Wide-band coupling of Earth’s normal modes due to anisotropic inner core structure

J. C. E. Irving, A. Deuss and J. Andrews

University of Cambridge, Institute of Theoretical Geophysics & Bullard Laboratories, Madingley Road, Cambridge CB3 0EZ, UK. E-mail: jcei2@cam.ac.uk

Accepted 2008 April 10. Received 2008 April 7; in original form 2007 December 18

SUMMARY
We investigate the importance of wide-band coupling of normal modes due to inner core anisotropy. We compare four different seismic models of inner core anisotropy, which were obtained by others using the splitting of Earth’s normal modes. These models have been developed using a self-coupling (SC) approximation, which assumes that coupling between nearby modes through anisotropic inner core structure is negligible. We test the SC approximation by comparing the frequencies and quality factors of 90 inner core sensitive modes, computed for these models using either the SC approximation or full-coupling (FC) among large groups of modes. We find significant shifts in the quality factors and frequencies for some modes. Groups of modes which significantly couple together are constructed for six target modes. These groups are model dependent and in some cases contain large numbers of modes. Synthetic seismograms are calculated to show that the difference between SC and FC is observable on the scale of seismograms and of the same order of magnitude as the difference between synthetic and observed seismograms. Thus, future models of inner core anisotropy should take cross-coupling between large groups of modes into account.

Key words: Composition of the core; Surface waves and free oscillations; Seismic anisotropy.

1 INTRODUCTION

1.1 Motivation
It has long been known that Earth’s normal modes are split due to rotation, ellipticity, 3-D heterogeneity and anisotropy (Dahlen & Tromp 1998). A number of modes with significant sensitivity to the core show anomalous splitting which cannot be attributed to ellipticity, rotation or mantle heterogeneity alone. Woodhouse et al. (1986) suggested inner core anisotropy as an explanation for this anomalous splitting of core-sensitive modes; it also provides an explanation for the directional variation in traveltimes of seismic body waves travelling through the inner core (Morelli et al. 1986). The majority of models of the inner core’s anisotropy have been produced using either body wave or normal mode data; see Creager (2000), Tromp (2001) and Song (1997) for recent overviews. While both normal modes and body waves sample the same inner core, there is still significant disagreement between the models which aim to explain either traveltime anomalies in body waves or anomalous splitting of inner core modes, and it has been difficult to reconcile the two data types (Durek & Romanowicz 1999).

Most normal mode studies have used isolated modes and ignored coupling with other modes, that is, the so-called self-coupling approximation (SC). SC also forms the basis for the splitting function technique. First steps in including coupling between different modes have been taken by Resovsky & Ritzwoller (1995, 1998), who coupled a few pairs of modes. The coupling of modes across wide frequency bands (i.e. full-coupling) was investigated by Deuss & Woodhouse (2001) who found that modes could couple across wider frequency bands than had previously been thought and that the impact of this coupling was an observable shift in both the amplitude and phase of synthetic seismograms. The calculations they performed did not include any inner core anisotropy. Andrews et al. (2006) showed that due to anelasticity, wide-band normal mode coupling causes significant changes in mode attenuation and frequency when inner core shear velocity is perturbed. The impact of full-coupling (FC) on mode characteristics when small changes are made to inner core properties illustrates the need to consider FC for inner core structure. The importance of the effect of FC due to inner core anisotropy is still unclear; here we try to quantify this effect.

As several different seismic models of inner core anisotropy exist, a FC treatment of the modes, where large numbers of modes are coupled, could cast some insight into anisotropy of the inner core. It is hoped that underlying similarities between the models could highlight the aspects of the anisotropy about which the models agree. In particular, the modes which couple together may be universal and independent of the model used. These universal groups could allow more rigorous inversions of seismic data, and production of better seismic models of inner core anisotropy and also provide definitive explanations of the anomalous coupling of some normal modes which are still unexplained.
1.2 Inner core anisotropy

The inner core of the Earth is known to be anisotropic: it exhibits different properties in different directions. Most inner core models, including the ones discussed below, assume cylindrical anisotropy, that is, anisotropy which is independent of longitude. There are a variety of different ideas about the dynamic origin of the anisotropy, these have been discussed in detail by several authors (see reviews by Tromp 2001; Song 1997). The crystal structure of the inner core, responsible for the anisotropic behaviour, is also currently unknown. Although ab initio calculations suggest that pure iron is stable in the hexagonal close packed structure under inner core conditions (Stixrude 1995; Vočadlo et al. 2003), the presence of lighter elements in the core may alter the energetic landscape sufficiently that another crystalline structure, such as body-centred cubic, is more stable (Oganov et al. 2005; Vočadlo 2007). Different seismic models suggest different crystal structures so that further knowledge concerning seismic anisotropy in the inner core would provide information about its crystal structure. Until the seismologically anisotropic structure of the Earth’s inner core is known in more detail the cause of the anisotropy and its implications will remain an enigma.

