

On the Optimal SINR in Random Access Networks with Spatial Reuse

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Abstract—In this paper we consider a simple model for a homogeneous random access system with spatial reuse. Our aim is to obtain insight into how physical layer parameters should be determined in order to maximize the total throughput-distance per unit area. In an infinitely dense network, we find that the optimal SINR threshold is asymptotically zero. Furthermore, we find that the optimal distance between transmitter and receiver is asymptotically non-zero. For finitely dense networks, we find that the optimal SINR threshold for packet detection is considerably smaller than that used in many current systems, and that the optimal distance between transmitters and intended receivers is larger than would be expected from prevailing conventional wisdom. These results suggest that the throughput performance of current systems may be improved by operating at lower values of SINR.

I. INTRODUCTION

In a wired network, there is typically no significant interaction between communication links, so they can be designed independently. For example, according to Shannon's famous capacity formula [1] for the additive white Gaussian noise channel over a band-limited channel with bandwidth W Hz, the capacity in bit's per second is

$$C = W \log_2(1 + \gamma) \quad (1)$$

where γ is the signal to noise ratio (SNR), i.e. the ratio of signal power at the receiver to noise power at the receiver. Thus, the capacity of a communication link is limited by either the inherent bandwidth available, W , or the transmission power available which determines the maximum value of γ . Depending on the context of the constraints involved, different tradeoffs can then be made to independently design each communication link in a network.

In a wireless network, transmissions may interfere with each other, causing a coupling between different communications links. It is generally difficult to identify channel capacity in this context, but by treating the signal interference as noise at each receiver, and assuming the interference is Gaussian we can use (1) to obtain an estimate of the communication rate possible. In this case γ is interpreted as a signal to interference and noise ratio (SINR). Here there is an interesting tradeoff involved. Roughly speaking, for a given fixed bandwidth W , we can increase the transmission power for a given link, which will increase the possible rate of transmission for this link, but doing so will increase the interference for other links, and hence

decrease the SINR for other links. Generally, from a capacity perspective, it may be inefficient to have all communication links active at the same time, and scheduling transmissions will increase the system throughput (for example, see [2] and [3]).

Since centralized scheduling approaches may suffer from scalability issues (though there are distributed hierarchical approaches to address scalability, as in [4]), we now discuss an analysis of this tradeoff in the context of a random access system with spatial reuse, as reported in [5]. An interesting aspect of random access systems with spatial reuse is that simultaneous transmissions can be successful provided they are far enough apart in space. This is in contrast to random access systems over wired systems, which usually operate such that packet transmissions are successful only if no other simultaneous transmission occurs.

As such, a key issue in wireless random access systems is the degree to which simultaneous transmissions should be allowed. Previous capacity studies of wireless networks [6] consider packet reception models based on SINR, but are focused on asymptotics which do not consider *optimization* of SINR. In this paper, we consider optimization of physical layer parameters, for example SINR and packet data rate, in the context of random access with spatial reuse. A similar optimization of physical layer parameters in the context of TCP traffic over wireless links was considered in [7].

Current random access systems with spatial reuse, for example IEEE 802.11 systems that operate with a Carrier Sense Multiple Access (CSMA) protocol, require high SINR at the receiver for packet reception. A typical value is 10dB. Such high SINR values are required for commonly deployed packet and timing acquisition approaches. A key finding of this paper is that for efficient operation in random access systems with spatial reuse, relatively low SINR values are required. This motivates recent efforts, for example [8], to find methods for packet and timing acquisition in low SINR environments.

A well-known previous study [9] found that the optimal transmission radius in a packet radio networks, which have spatial reuse, is such that approximately six nodes should be in communication range for optimal operation. This result contrasts with the findings of the present paper, which suggest that a much larger number of nodes may be in range for optimal operation. This apparent contradiction needs to be

investigated further, but may be a result of the disk based model for interference used in [9]. In the present paper, we consider a model for interference based on SINR, as considered in [6].

