Node-Based Distributed Optimal Control of Wireless Networks

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***Abstract*— We present a unified analytical framework within which power control, routing, and congestion control for wireless networks can be optimized on a node-by-node basis. We consider a multi-commodity flow model for interference-limited wireless networks, and develop distributed scaled gradient projection algorithms which iteratively adjust power control and routing schemes at individual nodes to minimize convex link costs. We provide locally computable scaling matrices which guarantee fast convergence of the algorithms to the global optimum from any initial condition. Furthermore, we show that congestion control can be seamlessly incorporated into our framework with the introduction of virtual overflow links.**

I. Introduction

Wireless networks differ fundamentally from wireline networks in that link capacities are variable quantities determined by transmission powers, channel fading, user mobility, the coding/modulation scheme, and other factors. In view of this, the traditional problem of routing and congestion control must be jointly optimized with power control and rate allocation at the physical layer. Moreover, the inherent decentralized nature of wireless networks mandates that efficient and distributed algorithms be developed to implement this optimization. In this paper, we present an analytical framework in which power control, routing, and congestion control for wireless networks can be optimized in an integrated manner. We then develop distributed network algorithms to achieve the joint optimum.

The study of network optimization initially concentrated on traffic routing in wireline networks. An elegant analysis of the optimal routing problem within a multi-commodity flow setting is given in [1]. Distributed algorithms using the gradient method are developed in [1], [2]. With the advent of variable-rate communications, congestion control has been an important topic of investigation. In [3]–[5], congestion control is optimized by maximizing the utilities of contending sessions with elastic rate demands subject to fixed link capacity constraints in wireline networks. The combination of congestion control and routing is studied in [6], [7].

The problem of power control has been extensively studied for CDMA networks. Previous work at the physical layer [8], [9] generally focus on the optimal trade-off between transmission powers and Signal-to-Interference-plus-Noise-Ratios (SINR). More recently, cross-layer optimization for wireless networks has been investigated in [9], [10]. These papers typically assume that all available paths to the destinations are known at the source nodes, which make the routing decisions. Due to frequent changes in network topology and node activity, however, the source routing approach may not be practical nor even desirable for large-scale wireless networks.

In this work, we present a framework in which the power control, routing, and congestion control functionalities at the physical, MAC, network, and transport layers of a wireless network can be jointly optimized. We perform this joint optimization on a *node-by-node* basis, i.e., each node decides on its total transmission power, power allocation, and traffic allocation on its outgoing links based on a limited number of control messages from other nodes in the network. We adopt interference-limited physical-layer models where link rates are functions of the SINR at the receivers. These include CDMA network models as a special case. We use a multi-commodity flow model to analyze the data traffic. We investigate the case where power control and routing variables are chosen to minimize convex link costs reflecting, for instance, average queueing delays. We develop a class of distributed scaled gradient projection algorithms and show that with appropriate scaling matrices, the algorithms jointly converge to the global optimum from any initial configuration with finite cost. Furthermore, for each of the algorithms, we specify scaling matrices which lead to fast algorithm convergence and distributed computation. Finally, we demonstrate that congestion control for users with elastic rate demands can be seamlessly incorporated into our analytical framework. We consider a situation in which the network seeks to balance user demands and the network cost by maximizing the aggregate session utility minus the total network cost. With the introduction of virtual overflow links, we show that the problem of jointly optimizing power control, routing, and congestion control in a wireless network can be made equivalent to a problem involving only power control and routing in a virtual wireless network.

II. Network Model and Problem Formulation

A. Network and Flow Model

Let the wireless network be modelled by a directed and connected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ and $\mathcal{E}$ are node

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and edge sets, respectively. A node $i \in \mathcal{N}$ represents a wireless transceiver and an edge $(i,j) \in \mathcal{E}$ corresponds to a unidirectional wireless link from node i to j. We assume that the wireless network is *interference-limited*, so that the capacity of link (i,j), denoted by C_{ij}, is a nonnegative function of the signal-to-interference-plus-noise ratio (SINR) at the receiver of the link, i.e., $C_{ij} = C(SINR_{ij})$, where

$$SINR_{ij}(\boldsymbol{P}) = \frac{G_{ij}P_{ij}}{G_{ij}\sum_{n\neq j} P_{in} + \sum_{m\neq i} G_{mj}\sum_n P_{mn} + N_j}.$$

Here, P_{mn} is the transmission power on link (m,n), G_{mj} denotes the (constant) path gain from node m to j, N_j is the noise power at node j's receiver. We assume $C(\cdot)$ is increasing, concave, and twice continuously differentiable. For instance, in CDMA networks using single-user decoding, the information-theoretic link capacity per unit bandwidth in the high-SINR regime is approximately given by $C_{ij} = \log(SINR_{ij})$. Assume every node i is subject to an individual power constraint and denote the set of all feasible power vectors by $\Pi = \{\boldsymbol{P} \geq \boldsymbol{0} : \sum_j P_{ij} \leq \bar{P}_i, \forall i \in \mathcal{N}\}$.