Many different seismic models of inner core anisotropy are available in the literature, developed using either normal mode or body wave data. We are interested in models obtained using normal modes and we investigate the effect of coupling due to inner core anisotropy on four models: those produced by Woodhouse et al. (1986), Tromp (1993), Durek & Romanowicz (1999) and Beghein & Trampert (2003). All of these seismic models can be described using the same parameters: \( \alpha \), \( \beta \) and \( \gamma \) (as in Tromp 1993). \( \alpha \) represents the relative speeds of inner core P-waves travelling along and perpendicular to the Earth’s rotational axis. Similarly, \( \beta \) represents the relative speeds of inner core S-waves travelling along and perpendicular to the Earth’s rotational axis. \( \gamma \) describes the speeds of waves which travel at intermediate angles to the Earth’s rotational axis. These three parameters can be defined in terms of the Love coefficients (Love 1927) of the inner core, so that

\[
\alpha = \frac{C - A}{A_0}, \quad \beta = \frac{L - N}{A_0} \quad \text{and} \quad \gamma = \frac{A - 2N - F}{A_0},
\]

where \( A \) is related to P-wave velocity in the equatorial direction (and \( A_0 \) is the value of \( A \) at the centre of the inner core), \( C \) is related to P-wave velocity in the polar direction, \( L \) is related to S-wave velocity in the polar direction, \( N \) to S-wave velocity in the equatorial direction and \( F \) is dependent on off-axis properties.

The four different models that we compare are shown in Fig. 1. Woodhouse et al. (1986) (W,G&L) created two models of inner core anisotropy by inverting data from seven normal mode multiplets to get estimates of \( \alpha \), \( \beta \) and \( \gamma \). We use the W,G&L model (Fig. 1a) in which \( \alpha \), \( \beta \) and \( \gamma \) are constant throughout the inner core, as this is the most simple model of inner core anisotropy that exists. This simple model was extended by Tromp (1993), who inverted a data set of splitting functions for 18 normal modes to find depth dependent values for \( \alpha \), \( \beta \) and \( \gamma \). The Tr model (Fig. 1b) proposes a cylindrically anisotropic structure with little anisotropic structure at the centre of the core, because the sensitivity kernels of the modes tend to zero towards the centre of the inner core (see Fig. 2 for some examples of sensitivity kernels).

To improve the accuracy of seismic anisotropy models at the centre of the inner core, where normal modes have very little sensitivity, the Durek & Romanowicz (1999) (D&R) model (Fig. 1c) was created by adding body wave observations to the inversion of normal mode data from four shallow and four intermediate to deep
earthquakes (including the 1994 Bolivia event). Data concerning 20 different modes were used in the inversion. The use of body wave data, in addition to normal mode data, resulted in the placement of strong anisotropic structure in the centre of the inner core of the D&R model.

The Beghein & Trampert (2003) (B&T) model (Fig. 1d) was developed using the ‘Neighbourhood Algorithm’ (Sambridge 1999), a specific type of Monte Carlo technique which was used to explore possible inner core anisotropic models using normal mode splitting functions. The sensitivity kernels for normal modes tend towards zero at the centre of the Earth (Fig. 2) but, using the Neighbourhood Algorithm, Beghein & Trampert (2003) suggested an anisotropic structure towards the centre of the inner core, without using any body wave data.

The amplitudes of $\alpha$, $\beta$ and $\gamma$ are larger in the D&R and B&T models than in the W, G&L and Tr models. Whilst the models are in some agreement at the inner core boundary, further towards the centre of the core large discrepancies occur. The differences between the models are greatest there, with $\alpha$, $\beta$ and $\gamma$ varying in both magnitude and sign. These discrepancies can occur because the sensitivity of normal modes to this region is very limited.

2 Theory

2.1 Normal modes

The Earth’s free oscillations, or normal modes, can be observed after large earthquakes. Any arbitrary motion of the Earth can be written as a superposition of these normal modes. There are two types of normal mode, described using the notation $S_l^n$ for spheroidal modes and $T_l^n$ for toroidal modes, where $n$ is the overtone number and $l$ the angular order of the mode. The modes oscillate at discrete frequencies. Toroidal modes involve shear motion of the Earth and do not exist in the fluid outer core. Spheroidal modes are sensitive to both the outer and inner core and will be used in this study. For a spherically symmetric, non-rotating Earth model, each mode consists of a $(2l+1)$-fold degenerate multiplet, whose singlets all have the same frequency. In this case each mode would contribute a single $\delta$-function to the seismogram. The finite observational time window and attenuation by the anelasticity of the Earth mean that modes are not observed as $\delta$-functions but as broadened peaks. The properties of a normal mode singlet can be described by its frequency, $\omega$, and its quality factor, $Q$, which is the reciprocal of the seismic attenuation. A mode with a high $Q$ will have a low attenuation and will be observable long after it has been excited by an earthquake.

