IIP in Oxford Centre for Theoretical Physics:
Reflectometry and Minerva

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Oxford the Place

• In general, Oxford is full of tons of people from all over the world, and the whole city felt more like being on Princeton Campus than in another country

• Just an hour away from CCFE (a national lab with JET, the most important tokamak for ITER and the future of fusion)
The Work

• My work was under an Oxford Professor, Felix Parra Diaz (theoretical plasma physics professor / researcher)
• But I was working physically at CCFE, about a 20 minute train ride from the center of Oxford, or an hour by bike
• There, my advisor was Jon Hillesheim (a task force manager for JET-ITER connections), and I was debugging code for someone from Max Planck in Germany named Jakob Svensson
• I also met tons of friendly engineers and scientists around the lab willing to help, most importantly Luis Meneses, a Portuguese reflectometry specialist
Bottom Line

• You’ll meet people from all over the world
• If you’re interested in plasma physics and/or fusion this is geographically the place to be (at least until the fallout of Brexit likely shuts JET down in a couple years)
• Oxford is not generally very different from Princeton – it’s like being on any American college campus but with extra old buildings everywhere
Impact on Me

• This project showed me the power of bayesian inference and statistics for physics, and particularly for fusion
• It led me to be taking mostly stats and math courses this semester
• It also led me to talk to a professor at PPPL about a thesis on AI applications for disruption prediction / mitigation in tokamaks
The Work

• Feel free to email me at jabbate@princeton.edu with any questions you might have

• Also, if you’re applying to this IIP, you’re likely a physics major and may want to hear more about the work itself than the location

• If so, here you are:
Reflectometry:

• Goal: obtain the density profile of plasma by sending in microwaves and analyzing the reflection

• Using: the Minerva Framework
  – Based on: Bayesian analysis and forward modeling
  – Simultaneously takes in diagnostics to produce distribution of parameters
Outline

1. Theory of phase shifts in waves (what is the relationship between density profile and phase signal?)
2. Bayesian analysis (how do we get the density profile from the phase signal using forward modeling?)
3. Minerva Plug (why forward modeling, why the complex framework of Minerva?)
Outline

1. Theory of phase shifts in waves (what reflected signal do we expect?)
2. Bayesian analysis (how can we best arrive at and describe our “answer” - the density profile?)
3. Minerva Plug
Dispersion Relation

• Maxwell’s Equations tell us:

\[ \text{curl}(\text{curl} \mathbf{E}) + \frac{\partial}{\partial t} (\mu_0 \mathbf{j} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t}) = 0 \]

• Assuming 1) homogeneous plasma and 2) Linear Ohm’s Law, consider a plane wave solution:

\[ (\mathbf{k} \mathbf{k}^t - k^2 \mathbf{1} + \frac{\omega^2}{c^2} \epsilon) \mathbf{E} = 0 \]

\[ \left( \epsilon = 1 + \frac{i}{\omega \epsilon_0 \sigma} \right) \]

• For \( \mathbf{E} \) to be nontrivial, the determinant of the expression in parentheses must be 0
• If \( k \) is perp to \( B \), we have a dispersion relation:

\[
N^2 = 1 - \frac{\omega_p^2}{\omega^2} \frac{\omega^2 - \omega_p^2}{\omega^2 - (\omega_p^2 + \omega_c^2)}
\]

– O Mode:
  • Cutoff: \( \omega_p \)
  • Resonance: None

– X Mode: R polarized:
  • Cutoff: \( \omega_R = \frac{1}{2}[\omega_c + (\omega_c^2 + 4\omega_p^2)^{\frac{1}{2}}]\)
  • Resonance: \( \omega_h = \sqrt{\omega_p^2 + \omega_c^2} \)

– X Mode: L polarized:
  • Cutoff: \( \omega_L = \frac{1}{2}[\omega_c + (\omega_c^2 + 4\omega_p^2)^{\frac{1}{2}}]\)
  • Resonance: \( \omega_h = \sqrt{\omega_p^2 + \omega_c^2} \)
Cutoffs and Resonances
Cutoffs and Resonances

\[ \omega_R = \frac{1}{2} \left[ \omega_c + \left( \omega_c^2 + 4\omega_p^2 \right)^{\frac{1}{2}} \right] \]

\[ \omega_L = \frac{1}{2} \left[ -\omega_c + \left( \omega_c^2 + 4\omega_p^2 \right)^{\frac{1}{2}} \right] \]

\[ \omega_O = \omega_p \]

\[ \omega_p = \sqrt{\frac{n_e e^2}{m \epsilon_0}} \]

\[ \omega_c = \frac{qB}{m} \]
WKB Approximation

- Problem: our plasma isn’t homogenous

- Solution: WKB approximation
  - High frequency, so density change is slow enough to look locally constant
  - Phase shift due to travel then given by $\phi = \int_{r_1}^{r_2} k \cdot ds$ instead of $\phi = k \cdot s$
Phase Shift Formula

- So all that’s left is to account for the phase shift due to the reflection itself – which turns out to be a simple \( \frac{\pi}{2} \) correction – and for the dispersion within the waveguide itself:

\[
\phi(\omega) = 2\frac{\omega}{c} \int_{x_0}^{x_c(\omega)} N(\omega, x)dx + \frac{\pi}{2} + \phi_0(\omega)
\]

- In practice, what we really want is the derivative, \( \frac{d\phi}{d\omega} \)
What we really want is
\[ \frac{d\phi}{d\omega} = \frac{1}{d\omega} \frac{d\phi}{dt} \]

