# Some Optimization Trade-offs in Wireless Network Coding

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***Abstract*— In this paper, we consider different optimization trade-offs in wireless networks with saturated or possibly emptying packet queues and specify the resulting cross-layer interactions in medium access control (MAC) and network layers. We separately consider scheduled and random access for MAC layer, whereas network layer operations are modeled by network coding or plain routing. Our objective is to analyze the trade-offs among performance objectives of maximizing aggregate or minimum throughput over different source-destination pairs and minimizing (transmission and coding) energy costs using a tandem wireless network model with the assumptions of omnidirectional transmissions (and node costs), interference effects and single transceiver per node. We do not limit the throughput measures to a common minimum value for all source-destination pairs (such as the Max-flow Min-cut value) but specify the entire achievable throughput region that provides the optimization constraints for the general case of multiple sources. We also extend the network optimization problem to the case of possibly emptying queues by specifying the stability region as the constraint set. We consider different multicast communication problems (such as broadcasting and unicasting) and discuss the trade-offs with anycast communication (with arbitrary throughput rates).**

## I. Introduction

Wireless network operation involves different optimization trade-offs depending on the cross-layer interactions in MAC and network layers. The problem of joint MAC and wireless network coding (or plain routing as a special case) has been studied in [1]-[2] for the single source multicasting case. The main performance focus was the multicast throughput rate achievable by all destination nodes, which can be maximized by network coding to the Max-flow Min-cut bound compared to plain routing (originally shown for wired networks in [3]). Energy efficiency has been also introduced in [1]-[2], [4]-[6] to the network coding problem as the performance objective to minimize transmission energy costs per unit multicast rate.

The cross-layer design of wireless network coding proposed in [1]-[2] was based on a two-step solution of (a) constructing conflict-free periodic transmission schedules that optimize the multicast rate (or transmission energy per decoded packet) achievable by network coding (or plain routing) and (b) jointly deriving the content of network flows, namely the network codes, to achieve the Max-flow Min-cut rates.

However, the common multicast rate does not fully reflect the aggregate throughput performance (even for the single source case), since throughput demands of different destinations can conflict with each other. The average throughput achievable by different destinations has been studied in [7] for a single source node in wired networks. In contrast, we allow multiple source nodes and jointly consider the throughput rates achievable by different source nodes while imposing common throughput condition for destinations of any source node. For that purpose, we need to specify the entire achievable throughput region representing the multicast throughput rates for different source nodes that can strongly conflict with each other because of limited bandwidth resources. We also extend the analysis to anycast communication by allowing different throughput rates from any source to different destinations.

The classical formulation of (wired or wireless) network coding is based on saturated packet queues that guarantee always availability of packets for transmissions without risk of underflow or delay build-up. This is realized by allowing relay nodes to accumulate incoming packets periodically under the assumption that source nodes have always a packet to transmit. If we allow packet queues to empty, we need to specify the stability region as the joint set of stable packet generation rates at source nodes (such that the queue lengths do not grow to infinity). We evaluate the trade-offs between giving higher priorities to relay or source packets (in the case of underflow).

Because of the complexity introduced by multiple sources and possibly emptying packet queues, we restrict ourselves to a simple tandem line network. We make realistic wireless network assumptions of omnidirectional transmissions, destructive interference effects modeled by classical collision channels and single transceiver per node (preventing simultaneous transmission and reception by any node). For the MAC part, we separately consider scheduled and random access, and specify constraints on the achievable throughput and stability region as function of transmission schedules and probabilities (for both network coding and plain routing in network layer).

We define different throughput criteria to reflect aggregate and minimum throughput rates over all source-destination pairs (in multicast or anycast communication). Since energy efficiency has paramount importance in wireless access because of the limited energy resources, we also introduce transmission and coding energy costs and discuss the optimization trade-offs involving the throughput and energy measures.

The resulting optimization problem involves choosing transmission schedules or probabilities in MAC layer and deriving network flows through network coding (or plain routing) in network layer to optimize the proposed performance criteria subject to the constraints on the achievable throughput or stability region. We specify the optimization trade-offs arising from the cross-layer interactions between network layer and MAC operations, and evaluate the effects of saturated and non-saturated queues on the network optimization problem.

The paper is organized as follows. We present the wireless network model in section II and formulate the cross-layer optimization problem in section III. For scheduled access, we specify in sections IV and V the achievable throughput and stability regions under assumptions of saturated and possibly emptying queues. We discuss the throughput optimization trade-offs in section VI and incorporate the energy efficiency measures in section VII. Then, we extend the results to random access in section VIII and draw conclusions in section IX.

## II. Wireless Network Model

We consider a linear tandem network with node set $N$, as shown in Figure 1. We assume a slotted synchronous system, in which the transmission time of each packet is one time slot, and consider multihop packet propagation in a store-and-forward manner instead of continuous information flows. For the case of saturated queues, we define $\lambda_{i,j}$ as the achievable throughput rate from source node $i$ to destination node $j$ in multicast group $M_i$. For the case of non-saturated queues, each node $i$ generates packets (to be delivered to destination node $j$ in multicast group $M_i$) independently according to a Bernoulli process with rate $\lambda_{i,j}$. We separately consider multicast communication with $\lambda_{i,j} = \lambda_i, i \in N, j \in M_i$, and anycast communication with arbitrary $\lambda_{i,j}, i \in N, j \in N \backslash i$.