Only some modes are sensitive to the inner core, others have sensitivity only to the upper regions of the Earth’s interior. PKIKP modes are those which are sensitive to the $P$-wave velocity in the inner core (as PKIKP body waves are). PKIKP modes normally have high overtone number $n$ and low angular order $l$. Radial modes are those which have an angular order of zero, that is, $S_0^0$. For a radial mode the overtone number, $n$, corresponds to the number of nodes of the standing wave along the radius of the Earth. Examples of three radial modes ($2S_0^0$, $4S_0^0$, $6S_0^0$) and three PKIKP modes ($11S_1^1$, $13S_1^1$, $15S_1^1$) are shown in Fig. 2. This figure shows the sensitivity of the different modes to variations in $P$-wave velocity, $S$-wave velocity and density. It can be seen that some modes are more sensitive

![Figure 2. Sensitivity kernels calculated using PREM for three radial modes: (a) $2S_0^0$, (b) $4S_0^0$, (c) $6S_0^0$; and three PKIKP modes (d) $11S_1^1$, (e) $13S_1^1$, (f) $15S_1^1$. Solid line shows $P$-wave sensitivity, dashed line $S$-wave sensitivity, and dot–dashed line density sensitivity. The black bars at the edge of each kernel show the depths to which a $P$ wave (left-hand side) and $S$ wave (right-hand side) with sensitivity to the Earth’s structure equivalent to the normal mode would travel.](http://gji.oxfordjournals.org/)
to P-wave velocity in the uppermost inner core (e.g. $2S_0, 1S_1$) whereas other modes (e.g. $1S_0, 0S_0$) have a greater sensitivity to the intermediate region of the inner core. The symmetry exhibited by normal modes requires that no modes have any sensitivity to velocity or density perturbations at the very centre of the inner core.

Aspherical structure, rotation and ellipticity of the Earth all remove the degeneracy of the normal modes so that each mode multiplet becomes split into a set of $(2l + 1)$ singlets with different frequencies. These singlets are labelled by their azimuthal order, $m$, where $m$ takes integer values $-l \leq m \leq l$. In addition to splitting of individual modes, cross-coupling between different modes also occurs, which again changes the frequencies of the singlets. Rotation couples spheroidal modes with the same angular order and also pairs $nS_l$ and $n'T_l^\pm$ where $n$ and $n'$ can take any value. Ellipticity couples the same pairs of modes as rotation and also pairs of modes $nS_l$ and $n'T_l^\pm$.

If the frequencies of two multiplets are close, the modes will also be coupled as a result of the 3-D heterogeneous structure of the Earth. Coupling rules described by Luh (1974) show that heterogeneous structure of degree $L$ (where $L$ is even) will allow a normal mode $nS_l$ to couple with modes $n'S_{L-l}$ and $n'T_{l-L}$. This means that when a model contains only even structure (as is the case for the four inner core anisotropy models discussed here), coupling modes will always have an even difference in angular order. This coupling due to heterogeneity has been considered to be negligible by many authors. Deuss & Woodhouse (2001), on the other hand, have shown that coupling through heterogeneous mantle structure is important for normal modes sensitive to the mantle. Consequently, when the effect of inner core anisotropy is considered, mode coupling may also be significant. In this context we define significant changes to be those changes which are of the same magnitude as the differences between the different models using the SC approximation, or as large as the differences between the model predictions and data. The purpose of this paper is to investigate the effect of normal mode coupling for seismic models of inner core anisotropy.

### 2.2 Coupling approximations

The SC approximation assumes that frequency differences between modes are much greater than the magnitude of possible coupling due to heterogeneous structure, so that each mode is effectively independent and can be isolated from the other modes. SC has been used by most authors in normal mode studies and forms the basis for the use of splitting functions. Group-coupling (GC) approximations allow very small groups (or pairs) of modes to couple. In this approximation, coupling between modes in the groups is considered to be significant but coupling between modes in different groups is assumed to be insignificant. Resovsky & Ritzwoller (1995, 1998) used this technique to find small groups and make observations of generalised splitting functions. FC schemes (Deuss & Woodhouse 2001) allow all modes to couple together. If the assumptions of SC (or GC) were valid it would be expected that the SC, GC and FC schemes would give the same results, although GC and FC schemes would require greater computational power to obtain those results.

