Optimal Resource Allocation for OFDMA Downlink Systems

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Abstract— **This paper proposes efficient rate and power allocation algorithms for OFDMA downlink systems where each tone is taken by at most one user. Weighted sum rate maximization (WSRmax) and weighted sum power minimization (WSPmin) problems are considered. Since these resource allocation problems are non-convex, complexity of finding the optimal solutions is prohibitively high. This paper employs the Lagrange dual decomposition method to efficiently solve both optimization problems. Because of their non-convex nature, there is no guarantee for the solution obtained by the dual decomposition method to be optimal. However, it is shown that with practical number of tones, the duality gap is virtually zero and the optimal solutions can be efficiently obtained.**

I. Introduction

Successful deployment of discrete multi-tone (DMT) systems in DSL area has proven advantages of multi-tone systems over systems with a single tone. One of the major advantages in DMT systems is the high throughput achieved by optimal rate and power allocation over the tones [1]. With dramatically increasing demand in high data rate services, Orthogonal Frequency Division Multiplexing (OFDM) has drawn much attention as a promising modulation technique for the next generation wireless communication systems. In particular, OFDM-based Frequency Division Multiple Access (OFDMA) system, which assigns each tone to at most one user, has been widely applied and studied [2]-[4]. With perfect channel state information (CSI), the transmitter of OFDMA systems can dynamically allocate power and rate on each tone to satisfy various QoS (quality of service) of each user, which is essential in multi-user communication systems [5].

In this paper, efficient rate and power allocation algorithms are proposed for OFDMA broadcast channels (BC). Two resource allocation problems are considered: weighted sum rate maximization (WSRmax) and weighted sum power minimization (WSPmin). These are non-convex problems since it is required to find optimal set of tones for each user. This set selection is a combinatorial problem whose complexity increases exponentially with the number of tones [2]. To find efficient suboptimal algorithms, many works have considered convex relaxation methods by introducing time-sharing or frequency-sharing variables [3][4]. However, in order to find solutions, this approach employs a different system model from the original OFDMA system. Thus, it eventually requires a heuristic approximation that might lead to a significant error.

On the other hand, Yu and Lui [6] showed that in multicarrier applications, even though the original resource allocation problems are non-convex, the duality gap becomes zero as the number of tones goes to infinity. Therefore, with large number of tones, Lagrange dual decomposition method can be used to find the optimal solutions accurately. This argument is based on the fact that if the optimal value of an optimization problem is a concave (or convex) function of the constraint vector, duality gap vanishes regardless of convexity of the original problem. In this paper, motivated by this result, OFDMA downlink resource allocation problems are solved in the dual domain by using Lagrange dual decomposition, and efficient algorithms are developed for WSRmax and WSPmin problems. Existing duality gap is actually evaluated, and the results show that with practical number of tones, the optimal objective is virtually concave in terms of the constraint vector, which validates the proposed dual approach.

The organization of this paper is as follows: Section II presents the system model and problem formulation. In Section III, a general theory on the duality gap of non-convex optimization problems is introduced, and duality gap of OFDMA problems is closely investigated. Section IV presents efficient resource allocation algorithms for OFDMA downlink systems. Numerical results are discussed in Section V, and Section VI provides concluding remarks.

Notation: Vectors are bold-faced, and $\mathbb{R}^n$ denotes the set of real n-vectors.

II. System Model and Problem Formulation

In this section, OFDMA downlink system model is described and two resource allocation problems are mathematically formulated. Consider a downlink transmission system with K users and N tones where the base-station (BS) and each user are equipped with a single antenna. It is assumed that the inter-symbol interference (ISI) is completely removed by exploiting OFDM techniques, i.e. the frequency response is flat within each tone. The total transmit power is constrained to P_{tot}. For user k on tone n, the channel gain is denoted by $H_k(n)$, and a zero-mean independent and identically distributed (i.i.d.) Gaussian noise with variance $\sigma_k^2(n)$ is added at the receiver part. The channel signal-to-noise ratio (SNR)