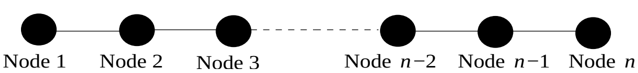


Fig. 1. Tandem Network Model with $n$ nodes.

We assume omnidirectional transmissions of at most one packet per time slot and single transceiver per node. Hence, nodes cannot simultaneously transmit and receive packets and it is necessary to partition them into disjoint sets of transmitters and receivers in every time slot. We consider the classical collision channel model. A packet transmission is successful, if it is the only transmission that reaches the intended receiver. We separately consider fixed conflict-free transmission schedules and random access in MAC layer.

(a) Scheduled Access: We order $n$ nodes from left to right and divide them into three groups such that node $i$ is included in group $m = (i-1)(\text{mod } 3) + 1$, where $1 \leq m \leq 3$. Nodes in different groups $m$ are separately activated for disjoint time fractions $t_m$, where $0 \leq t_m \leq 1$ and $\sum_{m=1}^{3} t_m = 1$.

(b) Random Access: Each node $i$ transmits a packet at any time slot with fixed probability $p_i$. The collided packets remain backlogged until they are successfully received.

We separately consider always availability of source and relay packets as well as possibly emptying queues. Each node $i$ has three separate queues of infinite capacities: the queue $Q_i^1$ stores the source packets it generates and two separate queues $Q_i^2$ and $Q_i^3$ store the relay packets that are incoming from its right and left neighbors, respectively. Each packet coming to node $i$ from one of its neighbors must be transmitted to the neighbor on the other side. In network layer, we separately consider (a) plain routing and (b) network coding operations.

(a) Plain Routing: Each node either transmits a packet from its source queue $Q_i^1$ or a packet from one of its relay queues $Q_i^2$ and $Q_i^3$ that can be combined to a single queue (in a first-come-first-served fashion).

(b) Network Coding: Each node either transmits a source packet, a relay packet or the coded combination of two relay packets one from each relay queue. We consider each packet as a vector of bits and assume $\text{F}_2$ as the field for linear network coding operations such that the bit-sum $x + y$ of two packets $x$ and $y$ is a modulo-2 vector addition of the corresponding vectors of each packet. Separation of transmissions of source and relay packets do not result in any performance loss.

## III. Cross-Layer Network Optimization Problem

We specify the constraints on the achievable or stable throughput rates $\underline{\lambda} = \lambda_{i,j},\ i \in N, j \in M_i$, separately for the cases of network coding and plain routing only. These constraints determine the achievable throughput region $\mathcal{A}$ or stability region $\mathcal{S}$ (depending on whether we consider saturated or non-saturated queues) as function of the transmission schedules $\underline{t} = \{t_m\}_{m=1}^3$ and transmission probabilities $\underline{p} = \{p_i\}_{i \in N}$ under scheduled access and random access, respectively. The regions $\mathcal{A}$ and $\mathcal{S}$ further depend on whether we employ multicast or anycast communication. In the multicast communication case, there are also additional constraints of $\lambda_{i,j} = \lambda_i$, $j \in M_i$, for any node $i$.

We use two separate performance criteria of $\lambda_\Sigma = \sum_{i \in N} \sum_{j \in M_i} \lambda_{i,j}$ and $\lambda_{\min} = \min_{i \in N, j \in M_i} \lambda_{i,j}$ to represent the total and minimum throughput values. The performance objectives will be extended to incorporate energy efficiency in section VII. The optimization problem is formulated as:

$$\begin{aligned} &\text{Select } \underline{\lambda} \text{ and } \underline{t} \text{ or } \underline{p} \\ &\text{to maximize } \lambda_\Sigma \text{ or } \lambda_{\min} \\ &\text{subject to } \underline{\lambda} \in \mathcal{A} \text{ or } \underline{\lambda} \in \mathcal{S} \end{aligned} \tag{1}$$

The dependence of the optimization constraints (namely the regions $\mathcal{A}$ and $\mathcal{S}$) on $\underline{t}$ or $\underline{p}$ leads to cross-layer optimization in MAC and network layers. We will derive the optimization constraints in sections IV and V for the cases of saturated and non-saturated queues under scheduled access. The extension to the random access case will follow in section VIII.

## IV. Achievable Throughput Region for Saturated Queues

We consider saturated packet queues under scheduled access. First, we look at multicast communication problem with $\lambda_{i,j} = \lambda_i$, $j \in M_i$, for any $i \in N$. We define $N_i^r$ as the set of nodes with packets that arrive at node $i$ from the right direction and need to be forwarded to the left neighbor of node $i$, and we define $N_i^l$ as the set of nodes with packets that arrive at node $i$ from the left direction (i.e. from node $i-1$) and need to be forwarded to the right neighbor of node $i$ (i.e. node $i+1$). Let $\Lambda_i^r = \sum_{j \in N_i^r} \lambda_j$ and $\Lambda_i^l = \sum_{j \in N_i^l} \lambda_j$ denote the rate of relay traffic incoming from right and left neighbor nodes of node $i$, respectively. Each node $i$ separately transmits packets it generates and (plain or coded) relay packets for $\tau_i$ and $1 - \tau_i$ fractions of time (whenever it is scheduled to transmit). For multicast communication with $\lambda_{i,j} = \lambda_i$, $j \in M_i$, for any $i \in N$, the achievable throughput rates $\underline{\lambda}$ satisfy