### 3 METHODOLOGY

The Preliminary Earth Reference Model (PREM) (Dziewonski & Anderson 1981) is used to calculate degenerate normal modes for a spherically symmetric and non-rotating Earth. Perturbation theory is then used to add mantle and inner core structure, rotation and ellipticity (Dahlen & Tromp 1998). We identified 90 inner core sensitive modes at frequencies under 10 mHz and performed our calculations comparing SC, GC and FC for all modes in this group. Whilst inves-
Figure 4. Eigenfrequencies and $Q$ for the radial modes (a) $2S_0$, (b) $4S_0$ and (c) $6S_0$ using (i) self-coupling and (ii) full-coupling. Mantle structure, ellipticity and rotation have not been included in these calculations—all variations in frequency and $Q$ are due to inner core anisotropy. For comparison, degenerate values are also shown for the isotropic Preliminary Reference Earth Model (PREM). For radial modes using the SC approximation, all five symbols lie on top of each other.

Figure 5. Eigenfrequencies and $Q$ for PKIKP modes (a) $11S_1$, (b) $13S_1$ and (c) $15S_1$ using (i) self-coupling and (ii) full-coupling, similar to Fig. 4. Mantle structure, ellipticity and rotation have not been included in these calculations—all variations in frequency and $Q$ are due to inner core anisotropy. For comparison, degenerate values are also shown for the isotropic Preliminary Reference Earth Model (PREM).
tigating the differences between FC and SC (Sections 4.1 and 4.2), and finding groups of modes which couple together (Section 4.3), mantle structure, rotation and ellipticity were not included in the computations so that the only differences between SC and FC would be those due to inner core anisotropy. When comparing the seismic models using SC and FC with real data (Section 4.4), mantle structure, rotation and ellipticity were used in addition to inner core anisotropy to achieve realistic seismograms.

Data from the 1994 June 9 Bolivia earthquake have been used for comparison with the synthetic data, which were calculated using the Centroid Moment Tensor solution for this event (Dziewonski et al. 1995). The 1994 Bolivia event provided some of the best quality normal mode data to be recorded thus far. As a result, data collected from this earthquake has been used in many different normal modes studies (He & Tromp 1996; Resovsky & Ritzwoller 1998; Deuss & Woodhouse 2001), as well as in the inversions for inner core anisotropy performed by Durek & Romanowicz (1999) and Beghein & Trampert (2003).

Model S2ORTS, developed by Ritsema et al. (1999), was used to provide mantle heterogeneity in shear wave velocity, $v_s$. The shear wave velocity perturbations were then scaled to obtain compressional velocity, $v_p$, and density, $\rho$, with scaling of the form $\delta v_p/v_p = \alpha(r)\delta v_s/v_s$, $\delta \rho/\rho = \beta(r)\delta v_s/v_s$. We use $\alpha = 0.5$ (Li et al. 1991) and $\beta = 0.3$ (Karato 1993).

### 4 RESULTS

#### 4.1 The differences between full-coupling and self-coupling

We first investigate the importance of FC due to inner core anisotropy, comparing it to SC. We calculate the eigenfrequencies and quality factors for each of the four anisotropy models (see Fig. 1) using different coupling schemes—(i) SC approximation for the individual inner core sensitive modes and (ii) FC between all 90 inner core sensitive modes. The effects of mantle heterogeneity, rotation and ellipticity were excluded so that it would be possible to isolate those modes which were coupling due to the anisotropy model used.

Fig. 3 shows the eigenfrequencies and quality factors ($Q$) for inner core sensitive modes using SC and FC for each model. The multiplets of many modes are split more strongly when FC is used than under the SC approximation. There are large changes in $Q$ for many modes, and there are also variations in frequency (although these are not easily visible at the scale used in Fig. 3). The frequency and $Q$ differences between FC and the SC approximation depend on the model used; the changes are not uniform between the four models.

We identified six modes which showed a marked difference between SC and FC, including three radial ($S_2$), four $S_0$ and four $S_0$ and three PKIKP modes ($S_0$, $S_0$ and $S_0$). It should be noted that these are not the only modes which change their properties, rather, they have been chosen to illustrate the differences between SC and FC. The sensitivity kernels of these six modes are shown in Fig. 2. All of these six modes have sensitivity to the structure of the mantle and the inner core. None of the modes are sensitive to the very centre of the inner core which is where the models disagree most strongly.