We adopt a *flow model* [11] to analyze the transmission of data inside the network. The flow model is reasonable for networks where the traffic statistics change slowly over time. Such is the case when each session consists of a large number of independent stochastic arrival processes, and no individual process contributes significantly to the aggregate session rate [11]. As we show, the flow model is particularly amenable to cost minimization and distributed computation. Consider a collection $\mathcal{W}$ of communication sessions. Each session w is identified by its source-destination node pair $(O(w), D(w))$. Assume the total incoming rate for session w is a positive constant r_w and denote session w's flow rate on link (i,j) by $f_{ij}(w)$. We then have the following flow conservation relations. For all $w \in \mathcal{W}$,

$$\begin{aligned}
&f_{ij}(w) \geq 0, && \forall (i,j) \in \mathcal{E},\\
&\sum_{j\in\mathcal{O}_i} f_{ij}(w) = r_w \equiv t_i(w), && \text{if } i = O(w),\\
&f_{ij}(w) = 0, && \text{if } i = D(w),\\
&\sum_{j\in\mathcal{O}_i} f_{ij}(w) = \sum_{j\in\mathcal{I}_i} f_{ji}(w) \equiv t_i(w), && \text{otherwise},
\end{aligned} \tag{1}$$

where $\mathcal{O}_i = \{j : (i,j) \in \mathcal{E}\}$ and $\mathcal{I}_i = \{j : (j,i) \in \mathcal{E}\}$, and $t_i(w)$ is the total incoming rate of session w's traffic at node i. For brevity, denote the set of all flow vectors $\boldsymbol{f} = (f_{ij}(w))_{(i,j)\in\mathcal{E}, w\in\mathcal{W}}$ satisfying (1) by $\mathcal{F}$. Finally, the total flow rate on link (i,j) is $F_{ij} = \sum_{w\in\mathcal{W}} f_{ij}(w)$.

B. Network Cost and Basic Optimization Problem

Let the network cost, denoted by D, be the sum of costs on all the links. The cost on link (i,j) is given by function $D_{ij}(C_{ij}, F_{ij})$. The network model and cost functions are illustrated in Figure 1. In previous literature on optimal routing in wired networks [1], [2], [12], where the link capacities are fixed, the link cost $D_{ij}(C_{ij}, \cdot)$ is usually assumed to be increasing and convex in F_{ij}. Wireless networks, on the other

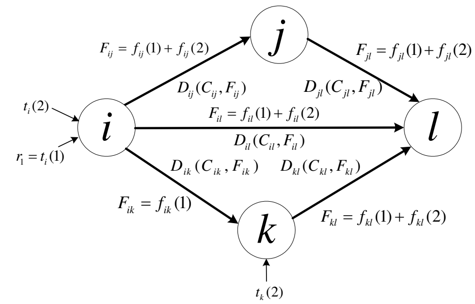

Fig. 1. An example network. Session 1 of rate r_1 originates from node i and ends at node l. Session 2, which originates elsewhere in the network and is destined also for node l, enters this part of the network at nodes i (at rate $t_i(2)$) and k (at rate $t_k(2)$). Node i routes session 1 to j, k, and l, and routes session 2 to j and l. Node k forwards session 2 directly to l.

hand, provide the possibility of controlling link capacities by, for instance, varying transmission powers. Since increasing link capacity reduces link cost such as queueing delay, we assume that $D_{ij}(\cdot, F_{ij})$ is a continuous, decreasing, and convex function of C_{ij} for each fixed F_{ij}. We further assume $D_{ij}(C_{ij}, F_{ij})$ is twice continuously differentiable in the region $\mathcal{X} = \{(C_{ij}, F_{ij}) : 0 \leq F_{ij} < C_{ij}\} \cup \{(0,0)\}$. Also we define $D_{ij}(C_{ij}, F_{ij}) = \infty$ for $F_{ij} \geq C_{ij}$ and $F_{ij} > 0$ to implicitly impose the link capacity constraint. To summarize, for all (i,j), the cost function $D_{ij} : \mathbb{R}_+ \times \mathbb{R}_+ \mapsto \mathbb{R}_+$ satisfies

$$\frac{\partial D_{ij}}{\partial C_{ij}} < 0,\ \frac{\partial D_{ij}}{\partial F_{ij}} > 0,\ \frac{\partial^2 D_{ij}}{\partial C_{ij}^2} \geq 0,\ \text{and}\ \frac{\partial^2 D_{ij}}{\partial F_{ij}^2} \geq 0,$$

if $(C_{ij}, F_{ij}) \in \mathcal{X}$, and $D_{ij}(C_{ij}, F_{ij}) = \infty$ otherwise. An example cost function satisfying the above conditions is

$$D_{ij}(C_{ij}, F_{ij}) = \frac{F_{ij}}{C_{ij} - F_{ij}},\ \text{for } 0 \leq F_{ij} < C_{ij}, \tag{2}$$

which gives the expected number of packets in the queue of link (i,j) under an $M/M/1$ approximation. Summing over all the links, the network cost gives the average number of packets in the network.[2]