The remainder of this paper is organized as follows. In Section II, we consider a model for a random access system with very high spatial node densities. We consider optimization of the average throughput-distance per unit area, and find that the asymptotically optimal SINR threshold is zero. Furthermore, we find that the asymptotically optimal transmission radius is non-zero, which contradicts the notion that for efficient operation, direct communication should only take place between closely located neighbors. In Section III we consider the same model for a random access network, but with a finite spatial node density. We consider some numerical examples which suggest that low SINR thresholds are optimal in this case as well. We conclude with some closing remarks in Section IV.

II. INFINITELY DENSE NETWORKS

A. System Model

Consider a slotted Aloha system with spatial reuse. In each time slot, the set of nodes which transmit packets is given by a two dimensional Poisson point process over the infinite plane, with spatial intensity λ . The spatial density of transmitting nodes λ is considered to be a parameter of operation of the system, and can be arbitrarily large. Thus the context here is an infinitely dense (or highly dense) population of users distributed throughout the infinite plane. The locations of the nodes can be different in each slot (i.e. mobility is allowed), but we do not require it for our model. Each packet transmission could be a new transmission, or a retransmission of a previously transmitted packet.

We assume that each node uses a common transmission power P . We assume that the separation between each transmitter and intended receiver is close to r length units. Since we are interested here in a very dense user population, we will assume that the separation between each transmitter and intended receiver is exactly r units. Because of the high node density, it is assumed that a given node transmits very infrequently, and therefore the intended receiver is not transmitting a packet with very high probability.

The transmission distance r is determined by higher level networking considerations (e.g. the routing layer), but here we consider it as a design parameter to be optimized for efficient system operation.

We assume a flat line-of-sight channel model for simplicity here, in which the signal power at the receiver is $Pl(r)$. Here $l(x)$ denotes the signal attenuation at a distance x , and we assume that $l(x) = (1 + Ax)^{-\alpha}$, where A is a constant and α is known as the propagation exponent which is commonly assumed to be between 2 and 4. This model has been suggested in [5].

Conditioned on the set of transmitting users in a slot, we model the interference as Gaussian, so that (1) can be used. The source of interference in each slot is random, due to the

random set of transmitters selected by the distributed protocol. Thus, the SINR γ seen by each intended receiver in each slot is random. Let I represent the random power of the interfering signals (assumed to be uncorrelated with each other) at an intended receiver. With our model, the interference power I is a Poisson shot noise. Using well known results [10] (Campbell's theorem), the first moment of I given by

$$E[I] = m_1 = \int_0^\infty 2\pi x \lambda Pl(x) dx$$

and the n^{th} central moment of I for $2 \leq n \leq 3$ is

$$E[(I - m_1)^n] = m_n = \int_0^\infty 2\pi x \lambda (Pl(x))^n dx .$$

Given a data rate μ for each packet transmission, and assuming the intended destination is not transmitting a packet (which happens with probability $1 - q$), there is a corresponding SINR threshold γ_{target} that determines whether or not a packet transmission is successful. That is, if $\gamma > \gamma_{\text{target}}$ then the intended receiver can receive the packet correctly, and if $\gamma < \gamma_{\text{target}}$, the packet is lost. A key parameter is the one-shot packet success probability, $P_{\text{succ}} = \text{Prob}(\gamma > \gamma_{\text{target}})$. To calculate $\text{Prob}(\gamma > \gamma_{\text{target}})$, we can use the information contained in the moments m_n for $n \geq 1$. Alternatively, since I is the superposition of a large number of independent components, we can model I as a Gaussian random variable¹. In this case, we can determine $\text{Prob}(\gamma > \gamma_{\text{target}})$ from m_1 and m_2 . Assuming a noise power σ_N^2 at each receiver, the total interference and noise power at the receiver is $\sigma_N^2 + I$, and the (random) SINR is given by

$$\gamma = \frac{Pl(r)}{\sigma_N^2 + I} .$$

Using the Gaussian approximation for the interference we can write,

$$\begin{aligned} 1 - P_{\text{succ}} &= \text{Prob}(\gamma < \gamma_{\text{target}}) \\ &= \text{Prob}(I + \sigma_N^2 > \frac{Pl(r)}{\gamma_{\text{target}}}) \\ &= Q\left(\frac{\frac{Pl(r)}{\gamma_{\text{target}}} - \sigma_N^2 - m_1}{\sqrt{m_2}}\right) , \end{aligned} \quad (2)$$

where $Q(\cdot)$ is the tail of the unit normal distribution, i.e. $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-y^2/2) dy$.