The eigenfrequencies and quality factors ($Q$) for each of these modes, together with those calculated using the 1-D isotropic PREM are shown in Figs 4 and 5. The shifts in $Q$ for FC are significant—up to a 35 per cent change in attenuation. The shifts of the frequencies of the modes are smaller, up to about 0.5 per cent. We also find that the effect of coupling on inner core sensitive modes is model dependent. The W, G&L model produces the largest shifts for modes $S_0$ and $S_0$ (Figs 4a and c), the Tr model for modes $S_0$ and $S_0$ (Figs 5a and c) and the B&T model for modes $S_0$ and $S_0$ (Figs 4b and 5c). The changes in the frequencies and attenuations of the radial modes are especially noteworthy. Radial modes are not sensitive to 3-D structure when SC is used (Fig. 4i), but FC allows coupling with other modes and allows the radial modes to respond to the anisotropic structure in the inner core (Fig. 4ii).

#### 4.2 Model-building modes

The four anisotropy models used here have all been developed by inverting data extracted from seismic records. The models inverted to form each model are shown in Table 1. Of the six modes which have been investigated in detail here, both the B&T and the D&R models used only mode $S_0$. Neither the W,G&L model, nor the Tr model use any of these six modes.

The four modes $S_0$, $S_0$, $S_0$, $S_0$ and $S_0$ were used in the construction of all four anisotropy models. Of these four modes, three ($S_0$, $S_0$ and $S_0$) show differences between SC and FC (frequency changes of 0.0005 mHz and changes in $Q$ of up to 42). More significantly, $S_2$ (Fig. 6a) shows frequency and $Q$ factor changes as large as those seen for the previous three PKIKP modes discussed in this paper. Sингlets in the mode $S_2$ have frequency changes of up to 0.003 mHz and $Q$ factor changes of up to 78 (27 per cent).

Mode $S_2$ (Fig. 6b) was used in the creation of three of the four anisotropy models (D&R, Tr and W&G&L). $S_2$ is known to be a poorly understood mode, about which there is some ambiguity concerning the splitting function (Durek & Romanowicz 1999). It was not used in the construction of the B&T model for this reason. It shows frequency changes of up to 0.0011 mHz for individual singlets.

