Power efficient Opportunistic p-persistent CSMA for Wireless Networks

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Abstract— **This paper proposes a power efficient p-persistent Carrier Sense Multiple Access (CSMA) employing multiuser diversity, called Opportunistic p-persistent CSMA (OpCSMA). At each idle time slot, an MT sends a packet if the corresponding channel gain is above the threshold which is determined such that the probability of accessing the medium is p for any idle slots. Also, the transmit power is controlled to maintain the constant signal-to-noise-ratio when the channel gain exceeds the cut-off fade depth. The analysis under the infinite user model shows that as traffic load grows, the transmit power consumption of the conventional p-persistent CSMA increases, whereas that of the OpCSMA decreases owing to multiuser diversity effect. Simulation results corroborate these findings in slow fading channels.**

I. Introduction

One of the main concerns in wireless networks such as wireless LAN (WLAN) is the limited power supply at mobile terminals (MTs). The medium access control (MAC) protocol employed by the 802.11 WLAN is Carrier Sense Multiple Access (CSMA) [1]. The 802.11 WLAN MAC protocol is shown to be closely approximated by the p-persistent CSMA [2], where the transmit power consumption of WLAN can be reduced by optimizing the access probability p [3]. In order to save the transmit power of WLAN, some other variants of the CSMA have been also proposed [4],[5]. Even though numerous transmit power control schemes can be utilized to prolong the battery life in wireless networks [6], the transmit power control in conjunction with the CSMA has not been thoroughly explored.

In scalar wireless multiple access channels, the system throughput is maximized by letting a user with the largest channel gain to access the wireless medium [7]. Moreover, the throughput increases as the number of active users rises because of the multiuser diversity effect. However, a centralized scheduler is generally required to utilize the multiuser diversity, which would not be feasible for random access channels. Recently in [8], the channel-aware ALOHA achieves the multiuser diversity gain by letting each MT access the medium only when the channel gain is larger than a threshold. Since [8] assumes independent and identically distributed (i.i.d.) block fading over time slots (i.e., fast fading channel), a single threshold value is assigned to each MT. Similar protocol was shown to achieve throughput comparable to that with a centralized scheduler [9]. Nonetheless, in a slow fading channel, the block fading assumption is no longer valid; therefore, the criterion on setting the threshold becomes more complicated.

This paper investigates transmit power reduction of CSMA-based WLANs whose channel gain are slowly-varying over time. The proposed method, Opportunistic p-persistent CSMA (OpCSMA), attains the multiuser diversity by selecting the MT with the largest channel gain in a distributed manner; then saves the transmit power consumption by applying a power control called truncated channel inversion (TCI) [10]. At each idle time slot, an MT accesses the medium only if the corresponding channel gain exceeds the threshold, which is predefined such that the probability of accessing the medium is maintained to be p for any idle slots. Different from the fast fading case, each user is assumed to have the constant channel gain for the subsequent time slots. Therefore, the threshold for the current idle slot takes a smaller value if there was no packet transmission in the previous idle slot. Also, the transmit power is controlled such that the constant signal-to-noise-ratio (SNR) is preserved at the receiver when the channel gain exceeds the cut-off fade depth.Similarly, the multiuser diversity can be utilized to increase the throughput of the network instead of saving the transmit power, which is discussed in [11].

Under the infinite user model, the OpCSMA is shown to substantially reduce the transmit power consumption compared with the p-persistent CSMA. As traffic load grows, the p-persistent CSMA requires more transmit power to achieve the target throughput; however, the required transmit power diminishes for the OpCSMA, which results in more prominent difference between two schemes. Besides, it is shown for the OpCSMA that the limit of transmit power consumption with $p \to 0$ monotonically decreases as traffic load becomes larger.