In order to determine the relationship between the target SINR γ_{target} and the data rate μ of each packet transmission, we can use Shannon's capacity formula (1). For normalization purposes, it is convenient to set the available bandwidth W to $W = 1$ Hz. From (1) we thus have

$$\mu = \log_2(1 + \gamma_{\text{target}}) . \quad (3)$$

Our aim is to optimize system throughput with respect to the system parameters. Since there on the average λ packet

¹Note that in this case, we are making *two* Gaussian approximations - that the interference itself is a Gaussian random process (conditioned on the set of active transmitters), and that the (unconditional) variance I of the interference is a Gaussian random variable.

transmissions per unit area in each slot, each transmission has data rate $\mu = \log_2(1 + \gamma_{target})$, each transmission moves information r units of length, and each transmission is successful with probability $P_{succ} = Prob(\gamma > \gamma_{target})$, the throughput-distance per unit area is given by $J = \lambda\mu r P_{succ}$. In order to maximize system throughput, we maximize J with respect to λ , μ , and r . The parameters σ_N^2 , P , α and A are considered to be fixed. The target SINR threshold γ_{target} is determined by the data rate μ according to (3). The optimal throughput-distance per unit area, J^* , is then defined by the following optimization problem:

$$\begin{aligned}
 J^* = & \sup\{\lambda\mu r P_{succ} : \\
 & 0 < \lambda < \infty, \\
 & 0 < \mu < \infty, \\
 & 0 < r < \infty, \\
 & \mu = \log_2(1 + \gamma_{target}), \\
 & P_{succ} = Prob(\gamma > \gamma_{target}) \}. \quad (4)
 \end{aligned}$$

We will present some closed form results on the value of J^* below, as well as the corresponding optimal values of λ , μ , and r . Before presenting these, we first consider a numerical example for a sub-optimization problem.

B. Optimal Operation with Fixed Density λ

Let $J^*(\lambda)$ be optimal throughput-distance per unit area for a fixed density of active transmitters λ , i.e.

$$\begin{aligned}
 J^*(\lambda) = & \sup\{\lambda\mu r P_{succ} : \\
 & 0 < \mu < \infty, \\
 & 0 < r < \infty, \\
 & \mu = \log_2(1 + \gamma_{target}), \\
 & P_{succ} = Prob(\gamma > \gamma_{target}) \}. \quad (5)
 \end{aligned}$$

We thus have $J^* = \sup\{J^*(\lambda) : 0 < \lambda < \infty\}$.

In this section, we present some numerical evaluations of $J^*(\lambda)$ for the scenario of an interference limited environment where $\sigma_N^2 = 0$. In this case, it is easy to see that the results are independent of the transmission power P . In the numerical examples here, we set $A = 1$ and $\alpha = 3$.

In Figure 1, we plot the optimum value of $J^*(\lambda)$ in problem (5), as a function of the density of active users λ , for $0 < \lambda < 30$. It is apparent that $J^*(\lambda)$ is monotone increasing in λ and that the supremum is approached as $\lambda \rightarrow \infty$. We shall later find a closed form expression for $J^* = \lim_{\lambda \rightarrow \infty} J^*(\lambda)$.

For each value of λ in the range $0 \leq \lambda \leq 30$, in Figures 2 - 5, we show how the supremum in $J^*(\lambda)$ is attained. For example, in Figure 2 we plot the optimal packet loss probability $1 - P_{succ}$ that attains the supremum in the definition of $J^*(\lambda)$ versus λ .

