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Microwave diagnostics of small plasma objects

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We suggest an approach for using microwave radiation in collisional, weakly ionized plasma diagnostics when plasma dimensions are relatively small compared with the microwave wavelength. We show that in this case the microwave diagnostics can be based on the measurement of the radiation scattered by an oscillating plasma dipole, similar to the Rayleigh scattering of an atom in light. Examples considered show possibilities of obtaining the decaying plasma parameters (time dependence of charge density and information about loss rates, for instance) from the measured scattered signal. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1996835]

I. INTRODUCTION

The interaction of microwave radiation with a plasma is a powerful method for plasma diagnostics.[1–3] Typical situations considered in the literature correspond to the case when $\lambda/L \ll 1$, where $\lambda = 2\pi c/\omega$ is the microwave wavelength and L is a plasma scale. In the opposite case, when $\lambda \gg L$, the use of traditional microwave diagnostics methods based on measurements of phase shifting, absorption, backscattering, and cutoff[1–3] no longer apply. Many problems with a relatively small scale plasma are of great interest, such as laser sparks, avalanche-streamer transitions, resonance-enhanced multiphoton ionization (REMPI) processes, etc. In this paper we suggest a method of using microwave radiation for the study of a small scale plasma.

II. DISCUSSION

A plasma volume in a relatively long-wavelength microwave field ($\lambda/L \gg 1$) is periodically polarized and therefore can be a source of scattered dipole radiation at the frequency of the incident microwave wave. Such a process is analogous to the elastic Rayleigh scattering of light.[4] At the distance R in the far field ($R \gg \lambda, L$) the scattered radiation can be considered as a wave from a point dipole source. Figure 1 shows the scattered geometry: the incident microwave field propagates in the x direction with the electric field polarized in the z direction. When there is no plasma and the primary microwave radiation is directed in the x direction, for instance, there is no scattered radiation in the z or y direction. But when the plasma is created and modulated by the microwave field, it emits scattered radiation. The measurement of this radiation can provide much information on the plasma parameters.

Consider an incident microwave of intensity $I_i = \frac{1}{2}\varepsilon_0 c E_{0,i}^2$ and frequency $f = \omega/2\pi$. This wave interacts with the quasineutral weakly ionized plasma column of length l, radius r, and charge density n. The primary radiation can be of any polarization, but the strongest effect will be when the microwave radiation is plain polarized with its electric field **E** along the plasma column (Fig. 1). If the skin-layer thickness $\delta = 2/\sqrt{2\mu_0\sigma\omega} > r$, where σ is a plasma conductivity, then the current is distributed throughout the plasma volume.

The plasma column becomes polarized with the microwave electric field, and the results in the induced dipole moment $\mathbf{d}(t) = \mathbf{l}q(t)$, where $q \approx en\Delta z(t)S$; $S = \pi r^2$ and $\Delta z(t)$ is the electron cloud displacement relative to the immobile ions.

The displaced electrons experience a restoring force $-(e^2/\varepsilon_0)n\Delta z(t)$. As a result, the equation for the plasma (and, therefore, q) oscillations is

$$\Delta\ddot{z} + \nu_m \Delta\dot{z} + \omega_p^2 \Delta z = \frac{e}{m} E_{0,i} \cos \omega t, \tag{1}$$

where $\omega_p = \sqrt{e^2 n/\varepsilon_0 m}$ and ν_m are the electron plasma and electron-neutral transport collision frequencies. Looking for a solution in a form $\Delta z(t) = \Delta z_0 \cos(\omega t + \varphi)$, from Eq. (1) it follows: $\tan \varphi = -\nu_m \omega/(\omega_p^2 - \omega^2)$ and $\Delta z_0 = e/mE_{0,i}/\sqrt{(\omega_p^2-\omega^2)^2 + (\nu_m\omega)^2}$. At $\nu_m\omega \gg |\omega_p^2 - \omega^2|$, $\Delta z_0 = \mu_e E_{0,i}/\omega$, where $\mu_e = e/m\nu_m$ is the electron mobility, and in this case Δz_0 corresponds to the amplitude of the drift electron oscillations in the radio frequency or microwave electric field.[5]

In the general case of an oscillating dipole, the instantaneous dipole radiation power is[6]

$$\Theta = \frac{\ddot{\mathbf{d}}^2}{6\pi\varepsilon_0 c^3}. \tag{2}$$

Substituting $d(t) = en\Delta z(t)lS = d_0 \cos(\omega t + \varphi)$ into Eq. (2) and averaging over the microwave period, the total averaged power radiated by the plasma dipole is

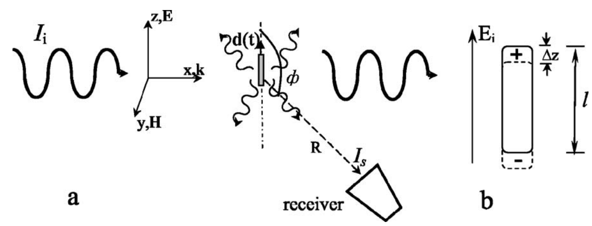

FIG. 1. A plasma dipole **d** scattering microwave radiation. (a) Scheme of experiment. (b) The plasma dipole in the microwave field.

