Modelling multi-path problems

(Invited Paper)

R.J. Gibbens
Computer Laboratory, University of Cambridge
William Gates Building, JJ Thomson Avenue, Cambridge, UK, CB3 0FD

***Abstract*—In this paper we consider the behaviour of both primal and dual multi-path algorithms for a simple network of three resources. We examine the equilibrium behaviour of our models as well as their transient response to the effect of a resource failing. The timescales over which the multi-path algorithms respond to changes in the network conditions are seen to be closely related to the round trip times of the different routes.**

I. Introduction

In this paper we study multi-path algorithms for the problem of joint routing and rate control in packet-based communication networks.

The basic algorithms are described in Section II. An example network is introduced in Section III and we discuss our experiments with this network in Section IV.

II. Algorithm descriptions

A. Network description

We suppose that the network consists of a collection of *resources* $j \in J$ and a collection of *sources* $s \in S$. Sets of resources are termed *routes*, r, and each source is uniquely identified with a set of *routes*. The notation $s(r)$ means the source identified with the route r. For each route r there is an associated flow $x_r(t)$. Let T_{rj} be the propagation delay from source $s(r)$ to resource j and T_{jr} the propagation delay from resource j to source $s(r)$. The *round trip time* for route r is then given by $T_r = T_{rj} + T_{jr}$ for all $j \in r$.

We use the notation $a = (b)^+_c$ to mean $a = b$ if $c > 0$ and $a = \max(0, b)$ if $c = 0$.

B. Primal algorithm

For our example of a primal algorithm we consider the routing extension to scalable TCP given in [1] and which we briefly summarize here. (See [4] for further discussion of scalable TCP and [3] for further background on primal multi-path algorithms.) In this model fluid flows, $x_r(t)$, operate as follows.

$$\dot{x}_r(t) = \frac{x_r(t-T_r)}{T_r}\left(\overline{a}(1-\lambda_r(t)) - b_r y_{s(r)}(t)\lambda_r(t)\right)^+_{x_r(t)} \tag{1}$$

where

$$\lambda_r(t) = 1 - \prod_{j\in r}\left(1-\mu_j(t-T_{jr})\right) \tag{2}$$

$$y_s(t) = \sum_{r\in s} x_r(t-T_r) \tag{3}$$

and

$$\mu_j(t) = p_j\left(\sum_{r:j\in r} x_r(t-T_{rj})\right). \tag{4}$$

Each resource j has a capacity C_j and a penalty function given by

$$p_j(z_j) = \left(\frac{z_j}{C_j}\right)^{\beta_j} \tag{5}$$

for some constant β_j.

1) Local stability condition: A sufficient condition [1] for local stability of the primal algorithm is that

$$\overline{a}(1+\beta) < \frac{\pi}{2}. \tag{6}$$

C. Dual algorithm

Our example of a dual algorithm uses the controlled splitting approach to the problem of joint routing and flow control presented in [2]. This scheme adopts a utility function

$$U_s(y) = \begin{cases} w_s\frac{y^{1-\alpha}}{1-\alpha} & \text{if } \alpha \neq 1 \\ w_s\log(y) & \text{if } \alpha = 1 \end{cases} \tag{7}$$

for $w_s > 0$ and $\alpha > 0$. The parameter q satisfying $q = p/(p+1)$ for some $p > 0$ is also used.

The fluid flows, $x_r(t)$, are then given by

$$x_r(t) = \lambda_r(t)^{-(p+1)} w_s^{p+1} y_{s(r)}(t)^{1-\alpha(p+1)} \tag{8}$$

where

$$\dot{y}_s(t) = \kappa_s y_s(t)^{\frac{1}{p+1}}\left(\sum_{r\in s} x_r(t-T_r)^q - y_s(t)^q\right)^+_{y_s} \tag{9}$$

and

$$\lambda_r(t) = \sum_{j\in r}\mu_j(t-T_{jr}). \tag{10}$$

The quantities $\mu_j(t)$ satisfy the following relations.

$$\dot{\mu}_j(t) = \kappa_j\mu_j(t)\left(z_j(t)-C_j\right)^+_{\mu_j(t)} \tag{11}$$

where

$$z_j(t) = \sum_{r:j\in r} x_r(t-T_{rj}). \tag{12}$$

The parameters κ_j and κ_s are small positive gain parameters.

978-1-4244-2247-0/08/$25.00 ©2008 IEEE.