In Figure 3 we show a plot of the optimal transmission rate μ attaining the supremum in $J^*(\lambda)$ as a function of λ . This is plotted on a log scale, and it is apparent that the optimal value of μ approaches zero as λ gets large. As we will see later, there is a non-zero limit to the product $\lambda\mu$ which attains

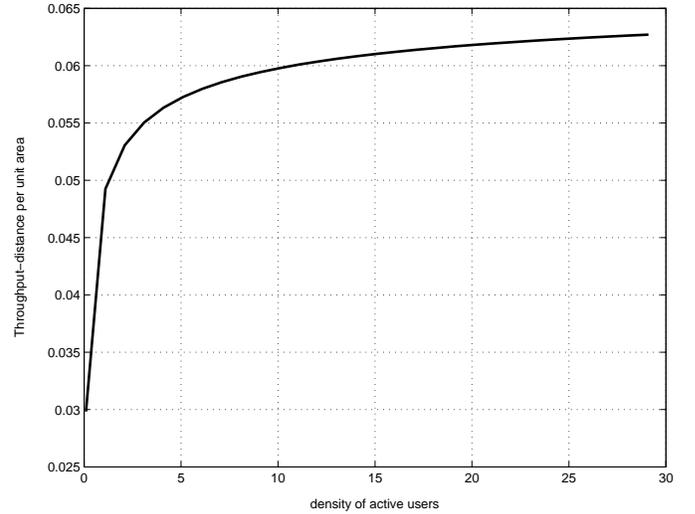


Fig. 1. Optimal throughput-distance per unit area $J^*(\lambda)$ versus λ , for an infinitely dense network.

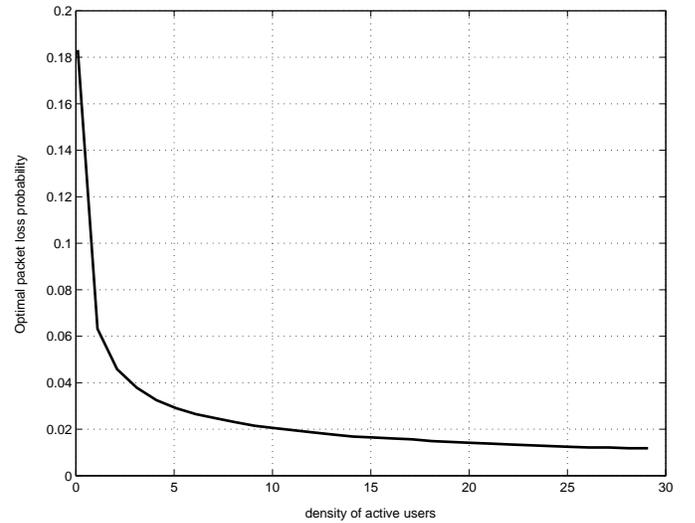


Fig. 2. Optimal packet loss probability $1 - P_{succ}$ achieving $J^*(\lambda)$ versus λ .

the supremum, and we shall find a closed form expression for this product.

In Figure 4, also on a log scale, we plot the optimal target SINR threshold γ_{target} that achieves the supremum in $J^*(\lambda)$ as a function of λ . As we will see later, it is noteworthy that for all values of λ , the optimal value of γ_{target} is relatively small. In particular it is less than 0 dB.

Finally, in Figure 5 we plot the optimal transmission distance r attaining the supremum in $J^*(\lambda)$ as a function of λ . As λ gets large, it is apparent that the optimal value of r approaches a non-zero limiting value. As mentioned in the introduction, this is in contrast to the result in [9], which would suggest that this limiting value should be zero. We shall find a closed form expression for the unconditionally optimal value of r .

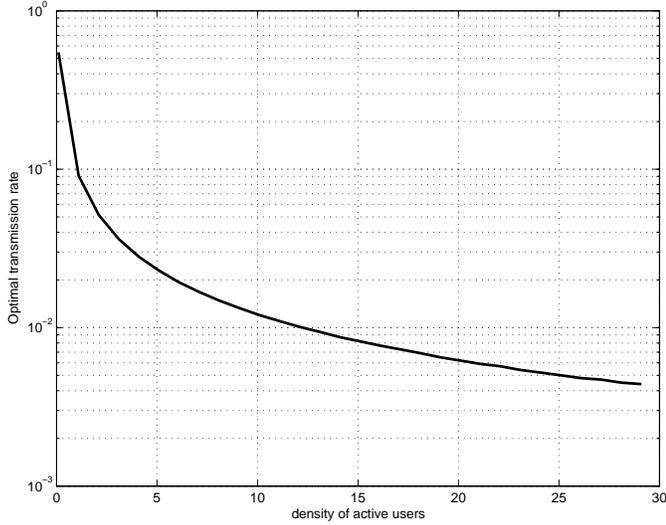


Fig. 3. Optimal transmission rate μ achieving $J^*(\lambda)$ as a function of λ .