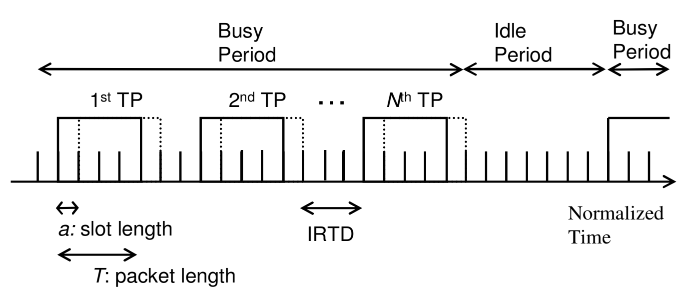

Fig. 1. p-persistent CSMA time-slot structure. (TP: transmission period, IRTD: initial random transmission delay)

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II. SYSTEM MODEL AND p-PERSISTENT CSMA

In the wireless network such as WLAN, a time-varying number of MTs communicate with an access point (AP) through slow fading channels. Assuming only one MT accesses the channel at each scheduling instance, the received signal of the AP at time t is given by

$$y(t) = \sqrt{h_i}x_i(t) + n(t), \tag{1}$$

where $x_i(t)$ is the transmitted signal from MT i over the bandwidth $2W$, h_i denotes the channel gain from MT i to the AP, and $n(t)$ is an additive white Gaussian noise (AWGN) with the power of N_0W. Without loss of generality, noise power N_0W and each MT's transmit signal power are assumed to be 1 throughout this paper. Then, each MT's SNR is i.i.d. according to the probability density function (pdf) $f_H(h)$. For a Rayleigh fading channel, $f_H(h) = e^{\frac{-h}{P_r}}/P_r$ where P_r is the average signal power at the receiver.

This paper considers the infinite user model that was first considered by Abramson [12], and then was used to evaluate the throughput of the CSMA [1]. The traffic sources are composed of an infinite number of MTs that collectively form a single Poisson process; thus each new packet always arrives at a new MT. In case of a collision or a back-off, the packet is delayed for a random interval so that it appears as a new arrival in the future. By aggregating arrival process including new and retransmitted packets, the average number of arrivals per unit time, G, is defined as *offered load*. The number of MTs in the network equals the number of packets waiting for transmission; hence the effect of time variation in the number of users is considered in the infinite user model. In this paper, it is also assumed that the channel gains of all users are i.i.d. according to the pdf $f_H(h)$, and the channel gains are invariant over time. Although this model may appear unnatural at first, it actually lower bounds the performance of a finite-user system since each user's packets are assumed to compete against each other [12]. In practice, these modeling hypotheses approximate a large finite population in which each MT transmits packets infrequently through slowly-varying channels.

Fig. 1 presents the operation of p-persistent CSMA [1] along a normalized time axis that is finely partitioned into slots of duration a. The slot duration is usually twice the maximum propagation delay between the AP and MTs. All MTs can start transmitting only at the beginning of a slot; each packet is of constant length T. To simplify the analysis, time axis is normalized by T so that the packet length is equal to 1. A beacon signal is placed at the beginning of each slot for channel estimation at the MTs. Provided the wireless channel is reciprocal in time division duplexed (TDD) WLANs [13], the estimated downlink channel gain can represent the uplink channel gain. When a packet arrives at an MT, the MT first checks whether any other MTs are transmitting a packet. If the channel is found idle, the MT accesses the wireless medium with probability p to reduce the collision probability. Therefore, even during a busy period, a silent period, referred to as *initial random transmission delay* (IRTD), appears between transmission periods (TPs). If an MT starts transmitting a packet at time $t = 0$, the other MTs wait until $t = T + a$ where the a term is owing to the propagation delay. Thus, as illustrated in the dotted boxes of Fig. 1, the duration of a TP is $(T + a)$. If an MT decides to defer transmission, it repeats the same procedure for the following idle slots. If all MTs have no packets to send, an idle period is maintained until new packets arrive.

III. TRANSMISSION POWER CONSUMPTION OF p-PERSISTENT CSMA

The average transmit power consumption, P_c, is defined as the expectation of the overall transmit power used by all MTs during a TP. If a centralized scheduler is used, only one MT is allowed to transmit during a given TP. Thus, P_c is equal to P, the average transmit power of individual MT. For a random access network, however, P_c is larger than P because more than one MT can simultaneously transmit packets when a collision occurs.