$$0 \leq \lambda_i \leq t_{m(i)}\tau_i, \Lambda_i^r \leq t_{m(i)}(1-\tau_i), \Lambda_i^l \leq t_{m(i)}(1-\tau_i) \quad (2)$$
$$0 \leq \lambda_i \leq t_{m(i)}\tau_i,\ \lambda_i + \Lambda_i^r + \Lambda_i^l \leq t_{m(i)}(1-\tau_i)$$

for network coding and plain routing, respectively, where $m(i) = (i-1) \pmod 3 + 1$, $i \in N$. After eliminating $\tau_i$ from (2), the achievable multicast throughput rates $\underline{\lambda}$ satisfy

$$\lambda_i + \Lambda_i^r \leq t_{m(i)}, \qquad \lambda_i + \Lambda_i^l \leq t_{m(i)} \quad (3)$$
$$\lambda_i + \Lambda_i^r + \Lambda_i^l \leq t_{m(i)}$$

for network coding and plain routing, respectively. The achievable multicast throughput region $\mathcal{A}$ is given by

$$\sum_{m=1}^{3} \max_{i:m(i)=m} [\lambda_i + \max(\Lambda_i^r, \Lambda_i^l)] \leq 1 \quad (4)$$
$$\sum_{m=1}^{3} \max_{i:m(i)=m} [\lambda_i + \Lambda_i^r + \Lambda_i^l] \leq 1$$

for network coding and plain routing. The achievable throughput region $\mathcal{A}$ does not depend on transmission schedules in tandem networks and involves only linear constraints.

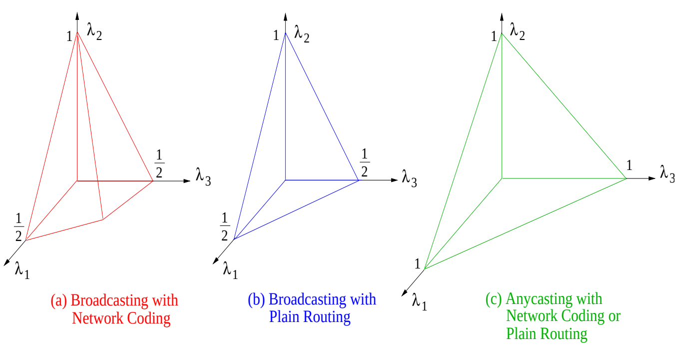


Fig. 2. Achievable throughput regions under network coding and plain routing solutions for broadcast and anycast communication.

Next, we consider anycast communication problem by relaxing the assumption $\lambda_{i,j} = \lambda_i$, $j \in M_i$, for any node $i$ such that node $i$ generates source packets with rate $\max_{j \in N \setminus i} \lambda_{i,j}$ and receives incoming traffic from right and left directions with rates $\sum_{k>i} \max_{j<i} \lambda_{k,j}$ and $\sum_{k<i} \max_{j>i} \lambda_{k,j}$, respectively. Therefore, if we redefine $\lambda_i = \max_{j \in N \setminus \{i\}} \lambda_{i,j}$, $\Lambda_i^r = \sum_{k>i} \max_{j<i} \lambda_{k,j}$ and $\Lambda_i^l = \sum_{k<i} \max_{j>i} \lambda_{k,j}$, the linear inequalities in (2)-(4) also describe the achievable throughput region $\mathcal{A}$ under anycast communication.

The region $\mathcal{A}$ is illustrated in Figure 2 for $n = 3$ under broadcast communication (with $M_i = N \setminus i$, $i = 1, 2, 3$) and anycast communication.

## V. Stability Region for Non-Saturated Queues

The absence of packets at any relay node will either limit the network coding operation to plain forwarding or force the relay nodes to accumulate incoming packets over subsequent time slots increasing the packet delay and reducing the achievable throughput. Therefore, conditions (2)-(4) provide upper bounds on the stability region $\mathcal{S}$ under scheduled access, if we allow packet queues to empty. We consider only stationary network operation, in which queue distributions reach steady state. We do not attempt to analyze the stability at boundary points of equality between average arrival and service rates.

We introduce dynamic network coding (and plain routing) strategies based on instantaneous packet contents and assign different priorities (in transmission order) to relay and source packets (rather than assuming fixed time-divisioned network coding decisions as in the case of saturated queues).