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for user k on tone n is defined as $c_k(n) = |H_k(n)|^2/\sigma_k^2(n)$. This paper assumes perfect CSI at both BS and each user, which enables BS to dynamically allocate power and rate on each tone according to channel conditions. Let S_i denote the set of tones allocated to user i. Each tone is allowed to be used by at most one user; hence, $S_i \cap S_j = \emptyset$ for $i \neq j$ and $\bigcup_{i=1}^{K} S_i \subseteq \{1, 2, \cdots, N\}$. The transmitter finds S_i for all $i = 1, \cdots, K$ and distributes power such that the objective of resource allocation is satisfied.

Let $r_k(n)$ and $p_k(n)$ denote the rate and the power of user k on tone n such that $r_k(n) = 0.5 \log_2(1+p_k(n)c_k(n))$ bits per dimension. Then, the WSRmax problem can be formulated as

$$\begin{aligned} \text{maximize} \quad & \sum_{k=1}^{K} \mu_k \sum_{n \in S_k} r_k(n) \\ \text{subject to} \quad & \sum_{k=1}^{K} \sum_{n \in S_k} p_k(n) \leq P_{tot}, \\ & S_i \cap S_j = \emptyset \quad \forall i \neq j, \\ & \bigcup_{k=1}^{K} S_k \subseteq \{1, 2, \cdots, N\}, \\ & p_k(n) \geq 0 \quad \forall k \quad \text{and} \quad \forall n, \end{aligned} \tag{1}$$

where $\mu_k \geq 0$ is the weight assigned to user k. Given the weight vector and the channel gains, this problem finds the power allocation that maximizes the weighted sum rate with total power constraint. The boundary of the achievable rate region can be traced by solving this problem for all possible weight vectors. In general, (1) is not a convex optimization problem since it needs to find the optimal set of tones for each user, which is a combinatorial problem whose complexity increases exponentially with N. This argument also holds for the WSPmin problem shown below.

The WSPmin is a dual problem of WSRmax that can be represented as

$$\begin{aligned} \text{minimize} \quad & \sum_{k=1}^{K} \lambda_k \sum_{n \in S_k} p_k(n) \\ \text{subject to} \quad & \sum_{n \in S_k} r_k(n) \geq R_k \quad \forall k, \\ & S_i \cap S_j = \emptyset \quad \forall i \neq j, \\ & \bigcup_{k=1}^{K} S_k \subseteq \{1, 2, \cdots, N\}, \\ & p_k(n) \geq 0 \quad \forall k \quad \text{and} \quad \forall n, \end{aligned} \tag{2}$$

where $\lambda_k \geq 0$ is the weight assigned to user k. Given the weight vector and the channel gains, this problem finds a power distribution that minimizes the weighted sum power with minimum rate constraint on each user. In the downlink, the inter-cell interferences can be reduced by solving the WSPmin problem. This problem is of particular interest in the uplink case since the battery life of the mobile terminal is critical. Because of the FDMA nature of OFDMA systems, the optimal solution of WSPmin in BC is equivalent to that in its dual MAC (Multiple Access Channel) where the role of transmitter and receivers in BC is reversed.

The next section shows that the duality gap for each of the aforementioned non-convex problems is virtually negligible with realistic number of tones, which makes it possible to develop efficient algorithms by using Lagrange dual decomposition.

III. Duality Gap of Non-Convex Optimization in OFDMA Downlink Systems

This section consists of two parts. The first part introduces the general theory on the duality gap in non-convex optimization problems. The second part evaluates and investigates the duality gap for resource allocation problems in OFDMA BC.