In this paper, the transmit power during a TP is controlled by the TCI [10]. If the channel gain is above a cutoff fade depth γ_o, the MT inverts the channel fading to maintain a constant received power level. Otherwise, no packet is transmitted and an outage is declared. During a packet transmission, all MTs keep the same transmission rate since each channel is transformed into a time-invariant AWGN channel by the TCI. Provided an outage probability p_o, the cutoff fade depth for a Rayleigh fading channel is

$$\gamma_o = -P_r \ln(1 - p_o). \tag{2}$$

Assuming the average transmit power to be 1, the received power when $h_i > \gamma_o$ becomes

$$P_d = \frac{P_r}{E_1(-\ln(1 - p_o))}, \tag{3}$$

where $E_1(x) = \int_x^\infty \frac{e^{-u}}{u} du$ is called the *exponential integral function*. Thus the transmit power is $P_t(h_i) = \frac{P_d}{h_i}$. The transmit power control may hamper the operation of CSMA because it can exacerbate the hidden-node problem [14]. However, its effect can be minimized in practical systems by employing methods such as busy tone signals.

The average transmit power of each MT is equal to 1 after the TCI. Since the transmit power and the number of transmitting MTs are mutually independent random variables, P_c is the product of the average transmit power of individual transmission and the average number of MTs transmitting a packet, which is

$$P_c = \sum_{n=0}^{\infty} \pi_n \mathbf{E}[l|n], \tag{4}$$

where n denotes the number of arrivals during previous TP, π_n is the probability of n arrivals, l represents the number of transmissions during a TP, and $\mathbf{E}[l|n]$ is the expected number of transmitted packets during a TP when n packets has been arrived. During the first TP in a busy period (see Fig. 1), the number of packet arrivals is Poisson distributed with the mean of $\lambda = aG$. Provided a is small, only one packet arrives with high probability; thus $\mathbf{E}[l|n] = 1$. For the following TPs, the number of arrivals, n, is Poisson distributed with a larger mean of $\lambda = (1+a)G$. In other words, $\pi_n = e^{-\lambda}\frac{\lambda^n}{n!}$ since arrived packets during the previous packet transmission have been backlogged until the channel is available. In addition, l is Binomial random variable with the mean of np; therefore the expectation of l given n is as follows:

$$\mathbf{E}(l|n) = \sum_{l=0}^{\infty} l \frac{{}_nC_l p^l q^{n-l}}{1-q^n} = \frac{np}{1-q^n}, \tag{5}$$

where $q = 1-p$ and ${}_nC_l$ denotes n choose l. Applying (5) to (4), Proposition 1 provides the analytic result for the average transmission power consumption.

Proposition 1: The average transmission power consumption of p-persistent CSMA becomes

$$P_c = e^{-\lambda}\left(1 + \lambda p \sum_{k=0}^{\infty} q^k e^{\lambda q^k}\right). \tag{6}$$

Proof: See appendix ■

From (6), the transmit power consumption of the network, P_c, always exceeds 1 because of possible collisions. In p-persistent CSMA, the collisions can be avoided by letting $p \to 0$ at the cost of longer wasted time during IRTD. In Proposition 2, it is shown that the transmission power consumption P_c approaches $P = 1$ as $p \to 0$.

Proposition 2: The asymptotic average transmit power consumption of p-persistent CSMA network approaches one as $p \to 0$.

Proof: To further simplify the analysis, the infinite sum in (6) is approximated by an integration assuming that p is small, then the approximation of the average transmission power consumption becomes

$$P_c \simeq e^{-\lambda}\left(1 + \frac{p(1-e^{\lambda})}{\ln q}\right) \tag{7}$$

From L'Hospital's rule, $\lim_{p\to 0}\frac{p}{\ln q} = -1$. Then, $\lim_{p\to 0} P_c = e^{-\lambda}(1 + e^{\lambda} - 1) = 1$. ■

Proposition 2 concludes that the use of transmit power control does not reduce the average power consumption of p-persistent CSMA network. At best, P_c could be maintained to be one at the cost of less throughput because of longer IRTDs per transmission. This result calls for a better method to save the power consumption in a CSMA network.