*Strategy 1:* Any node first attempts to transmit relay packet(s) by simply forwarding as in plain routing, if only one relay queue has packets, or by network coding, if both relay queues have packets. Otherwise, the node attempts to transmit a packet from the source queue. The queues $Q_i^2$ and $Q_i^3$ have arrival rates $\Lambda_i^r$ and $\Lambda_i^l$, respectively. Under network coding, the service rate for both queues is $t_{m(i)}$, since node $i$ can successfully transmit one packet from each relay queue (for $t_{m(i)}$ fraction of time). Therefore, queues $Q_i^2$ and $Q_i^3$ are empty with probability $1 - \frac{\Lambda_i^r}{t_{m(i)}}$ and $1 - \frac{\Lambda_i^l}{t_{m(i)}}$, respectively. Since a source packet is transmitted only if both relay queues are empty, the multicast stability conditions under network coding are

$$\lambda_i < t_{m(i)}(1 - \frac{\Lambda_i^r}{t_{m(i)}})(1 - \frac{\Lambda_i^l}{t_{m(i)}}) \quad (5)$$

The resulting stability region $\mathcal{S}$ is strictly suboptimal under network coding and equivalent to the achievable throughput region $\mathcal{A}$ (except the equalities) under plain routing, since both queues can be merged under plain routing such that the total arrival rate is $\Lambda_i^r + \Lambda_i^l$ and the service rate is $t_{m(i)}$.

*Strategy 2:* Source packets are given higher priority in transmission order. Only if the source queue is empty, the relay packets are transmitted by either simply forwarding as in plain routing, if only one relay queue has packets, or by network coding, if both relay queues have packets.

The source queue is empty with probability $1 - \frac{\lambda_i}{t_{m(i)}}$. Hence, each of the relay queues $Q_i^2$ and $Q_i^3$ has the service rate $t_{m(i)}(1 - \frac{\lambda_i}{t_{m(i)}})$. The resulting stability conditions are equivalent to the achievable throughput conditions without equalities. On the other hand, the stability regions under strategies 1 and 2 are equivalent under plain routing.

These results also apply for anycast communication after extending definitions of $\lambda_i, \Lambda_i^r$ and $\Lambda_i^l$, as done in section IV.

## VI. Throughput Optimization Trade-offs

In this section, we evaluate the trade-offs involving throughput measures $\lambda_\Sigma$ and $\lambda_{\min}$. We consider the saturated queue case, although the same optimization results can be approached by following strategy 2 for the case of non-saturated queues.

### A. Anycast Communication

For anycast communication problem, the best traffic demand (in terms of $\lambda_\Sigma$) is one-hop (closest-neighbor) communication, i.e. multicast communication with $M_i = \{i-1, i+1\}$, $i \in N \setminus \{1, n\}$, $M_1 = 2$ and $M_n = n-1$. For both network coding and plain routing solutions, the value of $\lambda_\Sigma$ is maximized to

$\frac{2n}{3}$ (by $t_2 = 1$), if $n (\mathrm{mod}\ 3) = 0$, to $\frac{2n-2}{3}$ (by $t_1 = 0$), if $n (\mathrm{mod}\ 3) = 1$, and to $\frac{2n-1}{3}$ (by $t_3 = 0$), if $n (\mathrm{mod}\ 3) = 2$.

The value of $\lambda_{\min}$ is maximized (by $t_m = \frac{1}{3}$, $m = 1, 2, 3$) to $\frac{1}{3}$ if $n \geq 3$ (and to $\frac{1}{2}$ if $n = 2$) such that $\lambda_\Sigma$ is $\frac{2(n-1)}{3}$ if $n \geq 3$ (and 1 if $n = 2$) . These results provide upper bounds on general multicast (e.g. broadcast and unicast) communication.

### *B. Broadcast Communication*

Next, we consider broadcast communication with $\lambda_{i,j} = \lambda_i$, $j \in M_i = N \backslash i$ and $i \in N$. Under network coding, $\lambda_\Sigma$ is maximized by $\lambda_1 = \lambda_n = \frac{1}{3}$, $\lambda_i = 0$, $i = N \backslash \{1, n\}$ (and $t_m = \frac{1}{3}, m = 1, 2, 3$) to the value of $\frac{2(n-1)}{3}$ for $n > 4$. Under plain routing, $\lambda_\Sigma$ is maximized by $\lambda_1 = \lambda_n = \frac{1}{6}$, $\lambda_i = 0$, $i = N \backslash \{1, n\}$ (and $t_m = \frac{1}{3}, m = 1, 2, 3$) to the value of $\frac{(n-1)}{3}$. The optimal values of $\lambda_\Sigma$ are $n - 1$ for $n \leq 4$ under both network coding and plain routing. Note that as $n$ increases, the value of $\lambda_\Sigma$ increases first for $2 \leq n \leq 4$. Then, the increasing interference effects decrease the value of $\lambda_\Sigma$ for $n = 5$ and slow the increase in $\lambda_\Sigma$ for $n > 5$.

Note that we can only achieve $\lambda_{\min} = 0$ for the optimal solutions in terms of $\lambda_\Sigma$. On the other hand, $\lambda_{\min}$ is maximized to $\frac{1}{3n-5}$, if $n (\mathrm{mod}\ 3) = 0$ or 1, and to $\frac{1}{3n-4}$, if $n (\mathrm{mod}\ 3) = 2$. The resulting value of $\lambda_\Sigma$ is $\frac{n(n-1)}{3n-5}$, if $n (\mathrm{mod}\ 3) = 0$ or 1, and $\frac{n(n-1)}{3n-4}$, if $n (\mathrm{mod}\ 3) = 2$, which can only approach half of the optimal value of $\lambda_\Sigma$, as $n$ increases. For plain routing, $\lambda_{\min}$ is maximized to $\frac{1}{3n}$, if $n > 4$ (and $\frac{1}{2}$, if $n = 2$, $\frac{1}{5}$, if $n = 3$, and $\frac{1}{9}$, if $n = 4$), i.e. the optimal values of $\lambda_{\min}$ under network coding and plain routing solutions approach each other, as $n$ increases. Figure 3 depicts the throughput rates per source-destination pair that are obtained by separately optimizing $\lambda_\Sigma$ and $\lambda_{\min}$ and compares the results with the throughput rates achievable under anycast communication.

