channels. Each modem may then implement the proposed scaled bit-loading algorithm to derive its intended PSD. If the channel and noise do not change, then the PSD computed by each modem will be as expected at the SMC. If however, there is a change, the modem will adapt and a minor difference will be reflected in the power distribution. As noted previously, a scaling factor is penalty information for a band, and this information may be nearly-optimal even when channel changes moderately. BPSM with scaling factors therefore enables DSL modems to rapidly adjust their power distribution (*i.e.* bit-swapping) to moderate changes without recomputation of all PSDs at the SMC.

## D. Scaling Factor Setting

This section explains a heuristic way to find scaling factors. Since adjacent tones are likely to have similar properties in a DSL channel, adjacent tones are grouped into a "subband", and one scaling factor is allocated to each subband. For this technique, a two-user problem in Fig.1 is considered. Assuming user 2 to be far-located, he is assumed to be a "weak" user and maximizes his rate while user 1 maintains his target rate. Then, according to the current ADSL standard [14], it is likely that user 2 uses power up to the PSDMASK on all tones. With user 2's power fixed at PSDMASK, the following bit-trade-off can be considered. If user 1 loads more bits on band 1, user 2 might lose some bits on the same band because of the increased interference from user 1. Thus, user 1 should consider loading bits on the bands where the loss of user 2 can be minimized. This bit-trade-off between users can be expressed as a cost table. A cost table calculated in this way significantly simplifies the optimization process since it conceals the details of bit-loading process.

Assume that  $\mathbf{p}_n^{(2)}$  is fixed as explained above. Then, a cost function can be defined as follows.

$$C(R_{1,k},k) = R_{2,k}(0) - R_{2,k}(\mathbf{p}_n^{(1)*}) \quad (k = 1, \dots, K)$$
$$R_{2,k}(\mathbf{p}_n^{(1)*}) = \sum_{n \in N_k} \log_2 \left( 1 + \frac{1}{\Gamma} \frac{|H_n^{(2,2)}|^2 \mathbf{p}_n^{(2)}}{\sigma_n^2 + |H_n^{(2,1)}|^2 \mathbf{p}_n^{(1)*}} \right)$$
(9)

with the optimal solution  $\mathbf{p}_n^{(1)*}$  of

$$\min \sum_{n \in N_k} \mathbf{p}_n^{(1)}$$
  
s.t. 
$$\sum_{n \in N_k} \log_2 \left( 1 + \frac{1}{\Gamma} \frac{|H_n^{(1,1)}|^2 \mathbf{p}_n^{(1)}}{\sigma_n^2 + |H_n^{(1,2)}|^2 \mathbf{p}_n^{(2)}} \right) \ge R_{1,k}, \quad (10)$$

where K is the total number of bands, and  $N_k$  is the set of tones on band k. Since  $\mathbf{p}_n^{(2)}$  is fixed, (10) is a convex problem and water-filling process finds the optimal solution.

With the above definitions, the following minimization problem with two users is considered.

min 
$$\sum_{k=1}^{K} C(R_{1,k}, k)$$
  
s.t 
$$\sum_{k=1}^{K} R_{1,k} \ge R_1^{target}$$
(11)

As explained previously, cost table is generated from (9) to solve (11). To reduce the size of cost table, incremental granularity,  $\Delta$  is introduced.

$$C_{\Delta}(i,k) = C(i \cdot \Delta, k), \tag{12}$$

where  $C_{\Delta}(i,k)$  is the (i,k) entry of the table. When user 1 can not load  $i \cdot \Delta$  bits any more,  $C_{\Delta}(i,k)$  is set as  $\infty$ . Table I shows one such example when K = 4 and  $\Delta =$ 10. For instance, if user 1 loads 30 bits in band 2, user 2 loses 5 bits in the same band. If user 1 loads 30, 30, 20, 20  $(R_{1,k} = \{30, 30, 20, 20\})$  bits on each band respectively, the total incurred cost becomes  $C_{\Delta}(3,1) + C_{\Delta}(3,2) + C_{\Delta}(2,3) + C_{\Delta}(2,4) = 15$ . If  $R_1^{target}$  is 70 and Table I is used, the optimal solution becomes  $R_{1,k} = \{10, 0, 60, 0\}$  with the minimum cost of 6. Note that a greedy algorithm does not work here because it finds  $R_{1,k} = \{30, 20, 10, 10\}$  as a solution, which costs 8 bits. In fact, this problem can be efficiently solved by Dynamic Programming [15] as follows.