IV. OPPORTUNISTIC p-PERSISTENT CSMA

In p-persistent CSMA, if the channel is sensed idle, each MT generates a Bernoulli random variable with the mean of p at the beginning of each slot. Then, an MT accesses the wireless medium if its random variable is equal to one. Though channel state information (CSI) is available at MT, p-persistent CSMA ignores the CSI in accessing the medium. On the contrary, the MT employing the OpCSMA accesses the channel only if its channel gain exceeds a predetermined threshold T_k during kth idle slot as shown in Fig. 2. Since the threshold is a decreasing function of k, an MT with the largest channel gain accesses the channel in the absence of a collision. During the transmission, the transmit power is controlled according to the TCI as in Section III; thus a constant data rate is maintained for all MTs at all time. In this manner, the multiuser diversity is attained without a centralized scheduler; then is used to reduce the transmit power consumption.

Each MT has a set of thresholds related with its own channel statistics. Since it is assumed that the channel statistics are the same for all MTs, every user shares a single set of thresholds, $\{T_0, T_1, \cdots, T_k, \cdots\}$, where the subscript denotes the index of idle slots. Assume that the ith MT has a packet to send, but $k-1$ idle slots have elapsed without its accessing the medium. Then, the ith MT sends the packet at the kth idle slot if its channel gain h_i exceeds the threshold T_k. Therefore, it can be easily seen that $T_{m-1} > T_m$ for all positive integer m. In the analysis, it is assumed that a is so small that no more packets arrive during an IRTD. As a result, k simply denotes the idle slot index. Since the channel gains of all MTs are smaller than T_{k-1} at the kth idle slot, the probability of accessing the wireless medium for the ith MT is

$$P(h_i \geq T_k | h_i < T_{k-1}) = \frac{F_H(T_{k-1}) - F_H(T_k)}{F_H(T_{k-1})}, k = 1, \cdots \tag{8}$$

where $F_H(h) = \int_0^h f_H(x)dx$ is the probability distribution function of a channel gain. To maintain the probability of transmission at each idle slot to equal p, $P(h_i \geq T_k | h_i < T_{k-1}) = p$ for all positive integer k. Then the following relation can be obtained from (8)

$$qF_H(T_{k-1}) = F_H(T_k). \tag{9}$$

In particular, the transmission probability on the 0th idle slot is $P(h_i \geq T_0) = 1 - F_H(T_0)$; so the threshold at slot k is $T_k = F_H^{-1}(q^{k+1})$ from (9) and the initial condition at $k = 0$. As an example, this paper considers a Rayleigh fading channel where $T_k = -P_r \ln(1 - q^{k+1})$.

Compared with the p-persistent CSMA, the average transmit power consumption is reduced by using the OpCSMA since an MT transmits a packet only when its channel gain is sufficiently large. Contrary to the conventional p-persistent

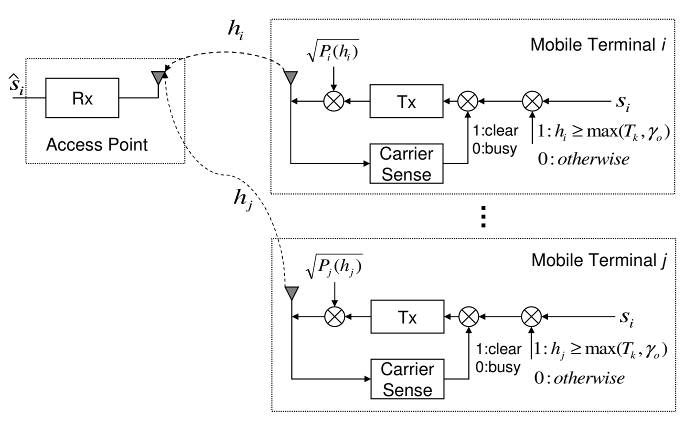

Fig. 2. Block diagram of the OpCSMA in uplink channels.

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CSMA analyzed in Section III, the average transmit power under the OpCSMA is a function of slot index k. Suppose that a packet is transmitted at the kth slot, then the average transmit power consumption at an MT is

$$
\begin{aligned}
P_t(k) &= \int_{T_k}^{T_{k-1}} f_H(h|T_{k-1} > h \geq T_k)\frac{P_d}{h}dh \\
&= \frac{P_d}{q^k p P_r}\left(E_1\left(\frac{T_k}{P_r}\right) - E_1\left(\frac{T_{k-1}}{P_r}\right)\right) \qquad (10)
\end{aligned}
$$

where $T_{-1} = \infty$. From $T_k = -P_r \ln(1 - q^{k+1})$ and (3), it follows that

$$
P_t(k) = \frac{E_1\left(-\ln(1-q^{k+1})\right) - E_1\left(-\ln(1-q^k)\right)}{q^k p E_1(-\ln(1-p_o))}. \qquad (11)
$$

If the channel gain h_i is below the cutoff fade depth γ_o, no packet is transmitted and the back-off ends for $T_k \leq \gamma_o$. Therefore, the largest slot index k_m is $\frac{\ln p_o}{\ln q} - 1$.