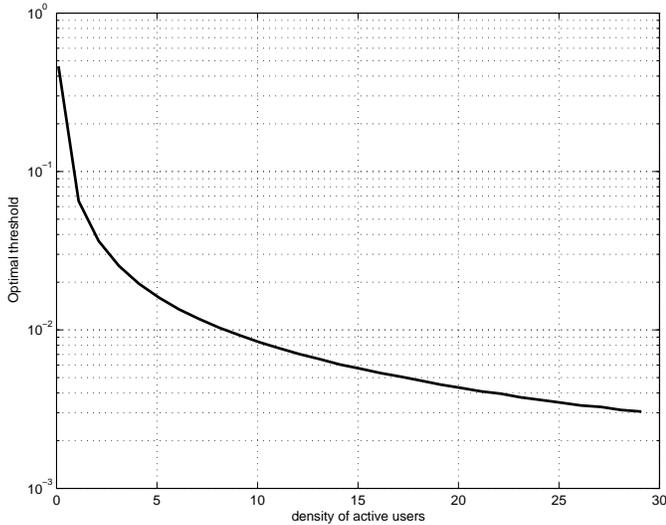


Fig. 4. Optimal SINR threshold γ_{target} achieving $J^*(\lambda)$ as a function of λ .

C. Unconditionally Optimal Operating Point for Infinitely Dense Networks

In this subsection, we present closed form results for the optimization problem (5).

Proposition 1: The optimum system throughput-distance per unit area J^* in problem (5) satisfies

$$J^* = \frac{(1 - 1/\alpha)^\alpha}{A(\alpha - 1) \ln(2) \int_0^\infty 2\pi x l(x) dx}. \quad (6)$$

Furthermore, the optimum is approached as $\lambda \rightarrow \infty$ with $\lambda\mu \rightarrow G^*$, where

$$G^* = \frac{(1 - 1/\alpha)^\alpha}{\ln(2) \int_0^\infty 2\pi x l(x) dx}. \quad (7)$$

Finally, the transmission distance r achieving the optimum is

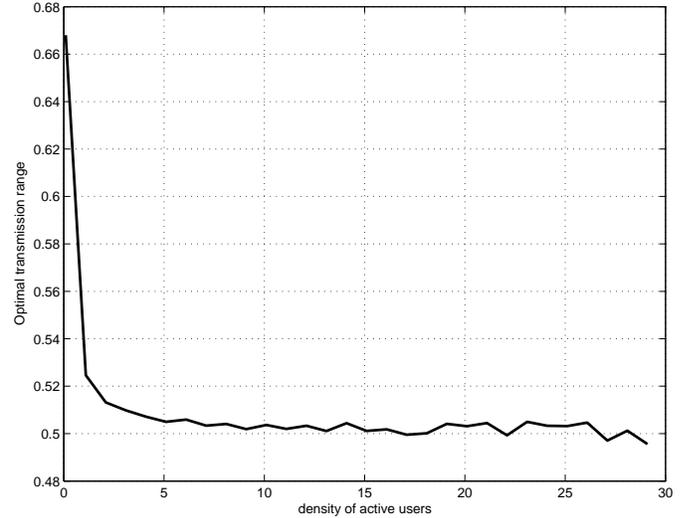


Fig. 5. Optimal transmission range r achieving $J^*(\lambda)$ as a function of λ .

r^* , where

$$r^* = \frac{1}{A(\alpha - 1)}. \quad (8)$$

Note that the product $\lambda\mu = G$ can be interpreted as the offered load to the system per unit area.

It is interesting to note that J^* does not depend on the thermal noise power σ_N^2 or the transmission power P . This is intuitively reasonable, since the density of users is high and we would expect the system to be interference limited in such a regime.