A. General theory on duality gap

This subsection introduces some conditions under which the duality gap is zero for general non-convex optimization problems in multi-tone systems. With N tones and K users, the optimization problem has the following general form.

$$\begin{aligned} \text{maximize} \quad & \sum_{n=1}^{N} f_n(\mathbf{x}_n) \\ \text{subject to} \quad & \sum_{n=1}^{N} \mathbf{h}_n(\mathbf{x}_n) \preceq \mathbf{P}, \end{aligned} \tag{3}$$

where $\mathbf{x}_n \in \mathbb{R}^K$ are vectors of optimization variables, $f_n(\cdot)$ are $\mathbb{R}^K \to \mathbb{R}$ functions which are not necessarily concave, and $\mathbf{h}_n(\cdot)$ are $\mathbb{R}^K \to \mathbb{R}^L$ functions also not necessarily convex. Constant $\mathbf{P}$ is an L-vector of constraints. The Lagrangian of (3) is defined as

$$\mathcal{L}(\{\mathbf{x}_n\}, \boldsymbol{\lambda}) = \sum_{n=1}^{N} f_n(\mathbf{x}_n) + \boldsymbol{\lambda}^T \left(\mathbf{P} - \sum_{n=1}^{N} \mathbf{h}_n(\mathbf{x}_n) \right), \tag{4}$$

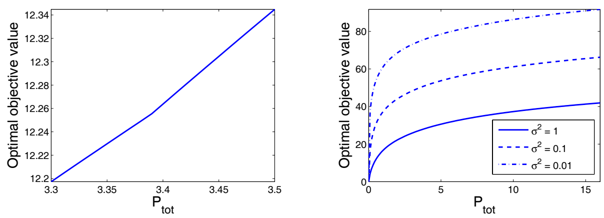
where $\boldsymbol{\lambda}$ is a vector of Lagrange dual variables. The dual objective $g(\boldsymbol{\lambda})$ is defined as an unconstrained maximization of the Lagrangian such that $g(\boldsymbol{\lambda}) = \max_{\{\mathbf{x}_n\}} \mathcal{L}(\{\mathbf{x}_n\}, \boldsymbol{\lambda})$. Then, the dual optimization problem becomes

$$\begin{aligned} \text{minimize} \quad & g(\boldsymbol{\lambda}) \\ \text{subject to} \quad & \boldsymbol{\lambda} \succeq 0. \end{aligned} \tag{5}$$

From duality theory, $g^* \geq f^*$ where f^* and g^* are primal and dual optimal values, respectively. The duality gap d^* is defined as $d^* = g^* - f^*$. When $f_n(\mathbf{x}_n)$'s are concave and $\mathbf{h}_n(\mathbf{x}_n)$'s are convex, (4) is a convex optimization problem, which guarantees zero duality gap. Zero duality gap implies that the globally optimal solution can be obtained by using Lagrange dual decomposition. Though the above optimization problem in (4) is non-convex, duality gap is zero if either of the following two conditions is satisfied [6][7].

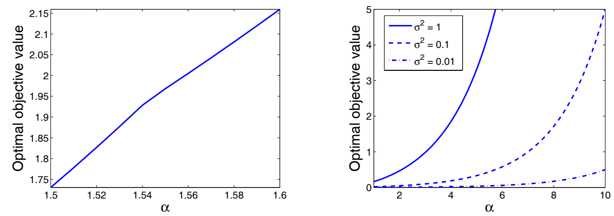
Theorem 1: If $\mathbf{x}_n^*(\boldsymbol{\lambda}) = \arg\max_{\mathbf{x}_n} \mathcal{L}(\{\mathbf{x}_n\}, \boldsymbol{\lambda})$, as a function of $\boldsymbol{\lambda}$, is continuous at $\boldsymbol{\lambda}^*$, the duality gap equals zero.

Theorem 2: Concavity of the optimal $\Sigma_n f_n$ in $\mathbf{P}$ implies zero duality gap.