At the first TP in a busy period, only one packet arrives with high probability. Hence, the average transmit power consumption is unchanged. Since the number of arrivals, n, is Poisson distributed with a mean of $\lambda = (1+a)G$ for the following TPs, the probability of transmitting packets at slot k is $P_K(k) = q^{kn}(1-q^n)$ where q^{kn} is the probability of no transmission before slot k and $(1-q^n)$ is the probability of at least one attempt to access the channel. Conditional expectation provides the average transmit power with the OpCSMA:

$$
P_o = \pi_0 + \sum_{n=1}^{\infty}\pi_n \frac{np}{1-q^n}\sum_{k=0}^{k_m} q^{kn}(1-q^n)P_t(k). \qquad (12)
$$

Using (12), Proposition 3 provides the analytic result of the average transmit power under the OpCSMA.

Proposition 3: Employing the OpCSMA, the average transmission power consumption becomes

$$
\begin{aligned}
P_o &= e^{-\lambda} + \qquad (13) \\
&\lambda e^{-\lambda}\sum_{k=0}^{k_m} e^{\lambda q^k}\frac{E_1\left(-\ln(1-q^{k+1})\right) - E_1\left(-\ln(1-q^k)\right)}{E_1(-\ln(1-p_o))}.
\end{aligned}
$$

Proof: Substitute (11) in (12). Then, change the order of summation. Finally, apply $\sum_{n=0}^{\infty}\frac{a^n}{n!} = e^a$ to obtain (13) ■

(6) can be obtained from (12) by replacing $P_t(k)$ by 1 after applying (11) to (12). For small k, $P_t(k) < 1$ since only MTs with sufficiently large channel gains transmit packets. Note that the summation interval is finite in (13) because there is no transmission for $k > k_m$ where $P_t(k) > 1$. Thus, average transmit power consumption of the OpCSMA (13) is smaller than that of the p-persistent CSMA (6).

As p approaches 0, the collision probability also approaches 0 because only one MT accesses the wireless medium with high probability. Disregarding the wasted time owing to the back-off, the transmit power consumption of p-persistent CSMA network as $p \to 0$ closely approximates that of a centrally-controlled network. The limit of P_o as $p \to 0$ is provided in Proposition 4.

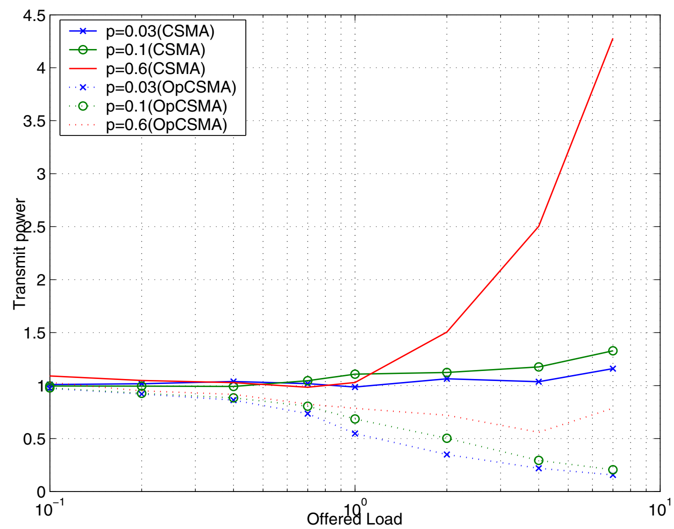

Fig. 3. Power consumption comparisons via simulations when a=0.01 and average SNR = 0dB.