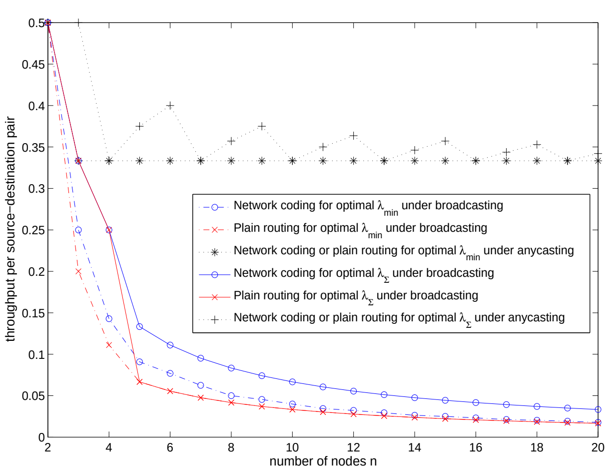


Fig. 3. Achievable throughput rates per source-destination pair under broadcast and anycast communication.

### *C. Unicast Communication*

Alternatively, we consider unicast communication with $|M_i| = 1$, $i \in N$. The least favorable unicast demand is that each destination is chosen as the node that has the largest hop distance from the source node. For network coding, we have $\lambda_\Sigma = n \lambda_{\min}$, where $\lambda_{\min}$ is $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{7}$ for $n = 2, 3, 4$, $\frac{1}{2n}$ for $n = 5, 6, 7$, and $\frac{1}{3\lceil \frac{n}{2} \rceil + 1}$ for $n \geq 8$. For plain routing, $\lambda_{\min}$ is $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{9}$ for $n = 2, 3, 4$, and $\frac{1}{3n-1}$ for $n > 4$.

For the best unicast demand, each destination is the one-hop neighbor of the source node. For both network coding and plain routing, the optimal value of $\lambda_{\min}$ is $\frac{1}{3}$, if $n \geq 3$ (and $\frac{1}{2}$, if $n = 2$), and the optimal value of $\lambda_\Sigma$ is $\lfloor \frac{n}{3} \rfloor$, for $n \geq 3$ (and 1, if $n = 2$). Next, we assume that the destination of each source node is randomly chosen from the rest of nodes. We evaluate in Figure 4 the throughput per source-destination pair for cases of optimal temporal allocation and suboptimal allocation with $t_m = \frac{1}{3}, m = 1, 2, 3$, and compare the results to lower bounds imposed by the least favorable unicast demand.

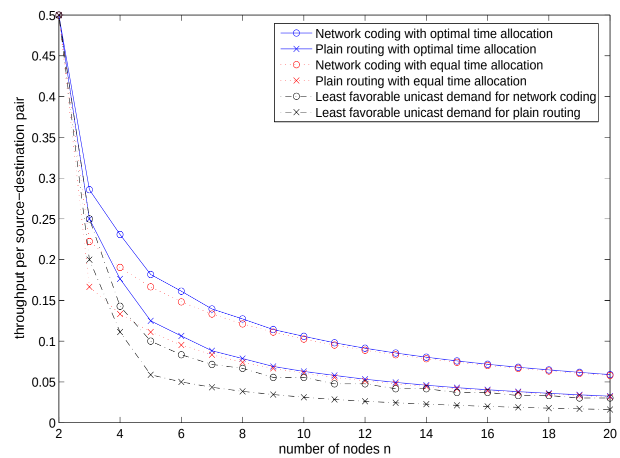


Fig. 4. Achievable throughput rates per source-destination pair under unicast communication.

### *D. Joint Optimization of $\lambda_\Sigma$ and $\lambda_{\min}$*

For broadcast communication, network coding doubles the value of $\lambda_\Sigma$ under plain routing, as $n$ goes to infinity, whereas the improvement in $\lambda_{\min}$ diminishes to zero. For unicast communication, network coding can double both $\lambda_\Sigma$ and $\lambda_{\min}$, as $n$ goes to infinity. This underlines the trade-offs depending on the communication demands. On the other hand, the performance objectives of maximizing $\lambda_\Sigma$ and $\lambda_{\min}$ can also conflict with each other, e.g. $\lambda_{\min}$ is limited to 0 for the optimal solutions in terms $\lambda_\Sigma$ under broadcast communication.