We now formulate the Jointly Optimal Power control and Routing (JOPR) problem: given the session input rates (r_w),

$$\begin{aligned}
\text{minimize} \quad & \sum_{(i,j)\in\mathcal{E}} D_{ij}(C_{ij}, F_{ij}) \\
\text{subject to} \quad & C_{ij} = C(SINR_{ij}(\boldsymbol{P})),\ \forall (i,j) \in \mathcal{E} \\
& F_{ij} = \sum_{w\in\mathcal{W}} f_{ij}(w),\ \forall (i,j) \in \mathcal{E}, \\
& \boldsymbol{P} \in \Pi, \quad (f_{ij}(w)) \in \mathcal{F}.
\end{aligned} \tag{3}$$

The objective function in (3) is convex in all flow variables. It is convex in $\boldsymbol{P}$ if every C_{ij} is concave in $\boldsymbol{P}$. Unfortunately, given that $C_{ij} = C(SINR_{ij})$ is strictly increasing, $\nabla^2 C_{ij}(\boldsymbol{P})$ cannot be negative definite. However, if

$$C''(x) \cdot x + C'(x) \leq 0, \quad \forall x \geq 0, \tag{4}$$

[2]Note that the cost function in (2) is not jointly convex in (C_{ij}, F_{ij}). For analytical purposes, we sometimes require joint convexity. For this case, an appropriate cost function is $D_{ij} = 1/(C_{ij} - F_{ij})$.

then with changes of variables $S_{mn} = \ln P_{mn}$ [13], $\nabla^2 C_{ij}(\boldsymbol{S})$ is negative definite and the objective function is convex in $\boldsymbol{S}$. This observation is first made in [14], where the capacity function is required to satisfy $-xC''(x)/C'(x) \in [1, 2]$. Our results, however, indicate that the upper bound 2 can be removed. The detailed proof is omitted here for brevity. In what follows, we assume that $C(\cdot)$ satisfies (4). An example of this is $C(x) = \log(x)$, the approximate capacity of a link in a high-SINR CDMA network.

III. Distributed Power Control and Routing

A. Node-Based Routing

To solve the JOPR problem, we first investigate distributed routing schemes for adapting link flow rates. Path-based *source routing* methods in wired networks [4]–[7] generally assume that source nodes have comprehensive information about all paths to their destinations. Wireless networks, however, are characterized by frequent topology changes. Thus, it is neither practical nor even desirable for sources to frequently obtain current path information. We therefore focus on *node-based* routing [1], where each node decides on its outgoing traffic allocation based on limited information from its neighboring nodes.

To decouple flow conservation constraints (1) across different nodes, we adopt the routing variables [1] defined for all $i \in \mathcal{N}$ and $w \in \mathcal{W}$ in terms of *link flow fractions*:

$$\text{Routing variables:} \quad \phi_{ik}(w) \equiv \frac{f_{ik}(w)}{t_i(w)},\ k \in \mathcal{O}_i.$$

Flow conservation translates into the following constraints for $\{\phi_{ik}(w)\}$: for all $i \in \mathcal{N}$ and $w \in \mathcal{W}$, $\phi_{ik}(w) \geq 0\ \forall k \in \mathcal{O}_i$, $\sum_{k \in \mathcal{O}_i} \phi_{ik}(w) = 1$ if $i \neq D(w)$, and $\phi_{ik}(w) = 0\ \forall k \in \mathcal{O}_i$ if $i = D(w)$.

B. Node-Based Power Control

As in the case of the routing variables, define the *power control* and *power allocation* variables as follows:

$$\text{Power allocation variables:} \quad \eta_{ik} \equiv \frac{P_{ik}}{P_i},\ \forall (i,k) \in \mathcal{E},$$

$$\text{Power control variables:} \quad \gamma_i \equiv \frac{S_i}{\bar{S}_i},\ \forall i \in \mathcal{N},$$

where $S_i = \ln P_i$ and $\bar{S}_i = \ln \bar{P}_i$. With appropriate scaling, we can always let $\bar{P}_i > 1$ for all $i \in \mathcal{N}$ so that $\bar{S}_i > 0$. Thus, the constraints for η_{ik} and γ_i are: $\eta_{ik} \geq 0,\ \forall (i,k) \in \mathcal{E}$, $\sum_{k \in \mathcal{O}_i} \eta_{ik} = 1, \gamma_i \leq 1,\ \forall i \in \mathcal{N}$.