### Table 1. Modes used in the creation of each inner core anisotropy model.

<table>
<thead>
<tr>
<th>Mode</th>
<th>B&amp;T</th>
<th>D&amp;R</th>
<th>Tr</th>
<th>W,G&amp;L</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_2$</td>
<td>$S_2$</td>
<td>$S_2$</td>
<td>$S_2$</td>
<td>$S_2$</td>
</tr>
<tr>
<td>$S_3$</td>
<td>$S_3$</td>
<td>$S_3$</td>
<td>$S_3$</td>
<td>$S_3$</td>
</tr>
<tr>
<td>$S_4$</td>
<td>$S_4$</td>
<td>$S_4$</td>
<td>$S_4$</td>
<td>$S_4$</td>
</tr>
<tr>
<td>$S_5$</td>
<td>$S_5$</td>
<td>$S_5$</td>
<td>$S_5$</td>
<td>$S_5$</td>
</tr>
<tr>
<td>$S_6$</td>
<td>$S_6$</td>
<td>$S_6$</td>
<td>$S_6$</td>
<td>$S_6$</td>
</tr>
<tr>
<td>$S_7$</td>
<td>$S_7$</td>
<td>$S_7$</td>
<td>$S_7$</td>
<td>$S_7$</td>
</tr>
<tr>
<td>$S_8$</td>
<td>$S_8$</td>
<td>$S_8$</td>
<td>$S_8$</td>
<td>$S_8$</td>
</tr>
<tr>
<td>$S_9$</td>
<td>$S_9$</td>
<td>$S_9$</td>
<td>$S_9$</td>
<td>$S_9$</td>
</tr>
<tr>
<td>$S_{10}$</td>
<td>$S_{10}$</td>
<td>$S_{10}$</td>
<td>$S_{10}$</td>
<td>$S_{10}$</td>
</tr>
<tr>
<td>$S_{11}$</td>
<td>$S_{11}$</td>
<td>$S_{11}$</td>
<td>$S_{11}$</td>
<td>$S_{11}$</td>
</tr>
<tr>
<td>$S_{12}$</td>
<td>$S_{12}$</td>
<td>$S_{12}$</td>
<td>$S_{12}$</td>
<td>$S_{12}$</td>
</tr>
<tr>
<td>$S_{13}$</td>
<td>$S_{13}$</td>
<td>$S_{13}$</td>
<td>$S_{13}$</td>
<td>$S_{13}$</td>
</tr>
<tr>
<td>$S_{14}$</td>
<td>$S_{14}$</td>
<td>$S_{14}$</td>
<td>$S_{14}$</td>
<td>$S_{14}$</td>
</tr>
<tr>
<td>$S_{15}$</td>
<td>$S_{15}$</td>
<td>$S_{15}$</td>
<td>$S_{15}$</td>
<td>$S_{15}$</td>
</tr>
<tr>
<td>$S_{16}$</td>
<td>$S_{16}$</td>
<td>$S_{16}$</td>
<td>$S_{16}$</td>
<td>$S_{16}$</td>
</tr>
<tr>
<td>$S_{17}$</td>
<td>$S_{17}$</td>
<td>$S_{17}$</td>
<td>$S_{17}$</td>
<td>$S_{17}$</td>
</tr>
<tr>
<td>$S_{18}$</td>
<td>$S_{18}$</td>
<td>$S_{18}$</td>
<td>$S_{18}$</td>
<td>$S_{18}$</td>
</tr>
<tr>
<td>$S_{19}$</td>
<td>$S_{19}$</td>
<td>$S_{19}$</td>
<td>$S_{19}$</td>
<td>$S_{19}$</td>
</tr>
<tr>
<td>$S_{20}$</td>
<td>$S_{20}$</td>
<td>$S_{20}$</td>
<td>$S_{20}$</td>
<td>$S_{20}$</td>
</tr>
<tr>
<td>$S_{21}$</td>
<td>$S_{21}$</td>
<td>$S_{21}$</td>
<td>$S_{21}$</td>
<td>$S_{21}$</td>
</tr>
<tr>
<td>$S_{22}$</td>
<td>$S_{22}$</td>
<td>$S_{22}$</td>
<td>$S_{22}$</td>
<td>$S_{22}$</td>
</tr>
<tr>
<td>$S_{23}$</td>
<td>$S_{23}$</td>
<td>$S_{23}$</td>
<td>$S_{23}$</td>
<td>$S_{23}$</td>
</tr>
<tr>
<td>$S_{24}$</td>
<td>$S_{24}$</td>
<td>$S_{24}$</td>
<td>$S_{24}$</td>
<td>$S_{24}$</td>
</tr>
<tr>
<td>$S_{25}$</td>
<td>$S_{25}$</td>
<td>$S_{25}$</td>
<td>$S_{25}$</td>
<td>$S_{25}$</td>
</tr>
<tr>
<td>$S_{26}$</td>
<td>$S_{26}$</td>
<td>$S_{26}$</td>
<td>$S_{26}$</td>
<td>$S_{26}$</td>
</tr>
<tr>
<td>$S_{27}$</td>
<td>$S_{27}$</td>
<td>$S_{27}$</td>
<td>$S_{27}$</td>
<td>$S_{27}$</td>
</tr>
</tbody>
</table>

Note: Anisotropy models are B&T: Beghein & Trampert (2003); D&R: Durek & Romanowicz (1999); Tr: Tromp (1993) and W,G&L: Woodhouse et al. (1986).
and $Q$ changes of up to 33 (4 per cent). Likewise, mode $3S_3$ (Fig. 6c) was used in the inversions to produce three of the four anisotropy models (B&T, Tr and W,G&L). It was not used in the creation of the D&R model as it showed poor convergence in the inversion. Singlets of $3S_3$ display frequency changes of up to 0.0013 mHz and $Q$ changes of up to 96 (14 per cent).

Beghein & Trampert (2003) comment that the B&T model prediction for the $3S_3$ mode fits the data used in the inversion well. Durek & Romanowicz (1999) published the residual variances of each mode after subtracting the predictions made by their anisotropy model. For the D&R model, predictions for modes $3S_2$ and $13S_2$ fit the data used in the inversion better than the average, and $13S_1$ fit the data used less well than the average. Tromp (1993) fits $3S_2$ and $13S_2$ poorly, $11S_1, 11S_2$ and $13S_1$ reasonably well and $3S_1$ very well. Information about the degree to which the W,G&L model fits the data from which it was created is unavailable. The modes for which the SC approximation work poorly are in some cases fitted well by the SC derived anisotropy model.

The investigation of the precise variation of each inner core mode is beyond the scope of this paper, but all of the models have been produced using at least two modes which display large differences between FC and SC, and at least three modes which are also changed by the SC approximation.