Alternatively, we can formulate a new problem of maximizing a weighted sum of $\lambda_\Sigma$ and $\lambda_{\min}$. Equivalently, we can maximize $\lambda_\Sigma$ subject to $\lambda_{\min} \geq \alpha$ for some positive constant $\alpha$, i.e. $\lambda_{i,j} \geq \alpha$ for all $i \in N$ and $j \in M_i$. We can solve the resulting optimization problem with additional constraints by the Lagrange Multipliers method. For $n > 4$, the optimal value of $\lambda_\Sigma$ is $\frac{n-1}{3}(2 - \alpha(3n - c))$, where constant $c$ is 10, if $n (\mathrm{mod}\ 3) = 0$ or 1, and 8, if $n (\mathrm{mod}\ 3) = 2$.

### *E. Hybrid Network Coding and Plain Routing*

Next, we consider a hybrid network, in which nodes in group $N_R \subseteq N$ are limited to plain routing only (e.g. due to hardware restriction), whereas the rest of the nodes are capable of network coding. The resulting constraints on $\mathcal{A}$ (or $\mathcal{S}$ without equalities, if we follow strategy 2) are

$$\sum_{m=1}^{3} \max \Big( \max_{i \notin N_R : m(i)=m} [\lambda_i + \max(\Lambda_i^r, \Lambda_i^l)], \quad \max_{i \in N_R : m(i)=m} [\lambda_i + \Lambda_i^r + \Lambda_i^l] \Big) \leq 1 \tag{6}$$

For broadcast communication, we evaluate in Figure 5 the throughput rates per source-destination pair (by optimizing either $\lambda_\Sigma$ or $\lambda_{\min}$) for the case $|N_R| = 1$, and compare the results to the network coding case with $|N_R| = 0$ and the plain routing case with $|N_R| = n$.

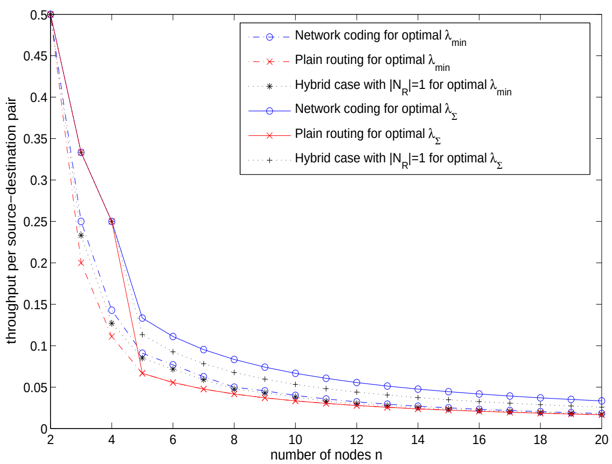


Fig. 5. Achievable (broadcast) throughput rates per source-destination pair in a hybrid network with one node limited to plain routing only.

## VII. Energy Efficiency Trade-offs

### A. Saturated Queues

First, we consider transmission energy costs $E_t(\underline{\lambda})$ per time slot to achieve throughput rates $\underline{\lambda}$. Let $\mathcal{E}_t$ denote the transmission energy cost of each transmission. Node $i$ transmits source packets with rate $\lambda_i \in \mathcal{A}$ incurring transmission energy cost $\mathcal{E}_t \lambda_i$ per time slot. Node $i$ receives relay packets with rates $\Lambda_i^r$ and $\Lambda_i^l$ from the right and left neighbors. For network coding, node $i$ consumes $\mathcal{E}_t$ amount of energy to transmit one coded packet from both relay queues $Q_i^2$ and $Q_i^3$. Since the relay packets arrive at queues $Q_i^2$ and $Q_i^3$ with rates $\Lambda_i^r$ and $\Lambda_i^l$, respectively, the total energy per time slot consumed to relay packets is $\mathcal{E}_t \max(\Lambda_i^r, \Lambda_i^l)$. As a result, we obtain $E_t(\underline{\lambda}) = \mathcal{E}_t \sum_{i=1}^n (\lambda_i + \max(\Lambda_i^r, \Lambda_i^l))$ for network coding. For plain routing, each packet from a relay queue is separately transmitted with cost $\mathcal{E}_t$ such that the total energy consumption per time slot to relay packets is $\mathcal{E}_t(\Lambda_i^r + \Lambda_i^l)$. As a result, we obtain $E_t(\underline{\lambda}) = \mathcal{E}_t \sum_{i=1}^n (\lambda_i + \Lambda_i^r + \Lambda_i^l)$ for plain routing.

Next, we consider the coding energy cost $E_c(\underline{\lambda})$ per time slot to achieve the throughput rates $\underline{\lambda}$. Let $\mathcal{E}_c$ denote the energy cost of a coding or decoding operation (namely the energy cost of binary vector addition). Then, $E_c(\underline{\lambda}) = 3\mathcal{E}_c \sum_{i=1}^n \min(\Lambda_i^r, \Lambda_i^l)$, since relay node $i$ performs coding operation with rate $\min(\Lambda_i^r, \Lambda_i^l)$ and each coding operation is accompanied with two decoding operations at neighbor nodes.

Network coding reduces the total (transmission and coding) energy cost per packet compared to plain routing, if $\mathcal{E}_c < \frac{\mathcal{E}_t}{3}$. For $\mathcal{E}_t = 3\mathcal{E}_c = 1$, we evaluate in Figure 6 the transmission and coding energy costs per source-destination pair to achieve optimal throughput rates (in terms of $\lambda_{\min}$) under unicast and broadcast communication. The results show that energy costs increase to achieve the rate improvement of network coding.