C. Cost Gradients

To solve the optimization problem with an iterative gradient projection method, it is necessary to compute the cost gradients with respect to optimization variables. For the routing variables, the gradients are given in [1] as follows.

$$\frac{\partial D}{\partial \phi_{ik}(w)} = t_i(w) \cdot \delta\phi_{ik}(w), \quad \forall k \in \mathcal{O}_i,$$

where the *marginal routing cost indicator* is

$$\delta\phi_{ik}(w) = \frac{\partial D_{ik}}{\partial F_{ik}}(C_{ik}, F_{ik}) + \frac{\partial D}{\partial r_k(w)}. \tag{5}$$

Here, $\frac{\partial D}{\partial r_k(w)}$ stands for the marginal delay due to a unit increment of session w's input traffic at k. It is computed recursively by [1]: $\frac{\partial D}{\partial r_k(w)} = 0$ if $k = D(w)$, and

$$\frac{\partial D}{\partial r_i(w)} = \sum_{k \in \mathcal{O}_i} \phi_{ik}(w) \left[\frac{\partial D_{ik}}{\partial F_{ik}} + \frac{\partial D}{\partial r_k(w)}\right],\ i \neq D(w). \tag{6}$$

The gradients in the power allocation variables are

$$\frac{\partial D}{\partial \eta_{ik}} = P_i \left[-\sum_{(m,n)} \frac{\partial D_{mn}}{\partial C_{mn}} \frac{C'_{mn} G_{mn} G_{in} P_{mn}}{IN_{mn}^2} + \delta\eta_{ik}\right],$$

where the *marginal power allocation cost indicator* is

$$\delta\eta_{ik} = \frac{\partial D_{ik}}{\partial C_{ik}} \frac{C'_{ik} G_{ik}}{IN_{ik}}(1 + SINR_{ik}). \tag{7}$$

In above equations, C'_{mn} stands for $C'(SINR_{mn})$ and IN_{mn} denotes the interference plus noise on link (m,n):

$$IN_{mn} = G_{mn}(P_m - P_{mn}) + \sum_{l \neq m} G_{ln} P_l + N_n.$$

The gradients in the power control variables are $\partial D/\partial\gamma_i = \bar{S}_i \delta\gamma_i$, where the *marginal power control cost indicator* is

$$\delta\gamma_i = P_i \left[-\sum_{(m,n)} \frac{\partial D_{mn}}{\partial C_{mn}} \frac{C'_{mn} G_{mn} G_{in} P_{mn}}{IN_{mn}^2} + \sum_{j \in \mathcal{O}_i} \delta\eta_{ij} \cdot \eta_{ij}\right]. \tag{8}$$

D. Conditions for Optimality

Theorem 1: For a feasible set of routing and power configurations $\{\phi_{ik}(w)\}_{w \in \mathcal{W}, (i,k) \in \mathcal{E}}$, $\{\eta_{ik}\}_{(i,k) \in \mathcal{E}}$ and $\{\gamma_i\}_{i \in \mathcal{N}}$ to be the solution of the JOPR problem in (3), the following conditions are necessary. For all $w \in \mathcal{W}$ and $i \neq D(w)$ with $t_i(w) > 0$, there exists a constant $\lambda_i(w)$ such that

$$\delta\phi_{ik}(w) \begin{cases} = \lambda_i(w), & \text{if } \phi_{ik}(w) > 0, \\ \geq \lambda_i(w), & \text{if } \phi_{ik}(w) = 0, \end{cases} \tag{9}$$

and for all $i \in \mathcal{N}$, all $\eta_{ik} > 0$,[3] and there exists a constant ν_i such that

$$\delta\eta_{ik} = \nu_i, \tag{10}$$

$$\frac{\delta\gamma_i}{P_i} \begin{cases} = 0, & \text{if } \gamma_i < 1, \\ \leq 0, & \text{if } \gamma_i = 1. \end{cases} \tag{11}$$

If the link cost functions $D_{ik}(C_{ik}, F_{ik})$ are also jointly convex in (C_{ik}, F_{ik}), then these conditions are sufficient for optimality if (9) holds at every $i \neq D(w)$ whether $t_i(w) > 0$ or not.

Due to limited space, the proof is omitted here and can be found in [15].

[3] Assumption (4) implicitly implies $C(0) = -\infty$. Thus, no link should have zero transmission power.

IV. Network Algorithms

After obtaining the optimality conditions, we come to the question of how individual nodes can adjust their local optimization variables to achieve a globally optimal configuration. We design a set of algorithms that update the nodes' routing, power allocation, and power control variables in a distributed manner, so as to asymptotically converge to the optimum. Since the JOPR problem in (3) involves the minimization of a convex objective over convex regions, the class of *scaled gradient projection* algorithms [16] is appropriate for providing a distributed solution with fast convergence rates.

