Modelling multi-path problems

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Multi-path flow control and routing

- Multi-path routing could improve both the performance and reliability of packet networks.
- Benefits accrue from load balancing and to responding through routing changes to congestion, failures and, perhaps, mobility.
- However, responding too quickly risks destabilizing the network whereas responding too slowly may miss potential benefits.
- An important challenge is to find our way between these opposing concerns.

Fluid models for networks

We use the following basic objects

- sources $s \in S$ comprising one or more routes $r \in R$
- route r is associated with source $s(r)$
- routes r comprising one or more resources $j \in r$
- resources have a capacity C_j, $j \in J$
- delays and round trip times $T_r = T_{rj} + T_{jr}$
- matrix $A_{jr} = 1$ if $j \in r$ and 0 otherwise.

We represent the flow on route r at time t by $x_r(t)$ for each $r \in R$.
Notation: define the function $a = (b)_c^+$ to mean $a = b$ if $c > 0$ and $a = \max(0, b)$ if $c = 0$.

Primal and dual algorithms

- Primal: multi-path variant of scalable TCP
  - T. Kelly, “Scalable TCP: improving performance in highspeed wide area networks,” *Computer Communication Review*, vol. 32, no. 2, pp. 83–91, 2003.
  - H. Han, S. Shakkottai, C. Hollot, R. Srikant, and D. Towsley, “Overlay TCP for multi-path routing and congestion control,” in *ENS-INRIA ARC-TCP Workshop*, 2003, Paris, France.
  - F. Kelly and T. Voice, “Stability of end-to-end algorithms for joint routing and rate control,” *ACM SIGCOMM Computer Communication Review*, vol. 35, no. 2, pp. 5–12, Apr. 2005.
- Dual: controlled splitting algorithm
  - T. Voice, “Stability of mulit-path dual congestion control algorithms,” *IEEE/ACM Trans. Netw.*, vol. 15, no. 6, pp. 1231–1239, Dec. 2007.

Primal algorithm [KV2005]

In this model fluid flows, $x_r(t)$, operate as follows

$$\dot{x}_r(t) = \frac{x_r(t - T_r)}{T_r}\left(\overline{a}(1 - \lambda_r(t)) - b_r y_{s(r)}(t)\lambda_r(t)\right)^{+}_{x_r(t)}$$

where

$$\lambda_r(t) = 1 - \prod_{j \in r}\left(1 - \mu_j(t - T_{jr})\right)$$

and

$$y_s(t) = \sum_{r \in s} x_r(t - T_r)\,; \qquad \mu_j(t) = p_j\left(\sum_{r : j \in r} x_r(t - T_{rj})\right)\,.$$

Each resource j has a capacity C_j and an associated penalty function, $p_j(\cdot)$, given (for some constant β_j) by

$$p_j(z_j) = \left(\frac{z_j}{C_j}\right)^{\beta_j}\,.$$

Primal stability conditions

A sufficient condition [KV2005] for local stability of the primal algorithm is that

$$\overline{a}(1+\beta) < \frac{\pi}{2}\,.$$

In comparison, the equivalent condition for (uni-path) scalable TCP is

$$\overline{a}(\beta) < \frac{\pi}{2}\,.$$

See also,

G. Vinnicombe. “On the stability of networks operating TCP-like congestion control,” *Proc. IFAC World Congress*, 2002, Barcelona, Spain.

Network utility maximization

Formulation for controlled splitting multi-path dual algorithm

Maximize over $x \geq 0$

$$\sum_{s \in S} U_s\left(x_s^{\frac{1}{q}}\right)$$

subject to

$$x_s \leq \sum_{r \in s} x_r^q \quad \forall s \in S$$

and

$$Ax \leq C$$

where

$$q = p/(p+1)$$

for some choice $p > 0$.