In order to prove Proposition 1, it is useful to consider a sub-optimization problem by first fixing the value of P_{succ} . Let $H(P_{succ})$ be the optimal throughput-distance constrained with a fixed value of P_{succ} , i.e. define

$$\begin{aligned} H(P_{succ}) = \sup\{J = \lambda\mu r P_{succ} : \\ 0 < \lambda < \infty, \\ \mu = \log_2(1 + \gamma_{target}), \\ 0 < r < \infty, \\ P_{succ} = Prob(\gamma > \gamma_{target})\} \end{aligned} \quad (9)$$

We thus have $J^* = \sup\{H(P_{succ}) : 0 \leq P_{succ} \leq 1\}$. In the next lemma we evaluate $H(P_{succ})$.

Lemma 1: We have

$$H(P_{succ}) = \frac{P_{succ}(1 - 1/\alpha)^\alpha}{A(\alpha - 1) \ln(2) \int_0^\infty 2\pi x l(x) dx}. \quad (10)$$

Note that Lemma 1 implies that $H(P_{succ})$ is increasing in P_{succ} , and hence that $J^* = H(1)$. Thus to prove Proposition 1, it suffices to prove Lemma 1.

Proof: Define $T_\mu = \gamma_{target}$. Since $\mu = \log_2(1 + T_\mu)$, it follows that $T_\mu = 2^\mu - 1$.

Let δ be such that $Q(\delta) = 1 - P_{succ}$. Using (2), we have

$$\begin{aligned} \frac{P \cdot l(r)}{T_\mu} - m_1 - \sigma_N^2 &\geq \delta \sqrt{m_2} \\ \Rightarrow P \cdot l(r) &\geq T_\mu(m_1 + \sigma_N^2 + \delta \sqrt{m_2}) \end{aligned} \quad (11)$$

Define

$$\begin{aligned} F(\lambda, P) &= \frac{m_1 + \sigma_N^2 + \delta\sqrt{m_2}}{\lambda P} \\ &= K_1 + \frac{\sigma_N^2}{\lambda P} + \delta\left(\frac{1}{\sqrt{\lambda}}\right)\sqrt{K_2}, \end{aligned} \quad (12)$$

where

$$K_1 = \frac{m_1}{\lambda P} = \int_0^\infty 2\pi x l(x) dx,$$

and

$$K_2 = \frac{m_2}{\lambda P^2} = \int_0^\infty 2\pi x l^2(x) dx.$$

We can then write (11) as

$$l(r) = T_\mu \lambda F(\lambda, P).$$

Using the fact that we are assuming $l(r) = (1 + Ar)^{-\alpha}$, this is in turn equivalent to

$$r = \frac{1}{A} [(T_\mu \lambda F(\lambda, P))^{-\frac{1}{\alpha}} - 1] \quad (13)$$

Note that if the right hand side of (13) is negative, then the feasible set of the optimization problem defined by (9) is empty.

We thus have that

$$\begin{aligned} H(P_{succ}) &= \sup_{\lambda, \mu} [\lambda \mu r P_{succ}] \\ &= \frac{P_{succ}}{A} \sup_{\lambda, \mu} \left[\lambda \mu [(T_\mu \lambda F(\lambda, P))^{-\frac{1}{\alpha}} - 1] \right] \\ &= \frac{P_{succ}}{A} \sup_{\lambda, \mu} \left[\lambda \mu [((2^\mu - 1)\lambda F(\lambda, P))^{-\frac{1}{\alpha}} - 1] \right] \\ &= \frac{P_{succ}}{A} \sup_{\lambda, G} \left[G [((2^{G/\lambda} - 1)\lambda F(\lambda, P))^{-\frac{1}{\alpha}} - 1] \right] \end{aligned} \quad (14)$$

where in (14) we have defined a change of variables with $G = \lambda \mu$.

We now argue that the optimum in (14) is approached with $\lambda \rightarrow \infty$. To see this note that $F(\lambda, P)$ is monotone decreasing in λ with

$$\lim_{\lambda \rightarrow \infty} F(\lambda, P) = K_1.$$

Furthermore note that for fixed G that $(2^{G/\lambda} - 1)\lambda$ is monotone decreasing in λ with

$$\lim_{\lambda \rightarrow \infty} (2^{G/\lambda} - 1)\lambda = G \ln(2).$$

It follows that

$$\begin{aligned} H(P_{succ}) &= \frac{P_{succ}}{A} \sup_{\lambda, G} \left[G [((2^{G/\lambda} - 1)\lambda F(\lambda, P))^{-\frac{1}{\alpha}} - 1] \right] \\ &= \frac{P_{succ}}{A} \sup_G \left[G [(G \ln(2) K_1)^{-\frac{1}{\alpha}} - 1] \right] \\ &= \frac{P_{succ}}{A} \sup_G \left[\frac{G^{1-1/\alpha}}{(\ln(2) K_1)^{\frac{1}{\alpha}}} - G \right]. \end{aligned}$$