A. Routing Algorithm (RT) [2]

Consider node $i \neq D(w)$. Omit the session index w for brevity. At the kth iteration, the routing algorithm in [2] updates the current routing vector $\boldsymbol{\phi}_i^k$ by

$$\boldsymbol{\phi}_i^{k+1} = RT(\boldsymbol{\phi}_i^k) = \left[\boldsymbol{\phi}_i^k - (M_i^k)^{-1} \cdot \delta\boldsymbol{\phi}_i^k\right]^+_{M_i^k}. \tag{12}$$

Here, $\delta\boldsymbol{\phi}_i^k$ is the vector $(\delta\phi_{ij}^k)$. The scaling matrix M_i^k is positive definite if $t_i^k > 0$, and M_i^k is the zero matrix if $t_i^k = 0$. The operator $[\cdot]^+_{M_i^k}$ denotes projection on the feasible set

$$\mathcal{F}^k_{\boldsymbol{\phi}_i} = \left\{\boldsymbol{\phi}_i \geq \mathbf{0} : \phi_{ij} = 0, \forall j \in \mathcal{B}_i^k \text{ and } \sum_{j \in \mathcal{O}_i} \phi_{ij} = 1\right\},$$

($\mathcal{B}_i^k$ represents the set of *blocked nodes* of i relative to the session under consideration at the kth iteration.[4]) relative to the norm induced by matrix M_i^k, i.e.,

$$[\tilde{\boldsymbol{\phi}}_i]^+_{M_i^k} = \arg\min_{\boldsymbol{\phi}_i \in \mathcal{F}^k_{\boldsymbol{\phi}_i}} \langle \boldsymbol{\phi}_i - \tilde{\boldsymbol{\phi}}_i, M_i^k(\boldsymbol{\phi}_i - \tilde{\boldsymbol{\phi}}_i)\rangle.$$

In order for node i to evaluate the terms $\delta\phi_{ik}(w)$, it needs to collect local measures $\frac{\partial D_{ik}}{\partial F_{ik}}$ as well as reports of marginal costs $\frac{\partial D}{\partial r_k(w)}$ from $k \in \mathcal{O}_i$ (cf. (5)). Moreover, it needs to calculate its own marginal cost $\frac{\partial D}{\partial r_i(w)}$ using (6), and provide the result to corresponding immediate upstream nodes.

B. Power Allocation Algorithm (PA)

At the kth iteration at node i, the current power allocation vector $\boldsymbol{\eta}_i^k$ is updated by

$$\boldsymbol{\eta}_i^{k+1} = PA(\boldsymbol{\eta}_i^k) = \left[\boldsymbol{\eta}_i^k - (Q_i^k)^{-1} \cdot \delta\boldsymbol{\eta}_i^k\right]^+_{Q_i^k}. \tag{13}$$

Here, $\delta\boldsymbol{\eta}_i^k = \left(\delta\eta_{ij}^k\right)_{j \in \mathcal{O}_i}$, Q_i^k is positive definite, and $[\cdot]^+_{Q_i^k}$ denotes projection on the feasible set $\mathcal{F}_{\boldsymbol{\eta}_i} = \{\boldsymbol{\eta}_i \geq \mathbf{0} : \sum_{j \in \mathcal{O}_i} \eta_{ij} = 1\}$ relative to the norm induced by Q_i^k.

Note that the derivation of marginal power allocation cost indicators $\delta\eta_{ik}$ *involves only locally obtainable measures* (cf. (7)). Thus, the power allocation algorithm does not need a protocol for collecting external control messages.

C. Power Control Algorithm (PC)

At the kth iteration, the vector of all nodes' power control variables $\boldsymbol{\gamma}^k = (\gamma_i^k)_{i \in \mathcal{N}}$ is updated by

$$\boldsymbol{\gamma}^{k+1} = PC(\boldsymbol{\gamma}^k) = \left[\boldsymbol{\gamma}^k - (V^k)^{-1} \cdot \delta\boldsymbol{\gamma}^k\right]^+_{V^k}. \tag{14}$$

Here, $\delta\boldsymbol{\gamma}^k = (\delta\gamma^k)$, V^k is a positive definite matrix, and $[\cdot]^+_{V^k}$ denotes projection on the feasible set $\mathcal{F}_{\boldsymbol{\gamma}} = \{\boldsymbol{\gamma} : \gamma_i \leq 1, \forall i\}$.

In general, (14) represents a coordinated network-wide power control algorithm. It becomes amenable to distributed implementation if and only if a diagonal scaling matrix is used, i.e., $V^k = \mathrm{diag}(v_i^k)_{i \in \mathcal{N}}$. In this case, (14) is then transformed to $|\mathcal{N}|$ parallel local sub-programs, each having the form

$$\gamma_i^{k+1} = PC(\gamma_i^k) = \min\{1, \gamma_i^k - (v_i^k)^{-1}\delta\gamma_i^k\}. \tag{15}$$

The formula for $\delta\gamma_i$ from (8) involves measures from all links in the network. We thus need to design a procedure to let every node i compute $\delta\gamma_i$ prior to the algorithm iteration. The following protocol is based on a rearrangement of (8):

$$\frac{\delta\gamma_i}{P_i} = \sum_{n \neq i}\left[-G_{in} \sum_{m \in \mathcal{I}_n} \frac{\partial D_{mn}}{\partial C_{mn}} \frac{C'_{mn} SINR_{mn}}{IN_{mn}}\right] + \sum_{n \in \mathcal{I}_i} \delta\eta_{in} \cdot \eta_{in}.$$

Power Control Message Exchange Protocol: Let each node n sum up the measures from all its incoming links (m, n) to form the power control message:

$$MSG_n = \sum_{m \in \mathcal{I}_n} \frac{\partial D_{mn}}{\partial C_{mn}} \frac{C'_{mn} SINR_{mn}}{IN_{mn}},$$

which is then broadcast to the whole network. Upon obtaining MSG_n, node i processes it as follows: if n is a next-hop neighbor of i, node i multiplies MSG_n with path gain G_{in} and subtracts the product from the value of local measure $\delta\eta_{in} \cdot \eta_{in}$; otherwise, node i multiplies MSG_n with $-G_{in}$. Finally, node i adds up all the processed messages, and this sum multiplied by P_i equals $\delta\gamma_i$. Note that this protocol requires *only one message from each node in the network* per iteration.