$$f_{1}(x_{1}) = C_{\Delta}(x_{1}, 1)$$

$$f_{k}(x_{k}) = \min_{0 \le m_{k} \le x_{k}} \{C_{\Delta}(m_{k}, k) + f_{k-1}(x_{k} - m_{k})\}$$

$$f_{K}(M) = \min_{0 \le m_{K} \le M} \{C_{\Delta}(m_{K}, K) + f_{K-1}(M - m_{K})\},$$
(13)

TABLE I COST TABLE BETWEEN TWO USERS

Γ	i	$C_{\Lambda}(i,1)$	$C_{\Lambda}(i,2)$	$C_{\Lambda}(i,3)$	$C_{\Lambda}(i,4)$
	1	0	1	3	2
	2	1	2	4	5
	3	1	5	6	6
	4	5	7	6	8
	5	8	9	6	10
	6	9	10	6	12
		$\infty$	$\infty$	$\infty$	$\infty$
С	5km User 2				
0		4km		3km	_

Fig. 1. Simulation Set up

User 1

RT

where M is  $R_1^{target}/\Delta$ ;  $x_k = \sum_{j=1}^k R_{1,j}$ ;  $f_k(x_k)$  is the minimum cost to load  $x_k$  bits from band 1 to k. The determination of  $f_k(x_k)$  can be divided into subproblems. The subproblems are to find  $f_{k-1}(x_k - m_k)$  when  $m_k$  bits are loaded on band k. Since  $m_k$  could be 0 to  $x_k$ ,  $x_k + 1$  subproblems are created. This dynamic program can be solved recursively. Once the solution of (11) is obtained, it can be converted to scaling factors.

## E. Simulation Results

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The performance of IWF, OSB and the proposed BPSM algorithm are compared by simulation. Fig. 1 illustrates a twouser ADSL downstream scenario, where user 2 communicates directly with the Central Office (CO) and user 1 is connected to a Remote Terminal (RT). Note that the downstream signal from the RT to user 1 operates as a strong interference to user 2.

In this simulation, Noise A in addition to -140 dBm/HzAWGN is injected, where Noise A is a mixture of 16 ISDN, 4 HDSL and 10 ADSL disturbers [16]. Fixed-Margin Water-Filling (FM-WF) is performed for user 1 while Rate-Adaptive Water-Filling (RA-WF) [17] is performed for user 2.

Fig. 2 shows the rate region, where a large gap between IWF and OSB can be observed. The primary reason for this gap is that the signal from the RT induces strong interference to the CO users in the low frequency region. OSB similarly avoids this phenomenon by allocating less power in the low frequency region of user 1. Note the PSD difference between OSB and IWF in Fig. 3. Fig. 2 also shows that BPSM's performance is very close to that of OSB even when K = 6. Also, note that the PSD of BPSM resembles that of OSB in Fig. 3.

Fig. 4 shows user 2's data rate when user 1's target rate is fixed to 6Mbps as K grows. The steep slope in the figure demonstrates the effectiveness of the subband grouping, which



Fig. 2. BPSM result when  $K=6, \Delta = 5$  with Noise A + AWGN Noise



Fig. 3. Power distribution profile of IWF, OSB, and BPSM when  $R_1 = 6$  Mbps and  $\Delta = 5$ , with noise A + AWGN Noise

implies the OSB performance can be approached by BPSM even with a small number of subbands.

#### **IV. CONCLUSION**

This paper has proposed a low-overhead band preference algorithm for distributed control of modem PSDs in a DSL network. Because it utilizes power scaling factors instead of directly controlling PSDs, the proposed algorithm leverages the modems' natural adaptive capability to respond to channel and noise fluctuations. An ad-hoc algorithm for choosing bandpreference parameters at the SMC was presented. Numerical simulation of this algorithm shows that significant gains (approaching optimal Level 2 DSM limits) can be achieved.



Fig. 4. Comparison of IWF, OSB, and BPSM with different number of bands when  $R_1 = 6$  Mbps and  $\Delta = 1$ , with noise A + AWGN Noise

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