We have the signal of phase \( \phi(t) \)

So if \( \phi(t) \) is changing very quickly relative to the frequency sweep (it is) then we’ll be able to see oscillations whose instantaneous frequency is a measure of \( \frac{d\phi}{dt} \)

Practically, then, we take Fourier Transforms in windows of time to estimate \( \frac{d\phi}{dt} \)
Outline

1. Theory of phase shifts in waves (what reflected signal do we expect?)

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3. Minerva Plug
Forward Modeling

• “Backward Modeling”: going from data collected like \( \frac{d\phi}{d\omega} \) to plasma parameters like the density profile

\[
f(N(\omega, x), x_c(\omega)) = \frac{d\phi}{d\omega}(\omega)
\]

• “Forward Modeling”: going from plasma parameters to the diagnostic data
Bayesian Analysis

• Bayes Rule: \[ \mathbb{P}(A|B) = \frac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B)} \]

• Us: \[ p(\text{density} | \frac{d\phi}{d\omega}) \sim p\left(\frac{d\phi}{d\omega} | \text{density}\right)p(\text{density}) \]

• To obtain likelihood, numerically differentiate

\[ \phi(\omega) = 2\frac{\omega}{c} \int_{x_0}^{x_c(\omega)} N(\omega, x)dx + \phi_0(\omega) \]
Spectrogram with Max and Dev
Bayesian Analysis

• Obtain posterior, the joint probability over all of the density parameters

• Integrate out all parameters but one to get its marginal distribution

• E.g., 6 parameter model:

\[
f(x) = n_{\text{sol}} + \frac{1}{2} (n_{\text{ped}} - n_{\text{sol}}) \left( \frac{e^{R'(x)}(1 + R'(x)s_{\text{ped}}) - e^{-R'(x)}(1 + R'(x)s_{\text{sol}})}{e^{R'(x)} + e^{-R'(x)}} + 1 \right)
\]
Minerva Graphical Model Bare Bones
Density Profiles for Parameter Variation

\[ f(x) = n_{\text{sol}} + \frac{1}{2}(n_{\text{ped}} - n_{\text{sol}}) \left( \frac{e^{R'(x)(1 + R'(x)s_{\text{ped}})} - e^{-R'(x)(1 + R'(x)s_{\text{sol}})}}{e^{R'(x)} + e^{-R'(x)}} + 1 \right) \]
Spectrograms for Parameter Variation

\[ f(x) = n_{\text{sol}} + \frac{1}{2}(n_{\text{ped}} - n_{\text{sol}}) \left( \frac{e^{R'(x)}(1 + R'(x)s_{\text{ped}}) - e^{-R'(x)}(1 + R'(x)s_{\text{sol}})}{e^{R'(x)} + e^{-R'(x)}} + 1 \right) \]
Scaled Likelihood for Parameter Value

\[ f(x) = n_{sol} + \frac{1}{2} (n_{ped} - n_{sol}) \left( \frac{e^{R'(x)} (1 + R'(x)s_{ped}) - e^{-R'(x)} (1 + R'(x)s_{sol})}{e^{R'(x)} + e^{-R'(x)}} + 1 \right) \]
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Minerva Goal

• Ultimate goal: automate diagnostic process for reactors
  – Input all observations and their associated uncertainties
  – Input prior distributions
  – Computer spits out full (potentially highly complex) joint distribution, from which marginal distributions can be obtained
Minerva
Graphical Model All Channels
Why Forward Modeling / Minerva?

1) Going forward easier than backward
2) Clarity about assumptions
3) Ease of changing assumptions
4) Finds most consistent solution based on all diagnostics at once
5) Can use “useless” information
What I Changed
ReflectometerPrediction:

- Created simple test profiles for debugging (in JoeTest.java)

- Replaced LinearInterpolation with a simple private function (firstIndexLower) which handles getting nearest distance to given cutoff frequency
  - The issue was that there are multiple positions at which a given cutoff frequency occurs

- Fixed absorption at 2\textsuperscript{nd} cyclotron frequency - now only considers absorption occurring for finite plasma density (i.e. after first wall) so that early absorption no longer occurs
  - Trace through definition of the variable $f_{0\_w2cycl}$ to see what that means precisely
Bug Fix (On Simple Test Profiles)
• ReflectometerPrediction (cont’d)
  – In updatePhaseSweepUncalibrated... functions moved interpolations/calculations for magnetic field, density, and adjusted mass at each position outside of inner loop to make the loop in total about 5 times faster (from ~3.2 to ~.6 s)
• JetKG10ReflectometerDataSource
  – Simple logic bug fix: calibration spurious frequency being set in the normal rather than the spurious case
  – Changed variance in spectrogram from constant value to regular variance, but with
    • A cutoff value for the Fourier weight below which the points aren’t counted in the variance (between 10 and 20% of the max seems to do a good job qualitatively)
    • A threshold for the variance itself above which the variance is just sent to 200 MHz (to effectively ignore noise)
  – Created resizeArray method so that the returned variance would be the sweep frequency length (since it ultimately gets added to the calibration variance and therefore needs to be the same size)
Future Work

• Consider doing noise analysis with Fourier transform rather than with an arbitrary threshold on the variance
• Consider keeping stack of the potential maximums and considering shortest paths through them so that we don’t miss out on locations where reflections are occurring
• NEXT STEP: Gaussian process (nonparametric modeling)!
Acknowledgements

- Thanks IIP!