D. Convergence of Algorithms

Theorem 2: Assume an initial loop-free routing configuration $\{\boldsymbol{\phi}_i^0(w)\}$ and valid transmission power configuration $\{\boldsymbol{\eta}_i^0\}$ and $\boldsymbol{\gamma}^0$ such that $D\left(\{\boldsymbol{\phi}_i^0(w)\}, \{\boldsymbol{\eta}_i^0\}, \boldsymbol{\gamma}^0\right) \leq D_0 < \infty$. Then there exist valid scaling matrices $M_i^k(w)$, Q_i^k, and V^k for algorithms $RT(\cdot)$, $PA(\cdot)$, and $PC(\cdot)$ such that the sequences generated by these algorithms converge, i.e., $(\boldsymbol{\phi}_i^k(w)) \to (\boldsymbol{\phi}_i^*(w))$, $(\boldsymbol{\eta}_i^k) \to (\boldsymbol{\eta}_i^*)$, and $\boldsymbol{\gamma}^k \to \boldsymbol{\gamma}^*$ as $k \to \infty$. Furthermore, if the link cost functions $D_{ik}(C_{ik}, F_{ik})$ are jointly convex in (C_{ik}, F_{ik}), then $\{\boldsymbol{\phi}_i^*(w)\}$, $\{\boldsymbol{\eta}_i^*\}$ and $\boldsymbol{\gamma}^*$ constitute a set of jointly optimal solutions of JOPR (3).

The proof of Theorem 2, given in full in [15], shows that with appropriate scaling matrices, every iteration of every algorithm strictly reduces the network cost unless the corresponding equilibrium conditions (9)-(11) of the adjusted variables are satisfied. Because the network cost is bounded from below, the cost reduction from the algorithm iterations must

[4] To solve the problem that the routing pattern of a session may contain loops, the device of blocked node sets $\mathcal{B}_i^k$ was invented in [1], [2]. At the kth iteration, it prevents a node from forwarding flow to any neighboring node currently having higher marginal cost or routing positive flows to more costly downstream nodes. Such a scheme guarantees that each session's traffic flows through nodes in decreasing order of marginal costs at all times, thus precluding the existence of loops. For the exact definition of $\mathcal{B}_i^k$, see [2]. It can be shown [1] that if the input routing pattern is loop free, the output routing of $RT(\cdot)$ is also loop free.

tend to zero and asymptotically the equilibrium conditions (9)-(11) hold at all nodes. By Theorem 1, we can conclude that the limiting network configuration is jointly optimal. The proof also shows that global convergence does not require any particular order in running the three algorithms at all nodes. For convergence to the joint optimum, every node i only needs to iterate its own $RT(\boldsymbol{\phi}_i(w))$, $PA(\boldsymbol{\eta}_i)$ and $PC(\gamma_i)$ algorithms until $\boldsymbol{\phi}_i(w)$, $\boldsymbol{\eta}_i$, and γ_i satisfy (9)-(11).[5]

A major part of the algorithm description is the selection of appropriate scaling matrices which give fast convergence while being amenable to distributed implementation. To specify the scaling matrices for all algorithms, assume that there is a finite upper bound D^0 on the initial network cost as in Theorem 2.

At the kth iteration of RT for the routing of session w (index w omitted below) at node i, the scaling matrix M_i^k can be chosen as

$$M_i^k = \frac{t_i^k}{2}\text{diag}\left(A_{ij}^k(D^0) + |\mathcal{AN}_i^k| h_j^k A^k(D^0)\right)_{j\in\mathcal{AN}_i^k},$$

where

$$A_{ij}^k(D^0) \equiv \max_{F_{ij}:D_{ij}(C_{ij}^k,F_{ij})\le D^0} \frac{\partial^2 D_{ij}}{\partial F_{ij}^2},$$

$A^k(D^0) \equiv \max_{(m,n)\in\mathcal{E}} A_{mn}^k(D^0)$, $\mathcal{AN}_i^k \equiv \mathcal{O}_i\backslash\mathcal{B}_i^k$, and h_j^k is the maximum number of hops (or any upper bound on the maximum) on a path from j to $D(w)$. It is clear that M_i^k can be computed locally at i with a simple distributed protocol whereby the h_j^k's are determined.