4.3 Group-coupling

To discover the modes that are the most significant contributors to changes in frequency and quality factors in the FC computation, groups of modes were sought for each of the modes $2S_0, 4S_0, 6S_0, 11S_1, 13S_1$ and $15S_1$. These groups contain all modes which couple with the lead mode and reproduce the FC modal frequencies to within 0.01 per cent and the corresponding $Q$ values to within 0.04 per cent. The coupling groups were found by adding and removing individual modes from the group until the group was able to replicate the FC frequency and $Q$ of the lead mode to within the required accuracy. It was noted that some modes, whilst not coupling directly with the lead mode, coupled strongly with other modes in the group, so that their omission would have significantly decreased the ability of the coupling group to replicate the FC properties. These indirect interactions should be taken into account in any inversion scheme which is based on group coupling.

Fig. 7 displays the groups for the four different inner core anisotropy models as a function of frequency. It can be seen that the groups are model dependent and that they are of drastically different sizes: those for the Tr model are the smallest and those for the B&T model are the largest. This result suggests that the more complex models also exhibit more complex coupling in larger groups.

There are some modes which couple in every model, the common modes of the group; these common modes are also displayed in Fig. 7. The groups of common modes are much smaller than the groups needed to account for all of the coupling. The common groups already show significant wide-band coupling over ranges up to 5 mHz. The coupling needed to completely account for all effects occurs over even wider frequency bands—for the W,G&L, D&R and B&T models the bands for some modes are over 8 mHz in width. Similar coupling over broad frequency bands was previously found for mantle structure by Deuss & Woodhouse (2001) and for inner core anelasticity by Andrews et al. (2006).

Modes which couple within these groups always differ by an even angular order, as is required by the symmetries of the system and the corresponding coupling rules. The modes which couple together

![Figure 6.](http://gji.oxfordjournals.org/) Eigenfrequencies and $Q$ for PKIKP modes (a) $3S_2$, (b) $13S_2$ and (c) $9S_3$ using (i) self-coupling and (ii) full-coupling, similar to Figs 4 and 5. Mantle structure, ellipticity and rotation have not been included in these calculations—all variations in frequency and $Q$ are due to inner core anisotropy. For comparison, degenerate values are also shown for the isotropic Preliminary Reference Earth Model (PREM).
in the B&T and Tr models have angular order differences of up to six and eight respectively, whereas the D&R and W,G&L models have several groups where the greatest angular order (l) difference between coupling modes is only two.

Although the W,G&L model only includes degree 0, 2 and 4 structure, the angular order differences between the main mode in each group and the other modes in the same group are larger than would be expected by the coupling rules (Luh 1974). Consider the mode \(2S_0\). When the W,G&L model is used, this mode couples with fourteen other modes, including \(7S_2\) and \(4S_6\). Whilst coupling between \(2S_0\) and \(4S_6\) would be forbidden by the coupling rules, \(4S_6\) is allowed to couple to \(7S_2\). To successfully approximate the FC behaviour of \(2S_0\), the mode \(4S_6\), normally considered to be forbidden from coupling with \(2S_0\), must be added to the group to successfully replicate the interaction between \(2S_0\) and \(7S_2\), which does couple to \(4S_6\).

These results demonstrate that the inner core modes which couple together in a group differ depending on the inner core anisotropy model used. As the coupling groups are different for every seismic anisotropy model, inversions which use a GC approximation will clearly produce models which are dependent on the coupling groups selected.

4.4 Normal mode spectra - the consequences of full-coupling

Synthetic seismograms and corresponding normal mode spectra were calculated for the 1994 June 9 Bolivia event at various stations, using the SC approximation, GC for the common modes and FC to illustrate the differences between both the inner core anisotropy models and also the different coupling approximations. Here, we have also included mantle structure, ellipticity and rotation. The synthetic seismograms, as shown in Figs 8 and 9, are displayed along with vertical motion (VHZ) observed data seismograms. Both synthetic and observed seismograms are cosine tapered and padded with zeroes before Fourier transformation to the frequency domain.

![Diagram](http://gji.oxfordjournals.org/)

**Figure 7.** The groups of modes which couple with modes \(2S_0\), \(4S_0\), \(6S_0\), \(11S_1\), \(13S_1\) and \(15S_1\). The small dots each represent a mode which couples with the main mode, whilst the large diamonds mark the main mode on each line. The length of the line indicates the width of the group across which coupling occurs.

© 2008 The Authors, GJI, 174, 919–929
Journal compilation © 2008 RAS
Normal mode coupling in the inner core

Fig. 8 shows synthetic and observed spectra for the radial mode $2S_0$ using both SC and FC. It can be seen from Fig. 8a that there is a significant difference between the seismograms produced using SC and FC, in agreement with results for mantle structure by Deuss & Woodhouse (2001). For station KIP, using the W,G&L model, the differences between $2S_0$ mode when SC or FC is used are about twice that between SC and the data. Fig. 4(a) shows a 0.1 per cent frequency difference for this mode, which may seem small but is clearly significant in the synthetic spectrum of Fig. 8(a). The differences between FC and SC are significant and observable on the scale of seismograms from real events.