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$$\langle \Theta \rangle = \frac{d_0^2 \omega^4}{12\pi\varepsilon_0 c^3} = \frac{1}{6\pi c^4} \frac{I_i V^2 \omega_p^4 \omega^4}{(\omega_p^2 - \omega^2)^2 + (\nu_m \omega)^2}, \quad (3)$$

and the effective "Rayleigh" cross section

$$\sigma_R(\omega) = \frac{\langle \Theta \rangle}{I_i} = \frac{1}{6\pi c^4} \frac{V^2 \omega_p^4 \omega^4}{(\omega_p^2 - \omega^2)^2 + (\nu_m \omega)^2}, \quad (4)$$

where $V = lS$ is the plasma channel volume.

At a distance $R \gg l$, λ the scattered radiation from the plasma channel can be considered to originate from a point dipole source where the effective differential scattering cross section is[4]

$$\frac{\partial \sigma_R}{\partial \Omega} = \frac{3}{8\pi} \sigma_R \sin^2 \phi, \quad (5)$$

and the corresponding averaged intensity and electric-field amplitude are

$$I_s(R) = \frac{\partial \sigma_R}{\partial \Omega} \frac{1}{R^2} I_i = \frac{3}{8\pi} \langle \Theta \rangle \frac{1}{R^2} \sin^2 \phi, \quad (6)$$

$$E_{0,s}(R) = \sqrt{\frac{2I_s}{\varepsilon_0 c}} = \frac{\sin \phi}{2R} \sqrt{\frac{3\langle \Theta \rangle}{\pi \varepsilon_0 c}}, \quad (7)$$

where ϕ is the angle between the dipole vector and the direction of observation (Fig. 1).

At $\nu_m \gg \omega \gg \omega_p$, Eq. (3) tends to the classic expression for short antenna Hertz dipole radiation[6]

$$\langle \Theta \rangle \approx \frac{l^2 I_0^2 \omega^2}{12\pi\varepsilon_0 c^3} \approx \frac{V^2 \sigma^2 E_{0,i}^2 \omega^2}{12\pi\varepsilon_0 c^3} \propto I_i V^2 n^2 \omega^2, \quad (8)$$

where $I_0 \approx \sigma E_{0,i} S$ is the conductivity current amplitude in the channel; $\sigma = e^2 n / m\nu_m$ is a corresponding plasma conductivity. From Eqs. (7) and (8) it follows that

$$E_{0,s} \propto n\omega V I_i^{1/2} / R. \quad (9)$$

Note that Eqs. (3)–(8) are valid for an arbitrary plasma volume V, when both the skin layer $\delta(\omega, n)$ and microwave wavelength λ are much larger than any plasma dimension and, therefore, the oscillation of plasma electrons is coherent.

As an example of an application, consider a decaying weakly ionized plasma channel with $l = 5 \times 10^{-3}$ m; $r = 10^{-3}$ m in air at pressure $p = 1$ atm, equal electron and gas temperatures $T_e = T = 300$ K, and initial electron density $n_e(0) = 10^{20}$ m^{-3}. Such parameters are typical for short streamers in cold air.[7] In the general case, the plasma density is determined by ambipolar diffusion and the rate of electron losses. For short times, $t \ll r^2 / D_a$ (D_a is the ambipolar diffusion coefficient) we can neglect the ambipolar channel expansion (for time $t \ll 0.3$ s) and consider the scattering process at $r = $ const. At these conditions the electron-loss rate is determined by the dissociative ion-electron recombination and attachment and the electron number density in the plasma channel[8]