Proposition 4: The limit of P_o as $p \to 0$ is

$$
\begin{aligned}
\lim_{p\to 0} P_o &= e^{-\lambda} + \frac{\lambda e^{-\lambda}(E_1(\lambda p_o) - E_1(\lambda))}{E_1(p_o)} \qquad (14) \\
&+ \sum_{n=1}^{\infty}\frac{(-1)^{n-1}\lambda^n E_1(n\ln\lambda)}{E_1(p_o)(n-1)!}
\end{aligned}
$$

Proof: See Appendix. ■

Since (14) is a decreasing function of λ, it is noted that the average transmit power consumption diminishes as more packets arrive to the network. The average transmit power consumption of conventional p-persistent CSMA network, however, approaches 1 as $p \to 0$; therefore 14 shows that the multiuser diversity provides significant power saving at small p.

V. Numerical Results

The transmit power consumption of p-persistent CSMA and the OpCSMA is evaluated through simulations in a Rayleigh fading channel. Also, the simulation results are compared to the analytic results. The target outage probability p_o during power control is set to be 2%; the cutoff fade depth is set using (2), accordingly. The slot length a is 0.01 and the average SNR of fading channel is set as 0dB. However, the average transmit power consumption is unaffected by the average SNR as shown in (6) and (13). Contrary to the analysis, the simulation considers new packet arrivals during ITRDs; as a result simulated transmit power consumption is usually smaller than the analytic result because the channel gains of newly-arrived packets can be even larger than T_{k-1}. In other words, the analytic result is often the upper bound of actual transmit power consumption.

In Fig. 3, the average transmit power consumption of the OpCSMA is compared with that of the p-persistent CSMA. As the offered load G grows beyond 0.4, the OpCSMA starts to provide some power saving over the conventional method.

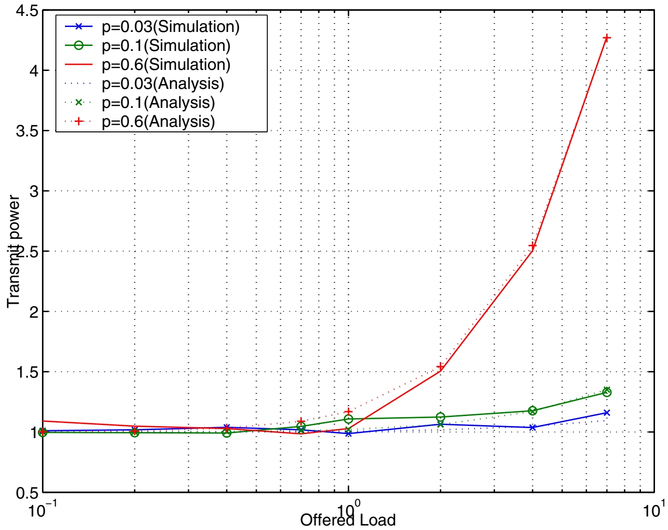

Fig. 4. Comparison of analyzed and simulated power consumption for conventional p-persistent CSMA when a=0.01 and average SNR = 0dB.

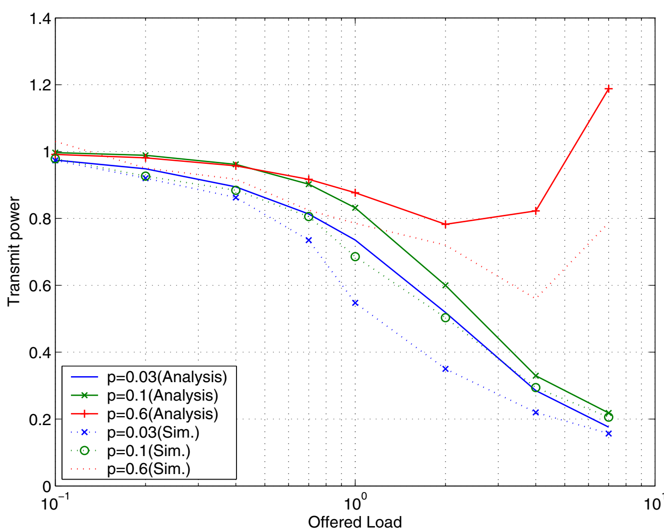

Fig. 5. Comparison of analyzed and simulated power consumption for the OpCSMA when a=0.01 and average SNR = 0dB.