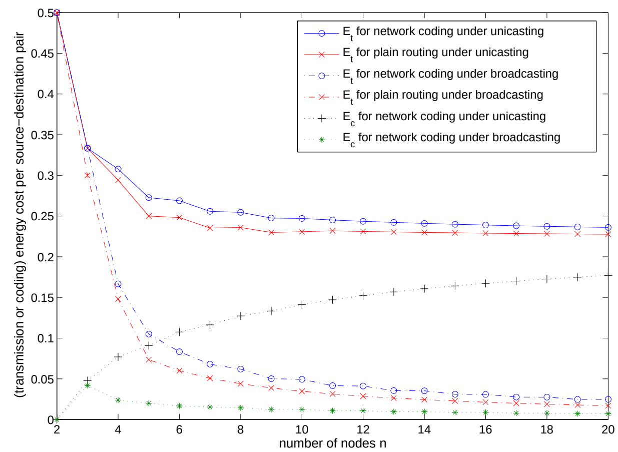


Fig. 6. Transmission and coding energy costs per source-destination pair to achieve optimal throughput rates under unicast and broadcast communication.

### B. Non-Saturated Queues

Next, we allow packet queues to empty. A source packet is transmitted by node $i$ with rate $\lambda_i$ incurring transmission energy cost $\mathcal{E}_t$. In strategy 1, a relay packet is transmitted by node $i$, if there exists a packet in relay queue $Q_i^2$ or $Q_i^3$. This occurs with probability $1-(1-t_{m(i)}^{-1}\Lambda_i^l)(1-t_{m(i)}^{-1}\Lambda_i^r)$, since queues $Q_i^2$ and $Q_i^3$ are empty with probabilities $1-t_{m(i)}^{-1}\Lambda_i^l$ and $1-t_{m(i)}^{-1}\Lambda_i^r$. Hence, the transmissions of relay packets incur energy cost $\mathcal{E}_t(t_{m(i)}^{-1}(\Lambda_i^l+\Lambda_i^r) - t_{m(i)}^{-2}\Lambda_i^l\Lambda_i^r)$. Since node $i$ is activated for $t_{m(i)}$ fraction of time, the total transmission energy cost per time slot under network coding is given by $E_t(\underline{\lambda}) = \mathcal{E}_t \sum_{i=1}^n (\lambda_i + \Lambda_i^r + \Lambda_i^l - t_{m(i)}^{-1}\Lambda_i^r\Lambda_i^l)$ for strategy 1.

In strategy 2, a relay packet is transmitted by node $i$, if there exists no source packet at queue $Q_i^1$ and there exists at least one packet in $Q_i^2$ or $Q_i^3$. The service rate is $t_{m(i)}$ for queue $Q_i^1$ and $t_{m(i)}(1-t_{m(i)}^{-1}\lambda_i)$ for both queues $Q_i^1$ and $Q_i^3$, since queue $Q_i^1$ is idle with probability $1-t_{m(i)}^{-1}\lambda_i$ such that packets from $Q_i^2$ and $Q_i^3$ are transmitted only with probability $1-t_{m(i)}^{-1}\lambda_i$. Since queues $Q_i^1$, $Q_i^2$ and $Q_i^3$ are empty with probabilities $(1-t_{m(i)}^{-1}\lambda_i)$, $(1-\frac{t_{m(i)}^{-1}\Lambda_i^r}{1-t_{m(i)}^{-1}\lambda_i})$ and $(1-\frac{t_{m(i)}^{-1}\Lambda_i^l}{1-t_{m(i)}^{-1}\lambda_i})$, we obtain $E_t(\underline{\lambda}) = \mathcal{E}_t \sum_{i=1}^n (\lambda_i + \Lambda_i^r + \Lambda_i^l - \frac{t_{m(i)}^{-1}\Lambda_i^r\Lambda_i^l}{1-t_{m(i)}^{-1}\lambda_i})$ for strategy 2.

For plain routing, both strategies achieve the same transmission energy cost as in the case of saturated queues.

The coding energy cost per time slot is $3\mathcal{E}_c \sum_{i=1}^n t_{m(i)}^{-1}\Lambda_i^r\Lambda_i^l$ and $3\mathcal{E}_c \sum_{i=1}^n \frac{t_{m(i)}^{-1}\Lambda_i^r\Lambda_i^l}{1-t_{m(i)}^{-1}\lambda_i}$ for strategies 1 and 2, respectively.

For any given $\underline{\lambda}$, strategy 1 outperforms strategy 2 in terms of coding (energy) efficiency, although strategy 2 is more efficient in terms of throughput and transmission energy cost. If we consider the total (transmission and coding) energy cost, strategy 1 has better performance, if $\mathcal{E}_c < \frac{\mathcal{E}_t}{3}$ (which is the same as the coding efficiency condition of choosing network coding over plain routing in the case of saturated queues).

Note that the energy costs are functions of temporal allocation compared to the case of saturated queues leading to a non-linear objective function, whereas the optimization constraints are non-linear for strategy 1 and linear for strategy 2.

These results also apply for anycast communication after extending definitions of $\lambda_i, \Lambda_i^r$ and $\Lambda_i^l$, as done in section IV.