(a) $N = 2$ (b) $N = 8$

Fig. 1. Maximum weighted sum rate in OFDMA BC versus P_{tot} ($K = 2$, $\boldsymbol{\mu} = [1 \;\; 2]^T$ and channel SNR vectors are (a) $[10 \;\; 160]^T$ and $[160 \;\; 10]^T$ (b) $\frac{1}{\sigma^2}[1^2 \; 2^2 \; \cdots \; N^2]^T$ and $\frac{1}{\sigma^2}[N^2 \; (N-1)^2 \; \cdots \; 1^2]^T$)



(a) $N = 2$ (b) $N = 8$

Fig. 2. Minimum weighted sum power in OFDMA BC versus rate constraints ($K = 2$, $\boldsymbol{\lambda} = [1 \;\; 2]^T, \mathbf{R} = \alpha[2 \;\; 1]^T$ (bits/symbol) and channel SNR vectors are (a) $[40 \;\; 160]^T$ and $[10 \;\; 90]^T$ (b) The same as those defined in Fig. 1(b))

The condition in Theorem 1 is sufficient for that in Theorem 2 but the converse is not always true. Recently, [6] shows that in non-convex multi-carrier optimization problems with the general form of (4), the concavity condition in Theorem 2 is always satisfied when the number of tones goes to infinity. However, existing duality gap for a problem with practical number of tones cannot be estimated from this argument. In the next subsection, duality gap of OFDMA BC resource allocation problems is closely investigated.

B. Duality gap for WSRmax and WSPmin in OFDMA BC

First, consider the WSRmax problem given in (1). For any fixed subchannel assignments S_k, the optimal solution of this problem can be obtained by multilevel water-filling [2] that is given as follows.

$$p_k(n) = \left(\mu_k K - \frac{1}{c_k(n)}\right)^+, \quad \forall n, k, \tag{6}$$

$$K = \frac{P_{tot} + \sum_{k=1}^{K} \sum_{n \in \{S_k : p_k(n) > 0\}} \frac{1}{c_k(n)}}{\sum_{k=1}^{K} \mu_k |S_k|}. \tag{7}$$

Finding optimal subchannel assignment requires K^N searches. Hence, this overall optimization requires $\mathcal{O}(NK^N)$ operations, which is exponentially complex. If all users have equal weights, this is a sum-rate maximization problem where the optimal set selection is to assign each subchannel to the user with the highest channel SNR, and the optimal power allocation is a single level water-filling over this optimal set [8]. Since the optimal subchannel allocation is unique and a change in total power only affects the waterlevel, the optimal power allocation continuously changes as the waterlevel varies, which satisfies the condition in Theorem 1.

However, for the general case where weights are not equal, the optimal subchannel allocation can also change as total power varies, which may destroy the concavity of the optimal objective function in terms of total power. A simple example for this argument is illustrated in Fig. 1(a) when $N = 2$, $K = 2$, $\boldsymbol{\mu} = [1 \;\; 2]^T$ and user 1 and 2's channel SNR vectors are $[10 \;\; 160]^T$ and $[160 \;\; 10]^T$. The maximum weighted sum rate is plotted for $P_{tot} = 3.3 \sim 3.5$. At $P_{tot} = 3.39$, optimal subchannel assignment changes from $S_1 = \{2\}, S_2 = \{1\}$ to $S_1 = \emptyset, S_2 = \{1, 2\}$. Since the waterlevel is different for each user, discrete change in the slope occurs at the transition point, which is observed to break down the concavity at $P_{tot} = 3.39$. With the same subchannel allocation, optimal weighted sum rate is concave in total power, but whenever the optimal set of tones changes, sudden jump in the slope appears, which might make the curve non-concave with that total power. As the number of tones increases, changes in optimal subchannel allocation occur more frequently when the sum power varies. On the other hand, the amount of discrete slope change tends to decrease with more tones since the bandwidth affected by each set change becomes narrower. Therefore, the overall curve is expected to be more concave as the number of tones increases. Fig. 1(b) illustrates maximum weighted sum rate versus total power when $N = 8$, $K = 2$, $\boldsymbol{\mu} = [1 \;\; 2]^T$, and user 1 and 2's channel SNR vectors are $\frac{1}{\sigma^2}[1^2 \; 2^2 \; \cdots \; N^2]^T$ and $\frac{1}{\sigma^2}[N^2 \; (N-1)^2 \; \cdots \; 1^2]^T$. σ^2 denotes the noise power at each tone. As P_{tot} sweeps from 0 to 16, changes in optimal subchannel allocation occur at least five times on each of three plots in Fig. 1(b). However, discrete slope changes are almost undetectable in this figure. Thus, in practical OFDMA downlink systems with much more than 8 tones, duality gap of WSRmax in OFDMA BC is expected to be virtually zero and the optimal solution can be derived in the dual domain.