At $G = 0.4$, the gain is about $10\% \sim 15\%$, but it reaches as high as 85% at $G = 7$. Put differently, the OpCSMA consumes only 15% of the power used by the conventional method. As G grows, more collisions occur which increases the average transmit power with the p-persistent CSMA. On the contrary, the transmit power consumption under the OpCSMA is observed to decrease with larger G because the effect of multiuser diversity dominates that of collisions.

The analyzed transmit power consumption of the conventional p-persistent CSMA network is compared to the simulation result in Fig. 4. The analytic result also exhibits increasing power consumption with larger traffic load. For all G and p, the analytic results are very close to the simulation results. On the other hand, with the OpCSMA, the analysis tends to provide a rough upper bound for the simulation results

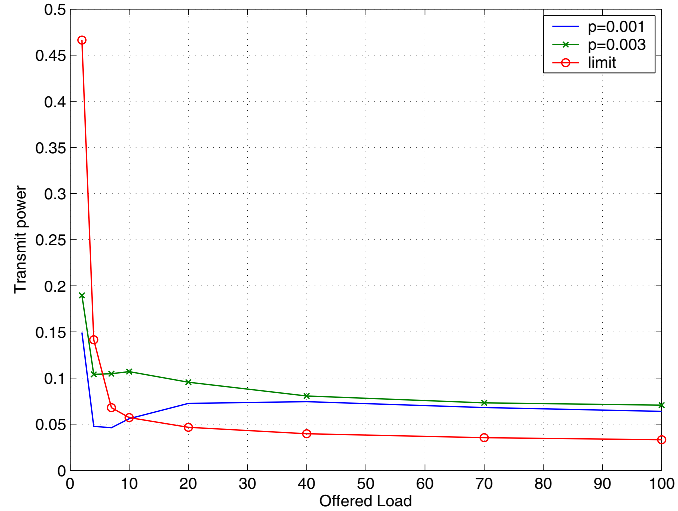

Fig. 6. The analyzed power consumption for the OpCSMA as $p \to 0$.

as shown in Fig. 5. The analytic results, however, become more accurate with smaller p. In addition, the analysis shows that the transmit power consumption under the OpCSMA diminishes as G increases.

As p approaches 0, the collision probability also converges to 0, which suggests that the system works as if a centralized scheduler existed. To gain the insight of multiuser diversity effect when a centralized scheduler is used, the limit of transmit power consumption as $p \to 0$ is compared to analytic results in Fig. 6. The limit shows that the transmit power saving reaches up to 96% with large G. However, there is no noticeable power reduction beyond $G = 20$ because the effect of higher collision probability cancels out the multiuser diversity effect.

VI. Conclusion

This paper proposes a variant of p-persistent CSMA called Opportunistic p-persistent CSMA (OpCSMA) that reduces the average transmit power consumption by utilizing multiuser diversity in a slow fading channel. Under the infinite user model, the analysis and simulation results show that the OpCSMA significantly saves the transmit power compared with the p-persistent CSMA. At small p, the required transmit power for the OpCSMA diminishes as the traffic load G grows; whereas the p-persistent CSMA requires more transmit power for larger G. The limit of transmit power consumption as $p \to 0$ approximates the maximum power saving in the presence of a central scheduler, which is shown to exceed 90%.

The proposed method is a promising MAC protocol for 802.11 WLAN since its MAC protocol is well approximated by p-persistent CSMA as well as the typical WLAN channel is slowly varying. Using the OpCSMA, the probability of accessing the wireless medium is still equal to p. Therefore, the MTs using the OpCSMA may be deployed without affecting MTs with conventional p-persistent CSMA. In practice, if the channels are not i.i.d., MTs with good channel condition tend to have higher probability of utilizing the wireless medium, so the OpCSMA needs to take into account the fairness issues.