Taking the derivative with respect to the offered load G , we have the following condition for optimality:

$$\frac{(1 - 1/\alpha)G^{-1/\alpha}}{(\ln(2) K_1)^{\frac{1}{\alpha}}} = 1.$$

The solution is

$$G^* = \frac{(1 - 1/\alpha)^\alpha}{\ln(2) K_1},$$

which proves (7). Therefore we have

$$\begin{aligned} H(P_{succ}) &= \frac{P_{succ}}{A} \sup_G \left[\frac{G^{1-1/\alpha}}{(\ln(2) K_1)^{\frac{1}{\alpha}}} - G \right] \\ &= \frac{P_{succ}}{A} \left[\frac{(G^*)^{1-1/\alpha}}{(\ln(2) K_1)^{\frac{1}{\alpha}}} - G^* \right] \\ &= \frac{P_{succ} G^*}{A} \left[\frac{(G^*)^{-1/\alpha}}{(\ln(2) K_1)^{\frac{1}{\alpha}}} - 1 \right] \\ &= \frac{P_{succ} G^*}{A(\alpha - 1)} \\ &= \frac{P_{succ} (1 - 1/\alpha)^\alpha}{A(\alpha - 1) \ln(2) K_1}, \end{aligned} \quad (15)$$

which proves (10), and hence (6).

Finally, the corresponding optimal distance between the transmitter and the intended receiver is given by substituting $\lambda = \infty$ and $G = G^*$ in (13). Doing this, we obtain the optimal distance r^* as in (8). ■

III. FINITELY DENSE NETWORKS

In this section, we consider a slightly different model than in the previous section. In particular, we suppose that the locations of nodes in a given slot form a Poisson point process with spatial density λ_0 , where λ_0 is a finite positive constant. The location of nodes may change from slot to slot, but that does not affect our analysis. We assume that all nodes are infinitely backlogged, i.e. each node always has a packet waiting to be transmitted. In each slot, each user transmits a packet independently with probability q . Thus, the set of active transmitters in each slot is a Poisson point process with spatial intensity $\lambda = \lambda_0 q$.

As before, all transmissions take place with a common power P . A packet is received correctly by the destination at distance r only if the SINR is above the target SINR threshold γ_{target} . In addition, for the packet to be received correctly, the destination node must not be transmitting in that slot. The probability of this event is $1 - q$. Thus the probability of a packet being received correctly in this model is $P_{succ} = (1 - q) Prob(\gamma > \gamma_{target})$. All other aspects of the model in this case are identical to the infinitely dense model considered before.

As before, we can optimize the system throughput-distance per unit area with respect to all the parameters. Specifically, let J_{λ_0} be the optimum value, i.e.

$$\begin{aligned}
J_{\lambda_0}^* &= \sup\{\lambda\mu r P_{succ} : \\
&0 < \lambda \leq \lambda_0, \\
&0 < \mu < \infty, \\
&0 < r < \infty, \\
&\mu = \log_2(1 + \gamma_{target}), \\
&q = \lambda/\lambda_0, \\
&P_{succ} = (1 - q)Prob(\gamma > \gamma_{target}) \}. \quad (16)
\end{aligned}$$

As before, we can evaluate $J_{\lambda_0}^*$ by first considering a sub-optimization problem where the value of λ is fixed to a value between 0 and λ_0 , which corresponds to a fixed value of q . Let $J_{\lambda_0}^*(\lambda)$ be the corresponding value of this sub-optimization problem, i.e.

$$\begin{aligned}
J_{\lambda_0}^*(\lambda) &= \sup\{\lambda\mu r P_{succ} : \\
&0 < \mu < \infty, \\
&0 < r < \infty, \\
&\mu = \log_2(1 + \gamma_{target}), \\
&q = \lambda/\lambda_0, \\
&P_{succ} = (1 - q)Prob(\gamma > \gamma_{target}) \}. \quad (17)
\end{aligned}$$