The optimal solution for the WSPmin problem formulated in (2) is achieved by the following steps: First, choose a subchannel assignment, and for each user, distribute enough power over its assigned tones in a water-filling fashion to satisfy its rate constraint R_k. Second, after iterating the first step for all K^N possible set selections, pick one of them which minimizes the weighted sum power (WSP). Therefore, WSPmin also requires $\mathcal{O}(NK^N)$ operations. If the optimal WSP turns out to be a convex function of the constraint vector $\mathbf{R}$, the duality gap will be zero from the condition in Theorem 2. Similar to WSRmax case, Fig. 2(a) shows that the convexity of the minimum WSP may break down when the optimal subchannel assignment changes. In this figure, $N = 2$, $K = 2$, $\boldsymbol{\lambda} = [1 \;\; 2]^T$, the rate constraint vector is $\mathbf{R} = \alpha[2 \;\; 1]^T$ (bits/symbol), and user 1 and 2's channel SNR vectors are $[40 \;\; 160]^T$ and $[10 \;\; 90]^T$. When α varies from 1.5 to 1.6, the optimal subchannel allocation changes from $S_1 = \{1\}, S_2 = \{2\}$ to $S_1 = \{2\}, S_2 = \{1\}$. This change causes sudden jump in the slope of the curve, which results in non-convexity at this transition point of $\alpha = 1.54$.

However, from the same argument provided in WSRmax case, the amount of slope change decreases when the number of tones rises as demonstrated in Fig. 2(b) where $N = 8$, $K = 2$, $\boldsymbol{\lambda} = [1 \;\; 2]^T, \mathbf{R} = \alpha[2 \;\; 1]^T$ (bits/symbol), and channel SNR vectors are the same as those defined in Fig. 1(b). When α sweeps from 1 to 10, discrete slope changes seem to be negligible in this figure. Therefore, in practice, the WSPmin problem in OFDMA downlink systems can be solved in the dual domain with much less computational complexity.

Based on the results in this section, Lagrange dual decomposition can be used to derive efficient algorithms for both WSRmax and WSPmin problems as shown in the following section.

IV. Efficient resource allocation algorithms

The Lagrangian of WSRmax problem in (1) is defined over domain $\mathcal{D}$ as

$$\mathcal{L}(\{p_k(n)\}, \{r_k(n)\}, \lambda) = \sum_{k=1}^{K} \mu_k \sum_{n=1}^{N} r_k(n) - \lambda \left(\sum_{k=1}^{K} \sum_{n=1}^{N} p_k(n) - P_{tot} \right), \quad (8)$$

where the domain $\mathcal{D}$ is defined as the set of all non-negative $p_k(n)$'s for $k = 1, \cdots, K$ and $n = 1, \cdots, N$ such that for each n, only one $p_k(n)$ is positive for $k = 1, \cdots, K$ (FDMA constraint). Then, the Largrange dual function is

$$g(\lambda) = \max_{\{p_k(n)\}, \{r_k(n)\} \in \mathcal{D}} \mathcal{L}(\{p_k(n)\}, \{r_k(n)\}, \lambda). \quad (9)$$