APPENDIX
PROOF OF PROPOSITION 1

Applying (5) to (4), P_c is expressed as

$$P_c = \pi_0 + \sum_{n=1}^{\infty} \pi_n \frac{np}{1-q^n}. \tag{15}$$

Since $\pi_n = e^{-\lambda}\frac{\lambda^n}{n!}$, the transmit power consumption is

$$P_c = e^{-\lambda} + \lambda p \sum_{n=1}^{\infty} e^{-\lambda} \frac{\lambda^{n-1}}{(n-1)!(1-q^n)}. \tag{16}$$

To simplify (16), it is multiplied by $1 = \sum_{k=0}^{-\infty} q^{kn}(1-q^n)$, the order of summation is exchanged, and then $e^a = \sum_{m=0}^{\infty} \frac{a^m}{m!}$ is used to obtain

$$P_c = e^{-\lambda}\left(1 + \lambda p \sum_{k=0}^{\infty} q^k e^{\lambda q^k}\right). \tag{17}$$

APPENDIX
PROOF OF PROPOSITION 4

By applying a continuous approximation to the summation over k in (13), the summation over k part is given by

$$\int_0^{k_m} e^{\lambda q^x}\left(E_a(-\ln(1-q^{x+1})) - E_1(-\ln(1-q^x))\right)dx. \tag{18}$$

Let $y = q^x$, the integral after change of variables is

$$\int_1^{\frac{p_o}{q}} \frac{e^{\lambda y}}{y \ln q}\left(E_a(-\ln(1-qy)) - E_1(-\ln(1-y))\right)dy. \tag{19}$$

The series expansion of exponential integral function is $E_1(x) = -\gamma - \ln x - \sum_{n=1}^{\infty}\frac{(-1)^n x^n}{n!n}$ where $\gamma = \lim_{n\to\infty}(\sum_{k=1}^{n}\frac{1}{k} - \ln n)$ denotes *Euler-Mascheroni* constant. Also, $-\ln(1-y)$ can be approximated as y for small y. Applying this series expansion and approximation to (19) and then changing the order of integration and summation yields

$$\int_{\frac{p_o}{q}}^{1} \frac{e^{\lambda y}}{y}dy + \sum_{n=1}^{\infty}\frac{(-1)^n(q^n-1)}{n!n\ln q}\int_{\frac{p_o}{q}}^{1} y^{n-1}e^{\lambda y}dy. \tag{20}$$

(20) can be simplified using Lemma 1.

Lemma 1 :

$$\int_c^1 y^{n-1}e^{\lambda y}dy = \frac{e^{\lambda}}{\lambda}\sum_{k=0}^{n-1}\frac{(-1)^k}{\lambda^k(n-k-1)!} \tag{21}$$

$$-\frac{e^{\lambda\frac{p_o}{q}}}{\lambda}\sum_{k=0}^{n-1}\frac{(-1)^k}{\lambda^k(n-k-1)!}\left(\frac{p_o}{q}\right)^{n-1-k}. \tag{22}$$

Proof: Omitted owing to page limit. ■

Since $e^{\lambda} \gg e^{\lambda\frac{p_o}{q}}$ for small p, the second summation in Lemma 1 can be dropped. Applying Lemma 1 to (20) yields

$$E_1\left(\frac{\lambda p_o}{q}\right) - E_1(\lambda) + \frac{e^{\lambda}}{\lambda\ln q}\sum_{n=1}^{\infty}\frac{(-1)^n(q^n-1)}{n^2}\sum_{k=0}^{n-1}\frac{(-1)^k}{\lambda^k(n-k-1)!}. \tag{23}$$

Approximating $q^n - 1 \approx -np$ for $p < 0.1$, and then rearranging the summation of (23) yields.

$$E_1\left(\frac{\lambda p_o}{q}\right) - E_1(\lambda) + \frac{e^{\lambda}}{\lambda\ln q}\sum_{k=0}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^k}{\lambda^k(n+k)(n-1)!}. \tag{24}$$

Using continuous approximation of summation over k, the transmit power consumption (13) is

$$P_o = e^{-\lambda} + \frac{\lambda e^{-\lambda}(E_1(\lambda p_o) - E_1(\lambda))}{E_1(p_o)} \tag{25}$$

$$-\frac{p}{\ln q}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\lambda^n E_1(n\ln\lambda)}{E_1(p_o)(n-1)!}.$$

Unlike (13), the terms related to p is outside of any integration or summation; thus the limit as $p \to 0$ can be readily applied to (25). Since $\lim_{p\to 0}\frac{p}{\ln q} = -1$, the limit of the transmit power consumption becomes (14).

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