Thus we have

$$J_{\lambda_0}^* = \sup\{J_{\lambda_0}^*(\lambda) : 0 \leq \lambda \leq \lambda_0\}. \quad (18)$$

It is easy to see that the problem (17) is closely related to the problem (5). In particular, for all λ satisfying $0 < \lambda \leq \lambda_0$ we have

$$J_{\lambda_0}^*(\lambda) = (1 - \lambda/\lambda_0)J^*(\lambda) \quad (19)$$

We can use (19) to obtain a plot of $J_{\lambda_0}^*(\lambda)$ versus λ for different values of λ_0 from a plot of $J^*(\lambda)$ versus λ , as in Figure 1. This is done to obtain the plot shown in Figure 6, where we consider the cases $\lambda_0 = 10, 20$, and 30 . Since we have set $\sigma_N^2 = 0$ in Figure 1, the plot in Figure 6 is for the interference limited case as well.

From examination of Figure 6, it is apparent that there is a finite optimal value of λ for each fixed value of λ_0 . The value of $J_{\lambda_0}^*$ can be determined from the graph for each given value of λ_0 . For example, we can see that the optimal active user density for the case $\lambda_0 = 30$ is about $\lambda^* = 3$.

Note that the optimal values of the other parameters μ , r , γ_{target} , P_{succ} that attain the supremum in problem (17) can also be obtained from the corresponding optimizing values in problem (6). For example, for the case $\lambda_0 = 30$, the optimal value of the packet loss probability $1 - P_{succ}$ is obtained by looking at the value from the plot in Figure 2 at $\lambda = 3$. This is seen to be approximately $0.96 = 1 - 0.04$. Similarly, the optimal target SINR for the case $\lambda_0 = 30$ is obtained from Figure 4 by looking at the value for $\lambda = 3$, which is about 0.025. In decibels, the optimal SINR is about $\gamma_{target}^* = -17.6$ dB, well below the typical values of SINR detection thresholds used in current wireless packet systems.

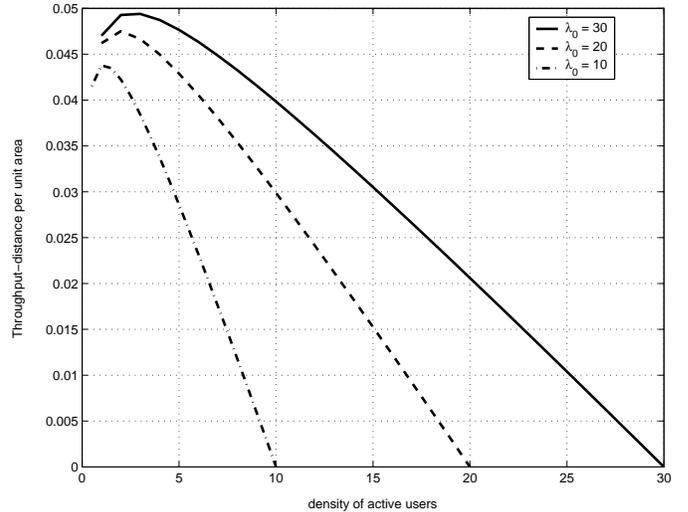


Fig. 6. optimal gain as a function of λ .

IV. CONCLUSION

In this paper, we considered a model for a random access system with spatial reuse, and optimization of system parameters to maximize the throughput-distance per unit area. Our results suggest that optimal SINR thresholds are much smaller than those used in many current systems, and there is potentially a significant throughput advantage possible by implementing timing and packet acquisition algorithms that can operate at low values of SINR.

Another finding suggested by our results is that it may be beneficial for nodes to communicate directly, even if they are separated much farther than their closest neighbors. This is in contrast to current conventional wisdom. This needs to be investigated further.

Possible extensions of our work include considering random distances between sources and destinations, instead of a fixed distance r . Another possible extension is to consider opportunistic routing, as in [5]. Finally, one direction for further study is to consider performance with specific coding and modulation schemes.

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