(8) suggests that the maximization of $\mathcal{L}$ can be decomposed into the following N independent optimization problems

$$g'_n(\lambda) = \max_{\{p_k(n)\} \in \mathcal{D}} \left\{ \sum_{k=1}^{K} \mu_k r_k(n) - \lambda \sum_{k=1}^{K} p_k(n) \right\} \quad (10)$$

for $n = 1, \cdots, N$. Then, the Lagrange dual function becomes

$$g(\lambda) = \sum_{n=1}^{N} g'_n(\lambda) + \lambda P_{tot}. \quad (11)$$

Assume user k is active on tone n. With a fixed λ, the object of max operation in (10) is a concave function of $p_k(n)$. By taking the derivative of this object regarding $p_k(n)$, the next optimality condition is obtained, which maximizes $g'_n(\lambda)$.

$$p_k(n) = \left(K_k - \frac{1}{c_k(n)} \right)^+, \quad (12)$$

where $K_k = \mu_k / (2 \log 2 \cdot \lambda)$. By searching over all K possible user assignments for tone n, $g'_n(\lambda)$ can be obtained as

$$g'_n(\lambda) = \max_k \left\{ \frac{\mu_k}{2} \log_2 \left(1 + \left(K_k - \frac{1}{c_k(n)} \right)^+ c_k(n) \right) - \lambda \left(K_k - \frac{1}{c_k(n)} \right)^+ \right\}, \quad n = 1, 2, \cdots, N. \quad (13)$$

Once above equation is solved for all n, the overall Lagrange dual function $g(\lambda)$ is derived from (11). Finally, it is required to find $\lambda^* \geq 0$ that minimizes $g(\lambda)$. The update of λ can be done by using a simple bisection method until the sum power converges [9]. Hence, $\mathcal{O}(NK)$ executions are required to find the optimal solution, which shows linear complexity of the proposed algorithm in N. If the converged sum power is equal to the total power constraint, the duality gap is zero and this solution is in fact globally optimal. From (13), the user selection at tone n can change at some level of λ where a quantum leap may occur in the sum power. Thus, if P_{tot} is within this gap, the sum power cannot converge to P_{tot} by using above bisection method on λ. However, the previous section shows that the duality gap quickly vanishes as the number of tones increases, and the solution obtained in the dual domain becomes a globally optimal solution. Therefore, the subchannel assignment at $\lambda = \lambda^*$ can be assumed to be optimal, and the global optimal solution can be found by doing multilevel water-filling with this set.

Similarly, the WSPmin problem can be also solved by using dual decomposition. The Lagrangian of WSPmin problem in (2) is defined over domain $\mathcal{D}$ as

$$\mathcal{L}(\{p_k(n)\}, \{r_k(n)\}, \boldsymbol{\mu}) = \sum_{k=1}^{K} \lambda_k \sum_{n=1}^{N} p_k(n) - \sum_{k=1}^{K} \mu_k \left(\sum_{n=1}^{N} r_k(n) - R_k \right). \quad (14)$$

Then, the Lagrange dual function is represented as

$$g(\boldsymbol{\mu}) = \min_{\{p_k(n)\}, \{r_k(n)\} \in \mathcal{D}} \mathcal{L}(\{p_k(n)\}, \{r_k(n)\}, \boldsymbol{\mu}). \quad (15)$$

From (14), the minimization of $\mathcal{L}$ can be decomposed into N independent optimization problems as follows

$$g'_n(\boldsymbol{\mu}) = \min_{\{p_k(n)\} \in \mathcal{D}} \left\{ \sum_{k=1}^{K} \lambda_k p_k(n) - \sum_{k=1}^{K} \mu_k r_k(n) \right\}, \quad (16)$$

for $n = 1, \cdots, N$. Thus, the Lagrange dual function is

$$g(\boldsymbol{\mu}) = \sum_{n=1}^{N} g'_n(\boldsymbol{\mu}) + \sum_{k=1}^{K} \mu_k R_k. \quad (17)$$

With a fixed $\boldsymbol{\mu}$, the object of min operation in (16) is a convex function of $p_k(n)$. Hence, taking the derivative of this object regarding $p_k(n)$ results in the following condition, which minimizes $g'_n(\boldsymbol{\mu})$.