We also compared synthetic spectra using GC with only the common modes. Fig. 8(b) shows that GC with the common modes is unable to completely replicate the effect of FC on $2S_0$, although GC with common modes produces seismograms which are much closer to FC than SC. Fig. 4(a) shows that the fully coupled D&R model produces a $2S_0$ mode which is quite close to the SC result, but even this small shift is still observable in Fig. 8(b).

We have computed synthetic seismograms for all four inner core anisotropy models for mode $13S_1$ using SC (Figs 9a and b) and FC (Figs 9c and d); the synthetic seismograms are shown together with data. The SC and FC synthetic seismograms produced from the four seismic anisotropy models are noticeably different from each other and the data. Using SC, the amplitudes of the $13S_1$ mode synthetic seismograms are similar for all of the models; this is because all models produce three singlets with $Q$ values of around 740. This high $Q$ results in large amplitude peaks even when a long time window, such as the window used here, is considered. There are substantial variations in the frequencies of the three singlets between the models, however all of the singlets fall within the window shown.

When FC is used (Figs 9c and d), the total amount of energy in this mode and therefore the overall amplitude of the mode is smaller than under SC, as the quality factor is lower for $13S_1$ when FC is used instead of SC (as can be seen in Fig. 5b). There is therefore less energy in the mode $13S_1$ in FC than in SC over the long time window used to observe normal modes. When the phase of mode $13S_1$ is considered, all four models show discrepancies between the SC approximation and FC. None of the models reproduce the phase observed in the data of the $13S_1$ mode accurately.

Using FC, the D&R model gives a peak at about 4.500 mHz because there is a singlet of the $13S_1$ mode at this frequency. This peak is at a higher frequency in the other models: 4.505 mHz in the Tr model, 4.507 mHz in the W,G&L model; it has been shifted out of the observation window to a frequency of 4.510 mHz in the B&T model. The widely varying location of this one singlet peak gives an indication of the size of the differences between the currently existing inner core anisotropy models and shows that these differences are readily observable.

5 CONCLUSIONS

We have shown that inner core anisotropy causes significant coupling between large numbers of inner core sensitive normal modes, and that the differences between SC and FC can be large. If the SC approximation was accurate enough, there would be no change in singlet frequency and quality factor when inner core modes are allowed to couple. As there is a significant change in the quality factor
and singlet frequencies of some inner core modes, we conclude that this approximation is not valid.

The modes that couple with each other are spread over a wide range of frequencies, in some cases up to 8 mHz. Coupled modes always differ by an even angular order, as required by the symmetries of the system for even order anisotropy models. The inner core modes which couple together differ depending on the inner core anisotropy model used. The WG&L model has the smallest group and the B&T model the largest group of modes which couple, suggesting that more complex models couple larger groups of modes together. Inversions which use a GC approximation will clearly produce models which depend on the coupling groups selected. An avenue for further investigation would be inversion schemes where the coupling groups are dynamically calculated between iterations of the inversion. This could considerably reduce the computational size of the inversion problem whilst retaining the significant interactions present in full-coupling.

The modes which interact with one another are not only those which would be expected to interact using coupling rules. Coupling between ‘intermediate’ modes permits some interaction between modes outside of those which would couple if quasi-degenerate perturbation theory was used. This means that full-coupling, in which no assumptions are made about the modes which couple, will ensure that unexpected interactions are possible and included in the calculations.

The differences between the modal eigenfrequencies and attenuations calculated using FC and the SC approximation are sufficiently large that they can be seen when synthetic seismograms are compared to observed seismograms from the 1994 June 9 Bolivia event. Using two modes sensitive to inner core structure, $\gamma S_0$ and $\gamma S_1$, the differences between the seismograms produced using the SC approximation, FC and the data from the Bolivia event are considerable.

It is clear that none of the models fit particularly well under the FC scheme. This is not surprising, since they have been created by inversion or forward modelling using the SC approximation. To better understand the causes of inner core anisotropy, the anisotropy must first be accurately described. Further work is clearly needed to determine better models of inner core anisotropy.

ACKNOWLEDGMENTS

We would like to thank Joe Resovsky, Caroline Beghein and Jeannot Trampert for their constructive reviews, and John Woodhouse for interesting discussions. JCEI was supported by NERC Studentship NER/S/A/2005/13491. AD was supported by the Nuffield Foundation. JA was supported by the Girdler scholarship awarded by the Earth Sciences Department, University of Cambridge.

REFERENCES


© 2008 The Authors, GJI, 174, 919–929

Journal compilation © 2008 RAS