For the power allocation algorithm, we assume that before the kth iteration of PA at node i, the local cost $\sum_{j\in\mathcal{O}_i} D_{ij}^k \le D_i^k$. The powers used by other nodes do not change over the iteration, and so C_{ij} depends only on η_{ij}:

$$C\left(\frac{G_{ij}P_i\eta_{ij}}{G_{ij}P_i(1-\eta_{ij})+\sum_{m\neq i}G_{mj}P_m+N_j}\right) \triangleq C_{ij}(\eta_{ij}).$$

It can be shown that there exists a lower bound (cf. footnote 3) $\underline{\eta}_{ij}$ on the updated value of η_{ij} such that $\underline{C}_{ij} = C_{ij}(\underline{\eta}_{ij})$ and $D_{ij}(\underline{C}_{ij}, F_{ij}^k) = D_i^k$. Accordingly, the possible range of $SINR_{ij}$, abbreviated as x_{ij}, is

$$\begin{aligned} x_{ij}^{min} &= \frac{G_{ij}P_i\underline{\eta}_{ij}}{G_{ij}P_i(1-\underline{\eta}_{ij})+\sum_{m\neq i}G_{mj}P_m+N_j} \le x_{ij} \\ &\le \frac{G_{ij}P_i}{\sum_{m\neq i}G_{mj}P_m+N_j} = x_{ij}^{max}. \end{aligned}$$

Define an auxiliary term

$$\begin{aligned} \beta_{ij} = \tfrac{1}{\underline{\eta}_{ij}^2}\Bigg[& B_{ij}(D_i^k) \max_{x_{ij}^{min}\le x\le x_{ij}^{max}} \{C'(x)^2 x^2(1+x)^2\} + \\ & + \tfrac{\partial D_{ij}}{\partial C_{ij}}\Big|_{D_{ij}(C_{ij},F_{ij}^k)=D_i^k} \cdot \min_{x_{ij}^{min}\le x\le x_{ij}^{max}} \{C''(x)x^2(1+x)^2\}\Bigg], \end{aligned}$$

where $B_{ij}(D_i^k) = \max_{D_{ij}(C_{ij},F_{ij}^k)\le D_i^k} \frac{\partial^2 D_{ij}}{\partial C_{ij}^2}$. Then, the scaling matrix can be chosen as

$$Q_i^k = \frac{1}{2P_i^k}\text{diag}\left\{(\beta_{ij})_{j\in\mathcal{O}_i}\right\}.$$

We see that all the terms in the formula of Q_i^k are local measures, and thus can be computed by node i independently.

Finally, at each iteration k of the power control algorithm, node i sets v_i^k in (15) to be

$$v_i^k = \frac{\bar{S}_i}{2}|\mathcal{N}||\mathcal{E}|\left[\bar{B}(D^0)\kappa + \underline{B}(D^0)\varphi\right],$$

where

$$\bar{B}(D^0) = \max_{(m,j)\in\mathcal{E}} \max_{D_{mj}\le D^0} \frac{\partial^2 D_{mj}}{\partial C_{mj}^2},$$

$$\underline{B}(D^0) = \min_{(m,j)\in\mathcal{E}} \min_{D_{mj}\le D^0} \frac{\partial D_{mj}}{\partial C_{mj}},$$

$\kappa = \max_{0\le x\le\bar{x}} C'(x)^2 \cdot x^2$, and $\varphi = \min_{0\le x\le\bar{x}} C''(x)\cdot x^2$. Here, $\bar{x}$ is a finite upper bound on the achievable $SINR$ on all links, which must exist due to the peak power constraints. Notice that v_i^k is independent of the iteration index k and is readily computable at node i as long as i knows the total numbers of nodes and links in the network. Since this information does not change as the iteration proceeds, it needs to be broadcast to all nodes only once.

V. Congestion Control

Thus far, we have focused on optimal link capacity and routing allocation for given user traffic demands. There are many situations, however, where the resulting network delay cost is excessive for given user demands even with the optimal configuration inside the network. In these cases, congestion control must be used to limit traffic input into the network. In this section, we extend our analytical framework to consider congestion control for sessions with elastic rate demands. We show that the problem of jointly optimal power control, routing, and congestion control can always be converted into a problem involving only power control and routing as studied above.

For a given session w, let the utility level associated with an admitted rate of r_w be $U_w(r_w)$. We consider the optimization problem of maximizing the *aggregate session utility minus the total network cost* [3], [5], that is

$$\text{maximize} \sum_{w\in\mathcal{W}} U_w(r_w) - \sum_{(i,j)\in\mathcal{E}} D_{ij}(C_{ij},F_{ij}). \qquad (16)$$

We make the reasonable assumption that each session w has a maximum desired service rate $\bar{r}_w$ so that the session utility $U_w(\cdot)$ is defined over the interval $[0,\bar{r}_w]$, where it is assumed to be twice continuously differentiable, strictly increasing, and concave. Taking the approach of [11], we define the *overflow rate* $F_{wb} \equiv \bar{r}_w - r_w$ for a given admitted rate $r_w \le \bar{r}_w$. Thus, at each source node $i = O(w)$, we have the modified flow conservation relation: $\sum_{j\in\mathcal{O}_i} f_{ij}(w) + F_{wb} = \bar{r}_w$.