$$p_k(n) = \left(M_k - \frac{1}{c_k(n)} \right)^+, \quad (18)$$

where $M_k = \mu_k / (2 \log 2 \cdot \lambda_k)$. By searching over all K possible user assignments for tone n, $g'_n(\boldsymbol{\mu})$ is obtained as

$$g'_n(\boldsymbol{\mu}) = \min_k \left\{ \lambda_k \left(M_k - \frac{1}{c_k(n)} \right)^+ - \frac{\mu_k}{2} \log_2 \left(1 + \left(M_k - \frac{1}{c_k(n)} \right)^+ c_k(n) \right) \right\}, \quad (19)$$

for $n = 1, \cdots, N$. After solving (19) for all n, $g(\boldsymbol{\mu})$ is derived from (17). In order to find $\boldsymbol{\mu}^* \succeq 0$ that maximizes $g(\boldsymbol{\mu})$, the update of $\boldsymbol{\mu}$ can be efficiently done by using the ellipsoid method until every user's rate converges [6][10]. Ellipsoid method converges in $\mathcal{O}(n^2)$ iterations where n is the number of variables [9]. A sub-gradient of this problem required for ellipsoid method is provided in the following proposition.

Proposition 1: For the WSPmin problem with a dual objective $g(\boldsymbol{\mu})$ defined in (15), the following choice of $\mathbf{d}$ is a subgradient for $g(\boldsymbol{\mu})$:

$$d_k = R_k - \sum_{n=1}^{N} r_k^*(n) \quad k = 1, \cdots, K, \tag{20}$$

where $\{r_k^*(n)\}$ and $\{p_k^*(n)\}$ optimize the minimization problem in the definition of $g(\boldsymbol{\mu})$.

Proof: Since $\{r_k^*(n)\}$ and $\{p_k^*(n)\}$ are already in $\mathcal{D}$, for any $\boldsymbol{\delta} \succeq 0$,

$$\begin{aligned} g(\boldsymbol{\delta}) &\leq \mathcal{L}(\{p_k^*(n)\}, \{r_k^*(n)\}, \boldsymbol{\delta}) \\ &= g(\boldsymbol{\mu}) + \sum_{k=1}^{K} (\delta_k - \mu_k) \left(R_k - \sum_{n=1}^{N} r_k^*(n) \right). \end{aligned} \tag{21}$$

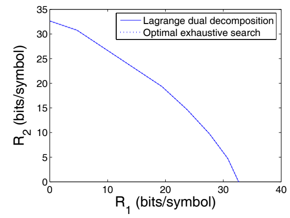
■

The overall optimization needs $\mathcal{O}(K^2)$ runs of optimization problem with complexity of $\mathcal{O}(NK)$. Hence, $\mathcal{O}(NK^3)$ executions are required to find the optimal solution of WSPmin by using the proposed algorithm. As discussed in WSRmax case, the discontinuity in power allocation can happen at $\boldsymbol{\mu}^*$ for the WSPmin problem as well. In this situation, find the subchannel assignment at $\boldsymbol{\mu}^*$, and allocate power in a water-filling fashion to satisfy rate constraint for each user. By doing so, the global optimal solution is derived from a dual domain in OFDMA downlink systems.

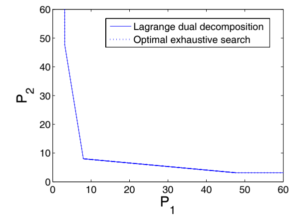
V. Numerical results and discussion

This section provides some simulation results generated by using proposed efficient resource allocation algorithms for OFDMA downlink systems. Fig. 3 shows achievable rate and power regions of OFDMA BC with $N = 8$ and $K = 2$. The user 1 and 2's channel SNR vectors are $10[1^2 \ 2^2 \ \cdots \ N^2]^T$ and $10[N^2 \ (N-1)^2 \ \cdots \ 1^2]^T$, respectively. Each region is generated by using both optimal exhaustive search and Lagrange dual decomposition methods. Fig. 3(a) illustrates the achievable rate region when $P_{tot} = NK = 16$. The boundary points are characterized by solving WSRmax for all possible $\boldsymbol{\mu}$. In this figure, the rate region obtained by employing Lagrange dual decomposition is indistinguishable from the optimal rate region, which implies zero duality gap in this case.