## VIII. Extension to Random Access

In this section, we consider random access instead of scheduled access and restrict the analysis to the case of saturated queues. Otherwise, the analysis should involve the problem of interacting queues [8]. This extension is beyond the scope of this paper but remains as a future research objective.

In multicast communication, a source packet may have to be delivered to both neighbors for throughput credit. We consider three transmission methods for source packets. In method A, nodes transmit new source packets without channel feedback. Only reception of a packet by both neighbors contributes to the throughput. Method B uses immediate channel feedback and allows a node to transmit a new source packet only if the previous source packet has been received by both neighbors. In method C, each node computes a linear combination of the source packets that have not been decoded yet by both neighbors, and transmits the resulting coded packet. For instance, if transmission of packet $x$ by node $i$ is only received by $i+1$, $i$ transmits $x+y$ (instead of $x$ as in method B). If $x+y$ is successfully received, $i$ needs to deliver $y$ only to $i-1$ (rather than to both $i-1$ and $i+1$ as in method B). The achievable multicast throughput rates $\underline{\lambda}$ are

$$\begin{aligned} s_i^l \lambda_i + \gamma_i(s_i^r, s_i^l)\Lambda_i^r &\leq p_i s_i^l \gamma_i(s_i^r, s_i^l) \\ s_i^r \lambda_i + \gamma_i(s_i^r, s_i^l)\Lambda_i^l &\leq p_i s_i^r \gamma_i(s_i^r, s_i^l) \end{aligned} \quad (7)$$

for network coding solutions, and

$$s_i^r s_i^l \lambda_i + s_i^r \gamma_i(s_i^r, s_i^l)\Lambda_i^r + s_i^l \gamma_i(s_i^r, s_i^l)\Lambda_i^l \leq p_i s_i^l s_i^r \gamma_i(s_i^r, s_i^l) \quad (8)$$

for plain routing solutions, where $\gamma_i(s_i^r, s_i^l) = s_i^r s_i^l$, $\frac{{s_i^l}^2 {s_i^r}^2 + {s_i^l}^2 s_i^r(1-s_i^r) + {s_i^r}^2 s_i^l(1-s_i^l)}{s_i^l s_i^r + {s_i^l}^2(1-s_i^r) + {s_i^r}^2(1-s_i^l)}$ or $\min(s_i^r, s_i^l)$ for method A, B or C, respectively. The region $\mathcal{A}$ is optimized by method C.

For anycast communication, we extend definitions of $\lambda_i$, $\Lambda_i^r$ and $\Lambda_i^l$, as we did in section IV. Since the delivery of a source packet from node $i$ to any neighbor node can result in throughput credit, we redefine $\gamma_i(s_i^r, s_i^l) = \max(s_i^r, s_i^l)$. Then, the achievable anycast throughput rates $\underline{\lambda}$ satisfy conditions (7)-(8) (with new definitions of $\lambda_i$, $\Lambda_i^r$, $\Lambda_i^l$ and $\gamma_i(s_i^r, s_i^l)$).

Next, we consider the problem of maximizing $\lambda_\Sigma$. Although the constraints on $\mathcal{A}$ are independent of $\underline{t}$ in scheduled access, they are non-linear functions of $\underline{p}$ in random access. We solve this non-linear constrained optimization problem using the Sequential Quadratic Programming method. The throughput rates per source-destination pair are depicted in Figure 7 for broadcast and anycast communication.

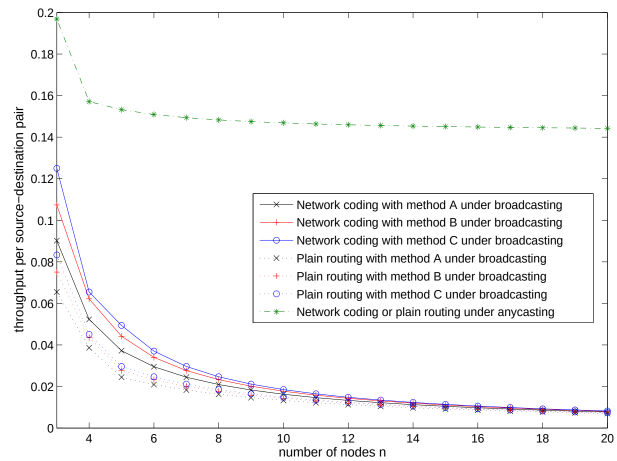


Fig. 7. Achievable throughput rates per source-destination pair in random access.

## IX. Conclusions

In this paper, we addressed different optimization trade-offs in wireless network coding (or plain routing as a special case) for separate MAC assumptions of scheduled and random access. We considered both cases of saturated and possibly emptying queues, and specify the achievable throughput and stability regions using a linear tandem network model. We proposed different problems of optimizing throughput measures and (transmission and coding) energy costs subject to constraints on achievable throughput and stability regions. We discussed the resulting optimization trade-offs that strongly depend on cross-layer interactions between MAC and network layers, assumptions of saturated or non-saturated queues and (multicast or anycast) communication demands. The optimal network operation relies on cooperation of nodes. As future work, network coding and MAC should be formulated as a non-cooperative game of optimizing individual performance.

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