[5]In practice, nodes may keep updating their optimization variables until further reduction in network cost by any one of the algorithms is negligible.

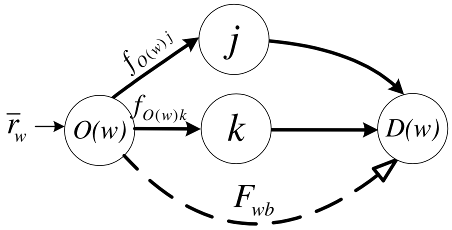

Fig. 2. Virtual Network with Overflow Link

Let $B_w(F_{wb}) \equiv U_w(\bar{r}_w) - U_w(r_w)$ denote the utility loss of session w resulting from rejecting F_{wb} from the network. If we imagine that the blocked flow F_{wb} is routed on a *virtual overflow link* directly from the source to the destination [11], then $B_w(F_{wb})$ can be interpreted as the cost incurred on that virtual link when its flow rate is F_{wb}. Moreover, because $B_w(F_{wb})$ is strictly increasing, twice continuously differentiable and convex in F_{wb} on $[0, \bar{r}_w]$, the dependence of $B_w(F_{wb})$ on F_{wb} is the same as the dependence of a real link cost function $D_{ij}(C_{ij}, F_{ij})$ on its flow rate F_{ij}. A virtual network including an overflow link is illustrated in Figure 2, where the overflow link wb is marked by a dashed arrow. Accordingly, the objective in (16) can now be written as

$$\sum_{w\in\mathcal{W}} U_w(\bar{r}_w) - \sum_{w\in\mathcal{W}} B_w(F_{wb}) - \sum_{(i,j)\in\mathcal{E}} D_{ij}(C_{ij}, F_{ij}).$$

Since $\sum_{w\in\mathcal{W}} U_w(\bar{r}_w)$ is a constant, (16) is equivalent to

$$\text{minimize} \sum_{(i,j)\in\mathcal{E}} D_{ij}(C_{ij}, F_{ij}) + \sum_{w\in\mathcal{W}} B_w(F_{wb}). \qquad (17)$$

Note that (17) has the same form as (3), except for the lack of dependence of $B_w(F_{wb})$ on a capacity parameter.[6] Thus, the problem of joint power control, routing, and congestion control in a wireless network is equivalent to a problem involving only power control on real links and routing on real and overflow links in a virtual wireless network.

To specify the optimality conditions for (17), we continue to use the capacity variables and routing variables, except for a modification of the routing variables $\boldsymbol{\phi}_i(w)$ when $i = O(w)$, $w \in \mathcal{W}$. In this case, define $t_i(w) \equiv \bar{r}_w$ and

$$\phi_{wb} \equiv \frac{F_{wb}}{t_i(w)}, \quad \phi_{ij}(w) \equiv \frac{f_{ij}(w)}{t_i(w)}, \quad \forall j \in \mathcal{O}_i.$$

The new routing variables are subject to the simplex constraint $\phi_{ij}(w) \ge 0$, $\phi_{wb} \ge 0$, $\sum_{j\in\mathcal{O}_i} \phi_{ij}(w) + \phi_{wb} = 1$. The optimality conditions for (17) are the same as in Theorem 1, except that the optimal routing conditions for all source nodes also include $\delta\phi_{wb} = \lambda_i(w)$ if $\phi_{wb} > 0$, and $\delta\phi_{wb} \ge \lambda_i(w)$ if $\phi_{wb} = 0$, where the marginal cost indicator $\delta\phi_{wb}$ of the overflow link is computed as $\delta\phi_{wb} = B'_w(F_{wb})$. That is, the flow of a session is routed only onto minimum-marginal-cost path(s) and the marginal cost of rejecting traffic is equal to the marginal cost of the path(s) with positive flow.

[6]Unlike $D_{ij}(C_{ij}, F_{ij})$, however, B_w has no explicit dependence on a capacity parameter. If we assume that $U_w(0) = -\infty$, so that there is an infinite penalty for admitting zero session w traffic, then $B_w(\bar{r}_w) = \infty$, and $\bar{r}_w$ could be taken as the (fixed) "capacity" of the overflow link.

VI. Conclusion

We have presented an analytical framework in which power control, routing, and congestion control can be jointly optimized in wireless networks. A set of distributed node-based gradient projection algorithms is developed where routing, power allocation, and power control variables are iteratively adjusted at individual nodes. We explicitly derive appropriate scaling matrices for each of the algorithms. The scaling matrices expedite the algorithms' convergence and are locally computable. Furthermore, they make the convergence independent of the initial condition and the ordering and synchronization of iterations at different nodes. Finally, we demonstrate that congestion control can be seamlessly incorporated into our framework, in the sense that the problem of joint capacity allocation, routing, and congestion control in a wireless network can be made equivalent to a problem involving only capacity allocation and routing in a virtual wireless network with the addition of overflow links.

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