Choice of utility function

We use the standard choice of isoelastic utility functions given by

$$U_s(y) = \begin{cases} w_s \frac{y^{1-\alpha}}{1-\alpha} & \text{if } \alpha \neq 1 \\ w_s \log(y) & \text{if } \alpha = 1 \end{cases}$$

for parameters $w_s > 0$ and $\alpha > 0$.
These functions exhibit constant relative risk aversion (CRRA) given by

$$\frac{-yU_s''(y)}{U_s'(y)} = \alpha$$

and elasticity of substitution given by $1/\alpha$.

Dual algorithm [V2007]

The fluid flows, $x_r(t)$, are then given by

$$x_r(t) = \lambda_r(t)^{-(p+1)} w_s^{p+1} y_{s(r)}(t)^{1-\alpha(p+1)}$$

where

$$\dot{y}_s(t) = \kappa_s y_s(t)^{\frac{1}{p+1}} \left(\sum_{r \in s} x_r(t - T_r)^q - y_s(t)^q \right)^+_{y_s}$$

and

$$\lambda_r(t) = \sum_{j \in r} \mu_j(t - T_{jr}) \, .$$

The dual variables, $\mu_j(t)$, obey the following relations

$$\dot{\mu}_j(t) = \kappa_j \mu_j(t) \left(z_j(t) - C_j \right)^+_{\mu_j(t)}$$

where

$$z_j(t) = \sum_{r : j \in r} x_r(t - T_{rj}) \, .$$

Here, the parameters κ_j and κ_s are (sufficiently small) positive gain parameters.

Dual stability conditions

When $\alpha(p+1) > 1$ we have local stability of the dual algorithm if the following sufficient conditions [V2007] hold for each resource j and for each source s

$$\kappa_j(p+1)T_jC_j < \frac{1}{2}$$

where the weighted RTT for flow through resource j is

$$T_j = \frac{1}{C_j}\sum_{r:j\in r} x_r T_r$$

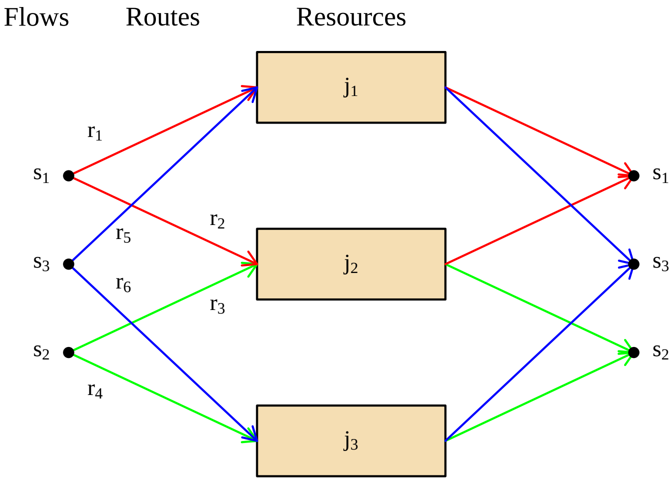
and

$$\kappa_s\left((p+1)\alpha - 1\right)T_s < \frac{1}{2}$$

where the weighted RTT for flow from source s is

$$T_s = \frac{q}{y_s^p}\sum_{r\in s} x_r^q T_r\,.$$

Network example



Network parameters

Delay and round trip time (RTT) parameters are as follows.

j	r	T_{rj}	T_{jr}	T_r
j_1	r_1	0.01	0.01	0.02
j_1	r_5	0.01	0.01	0.02
j_2	r_2	0.1	0.1	0.2
j_2	r_3	0.1	0.1	0.2
j_3	r_4	1.0	1.0	2.0
j_3	r_6	1.0	1.0	2.0

Capacities of $C_j = 10$ for each resource j.
Here, we have taken $T_{rj} = T_{jr}$ so that

$$T_r = T_{rj} + T_{jr} = 2T_{rj} = 2T_{jr} \,.$$

Primal parameters

Primal parameter values $\overline{a} = 0.1$, $b_r = 0.875$ and $\beta_j = 10$ for each route r and resource j, respectively.
Then,