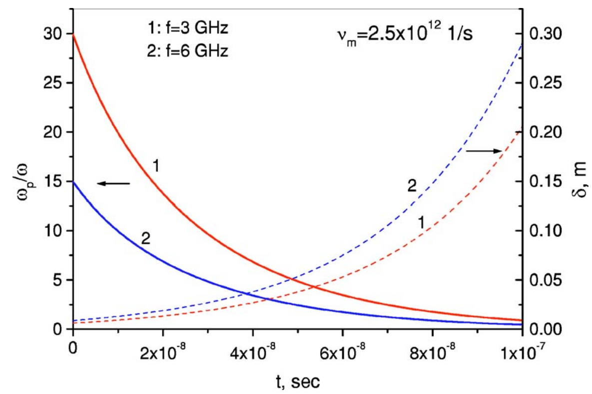

FIG. 2. (Color online) Plasma to microwave frequency ratios and corresponding skin-layer thicknesses at microwave frequencies 3 and 6 GHz in air at p=1 atm, T=300 K, and $n_e(0) = 10^{20}$ m^{-3}.

$$n_e(t) = \frac{n_e(0)\exp(-t/\tau_a)}{1 + \beta\tau_a n_e(0)[1 - \exp(-t/\tau_a)]}, \quad (10)$$

where $n_e(0) = n_e(t=0)$ is an initial plasma density and $\beta \approx 2 \times 10^{-13}$ m^3/s and $\tau_a \approx 8.85 \times 10^{42} / N^2$ s are a recombination coefficient and effective attachment time in air at $T_e = T = 300$ K; N is the molecular number density in air.[7]

A set of results for $I_i = 1$ and 10 W/m^2 at frequencies $f = 3$ and 6 GHz is shown in Figs. 2 and 3 (corresponding electric-field amplitudes $E_{0,i} = 27.4$ and 86.8 V/m are too small for ionization or additional heating of air at considered

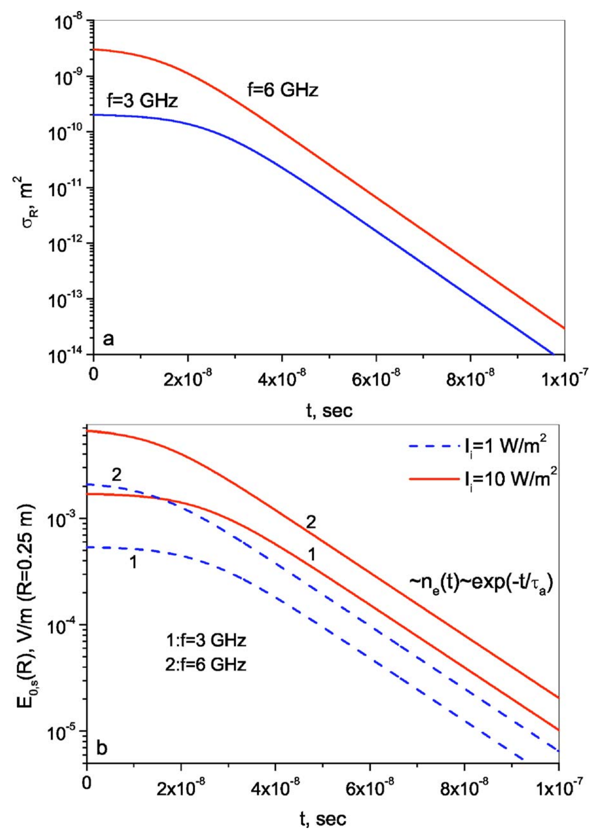

FIG. 3. (Color online) (a) Effective cross section for the plasma dipole scattering at different primary microwave frequencies, 3 and 6 GHz. (b) Corresponding electric-field amplitudes of scattered signal at distance $R = 0.25$ m from the plasma dipole at different primary microwave intensities, 1 and 10 W/m^2, and frequencies 3 and 6 GHz.

conditions). The scattering process we consider is in the collision dominated regime, when $\nu_m \gg \omega, \omega_p$ and, also, our assumption for the skin layer, $\delta(n_e, \omega) \gg l, r$ is valid (Fig. 2). For the modeled plasma channel $n_e(t) \propto \exp(-t/\tau_a)$ because the most important loss mechanism is the electron attachment. The same time dependence is shown by $E_{0,s}$ [Fig. 3(b)] (the values of the $E_{0,s}$ correspond to the angle $\phi = \pi/2$, related to the maximum intensity of the scattered radiation). Therefore, the measured scattered signal reflects information about the instantaneous plasma density in accordance with (9) and can be used to calculate the attachment rate.

III. SUMMARY

Microwave scattering from plasma structures that are much smaller than the microwave wavelength falls into the Rayleigh scattering regime. This leads to the relatively simple formulation of the detected signal and provides an approach to the measurement of the electron number density and electron-loss rate.

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