Fig. 3(b) shows the achievable power region when the target rate vector $\mathbf{R} = [19.36 \ \ 19.36]^T$ bits per symbol. The boundary points are characterized by solving WSPmin for all possible $\boldsymbol{\lambda}$. As in Fig. 3(a), Lagrange dual decomposition produces the optimal power region. Since the target rate vector lies on the boundary of rate region in Fig. 3(a), the minimum sum power to achieve this rate vector must equal $P_{tot} = 16$, which can be checked in Fig. 3(b). The results in this section suggest that in practical OFDMA downlink systems with much more than eight tones, the proposed dual approach can find optimal solutions with significantly lower computational complexity than the optimal exhaustive search.



(a) Rate region



(b) Power region

Fig. 3. Optimal exhaustive search versus Lagrange dual decomposition methods ($N = 8$, $K = 2$, channel SNR vectors are $10[1^2 \ 2^2 \ \cdots \ N^2]^T$ and $10[N^2 \ (N-1)^2 \ \cdots \ 1^2]^T$. In (a), $P_{tot} = NK = 16$ and in (b), $\mathbf{R} = [19.36 \ \ 19.36]^T$ (bits/symbol))

VI. Conclusion

In OFDMA downlink systems, efficient algorithms are developed for weighted sum rate maximization and weighted sum power minimization problems. Though these are originally non-convex problems with exponential complexity, the duality gap of each problem is shown to quickly vanish as the number of tones increases. From this observation, Lagrange dual decomposition method is employed to efficiently solve both problems. Simulation results show that with only eight tones, virtually optimal solutions are obtained by using the proposed dual approach.

References

[1] T. Starr, M. Sorbara, J.M. Cioffi, and P. Silverman, *DSL Advances*, Prentice Hall, 2003.

[2] L.M.C. Hoo, B. Halder, J. Tellado, and J.M. Cioffi, "Multiuser Transmit Optimization for Multicarrier Broadcast Channels: asymptotic FDMA capacity region and algorithms," *IEEE Trans. Comm.*, vol. 52, no. 6, pp. 922-930, June 2004.

[3] W. Rhee and J.M. Cioffi, "Increase in capacity of multiuser OFDM system using dynamic subchannel allocation," in *Proc. IEEE Vehicular Technology Conf. (VTC 2000)*, Tokyo, Japan, May 2000, pp. 1085-1089.

[4] Y.W. Cheong, R.S. Cheng, K.B. Lataief, and R.D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE J. Select. Areas Commun.*, vol. 17, no. 10, pp. 1747-1758, Oct. 1999.

[5] K. Seong, R. Narasimhan, and J.M. Cioffi, "Queue proportional scheduling via geometric programming in fading broadcast channels," *IEEE J. Select. Areas Comm.*, vol. 24, no. 8, Aug. 2006.

[6] W. Yu and R. Lui, "Dual Methods for Non-Convex Spectrum Optimization of Multi-carrier Systems," accepted in *IEEE Transactions on Communications*, 2005.

[7] M. Chiang, S. Zhang, and P. Hande, "Distributed rate allocation for inelastic flows," submitted to *IEEE/ACM Trans. Networking*, Nov. 2005.

[8] J. Jang and K.B. Lee,"Transmit power adaptation for multiuser OFDM systems," *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 2, pp. 171-178, Feb. 2003.

[9] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2003.

[10] M. Mohseni, R. Zhang, and J.M. Cioffi, "Optimized transmission for fading multiple-access and broadcast channels with mutiple antennas," *IEEE J. Select. Areas Comm.*, vol. 24, no. 8, Aug. 2006.