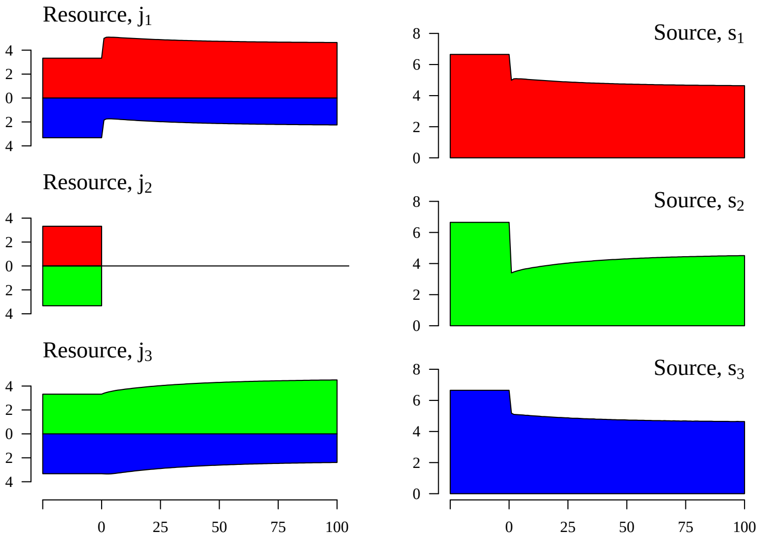
$$p_j(z) = \left(\frac{z}{C_j}\right)^{\beta_j}$$

implies that

$$p_{j_1}(z) = p_{j_2}(z) = p_{j_3}(z) = \left(\frac{z}{10}\right)^{10} .$$

Primal results

Flows by resources and sources



Dual parameters

The parameter p was 7 and thus $q = p/(p+1) = 7/8$.
Take $\alpha = 1$ corresponding to the utility function

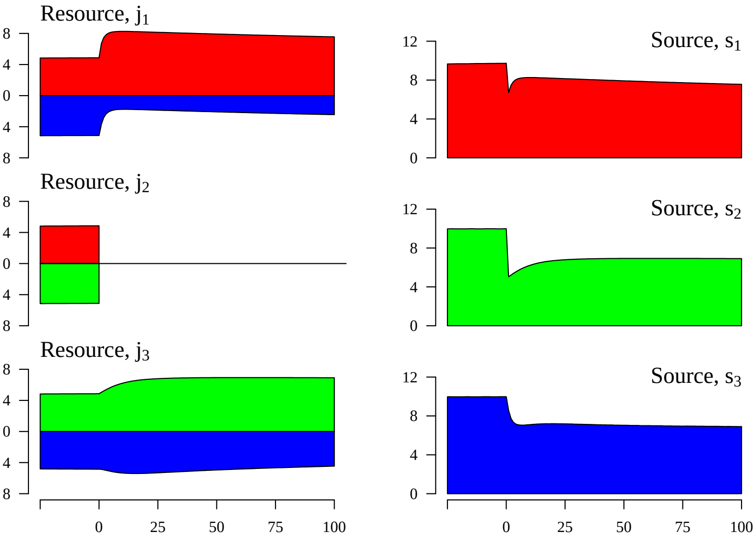
$$U_s(y) = w_s \log(y)\,.$$

The dual gain parameters were

j	κ_j	s	κ_s	w_s
j_1	0.1	s_1	0.25	0.1
j_2	0.01	s_2	0.01	0.1
j_3	0.001	s_3	0.01	0.1

Dual results

Flows by resources and sources



Dynamic route selection algorithms

Basic question

How do we dynamically adjust the active set of routes in the face of varying congestion, failures, transient overloads and mobility?

A possible approach is to use the behaviour of the multi-path congestion/flow control algorithm itself as the feedback signal within a sticky random algorithm, much as for circuit-switched telephone networks

R.J. Gibbens, F.P. Kelly and P.B. Key
Dynamic alternative routing — modelling and behaviour. In Twelfth International Teletraffic Congress. North-Holland (1988), Turin.

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Discussion