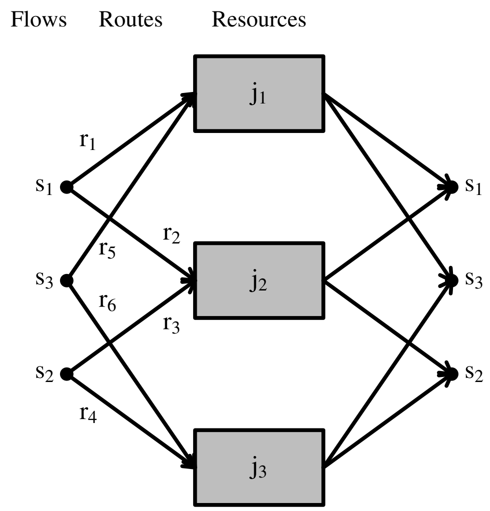

Fig. 1. The example network of three resources and a total of six routes.

1) Local stability conditions: When $\alpha(p+1) > 1$ we have local stability [2] of the dual algorithm if the following sufficient conditions hold for each resource j and for each source s

$$\kappa_j(p+1)T_jC_j < \frac{1}{2} \tag{13}$$

where

$$T_j = \frac{1}{C_j}\sum_{r:j\in r} x_r T_r \tag{14}$$

and

$$\kappa_s\left((p+1)\alpha - 1\right)T_s < \frac{1}{2} \tag{15}$$

where

$$T_s = \frac{q}{y_s^p}\sum_{r\in s} x_r^q T_r\,. \tag{16}$$

III. Example network

We consider a simple example network consisting of just three resources labelled j_1, j_2 and j_3. Each resource has a capacity of $C_j = 10$.

There are three sources labelled s_1, s_2 and s_3 and a total of six routes labelled $r_1,\ldots,r_6$. Source s_1 uses routes r_1 and r_2, source s_2 uses routes r_3 and r_4 and source s_3 uses routes r_5 and r_6.

Routes r_1 and r_5 consist of the single resource j_1, routes r_2 and r_3 consist of the resource j_2 and routes r_4 and r_6 consist of the single resource j_3.

Figure 1 shows the assignment of routes, resources and sources whereas Table I shows the propagation delays and round trip times. Note that the round trip times for the resources j_1, j_2 and j_3 vary widely and are in the proportions $1 : 10 : 100$.

TABLE I
Delay and round trip time parameters

j	r	T_{rj}	T_{jr}	T_r
j_1	r_1	0.01	0.01	0.02
j_1	r_5	0.01	0.01	0.02
j_2	r_2	0.1	0.1	0.2
j_2	r_3	0.1	0.1	0.2
j_3	r_4	1.0	1.0	2.0
j_3	r_6	1.0	1.0	2.0

TABLE II
Dual parameters

j	κ_j	s	κ_s	w_s
j_1	0.1	s_1	0.25	0.1
j_2	0.01	s_2	0.01	0.1
j_3	0.001	s_3	0.01	0.1

IV. Experiments

In this section we discuss a number of experiments conducted with the example network. We look at the equilibrium behaviour as well as the transient response generated by resource j_2 failing entirely. We arrange our experiments so that the flows have converged to their equilibrium values by time 0 when resource j_2 is then failed. As a consequence flows $x_{r_2}(t) = x_{r_3}(t) = 0$ for $t > 0$.

The fluid flow models are solved by a simple integration approach with a fixed step size of $\delta t = 0.0001$.

A. Primal algorithm

The primal algorithm requires the selection of several parameters and in these experiments we used the parameter values $\overline{a} = 0.1$, $b_r = 0.875$ and $\beta_j = 10$ for each route r and resource j, respectively.

Figures 2 and 3 show our the flows were seen to evolve over time. In Figure 2 observe how flows x_{r_1} and x_{r_5} react quickly when at time 0 resource j_2 is failed since they both use resource j_1 with the short round trip time. In contrast, flows x_{r_4} and x_{r_6} both use resource j_3 with the longer round trip time and react more slowly.

We see from Figure 3 that the total flow for source s_2 (that is, $x_{r_3} + x_{r_4}$) declines rather severely when resource j_2 fails and only recovers rather slowly to a new equilibrium value. The flows for sources s_1 and s_3 are able to react more quickly since they both can route load to resource j_1 which has the short round trip time. Indeed, the joint flows for sources s_1 and s_3 are indistinguishable on the graph.

B. Dual algorithm

For the dual algorithm we used the parameter values given in Table II. The various gain parameters were chosen to ensure local stability of our solutions in accordance with the stated conditions given above. The parameter p was taken to be 7 and thus $q = 7/8$.

Figures 4 and 5 show the equivalent results with the operation of the controlled splitting dual algorithm.

We again find that flows x_{r_1} and x_{r_5} react quickly since they use the resource with the short round trip time. This example

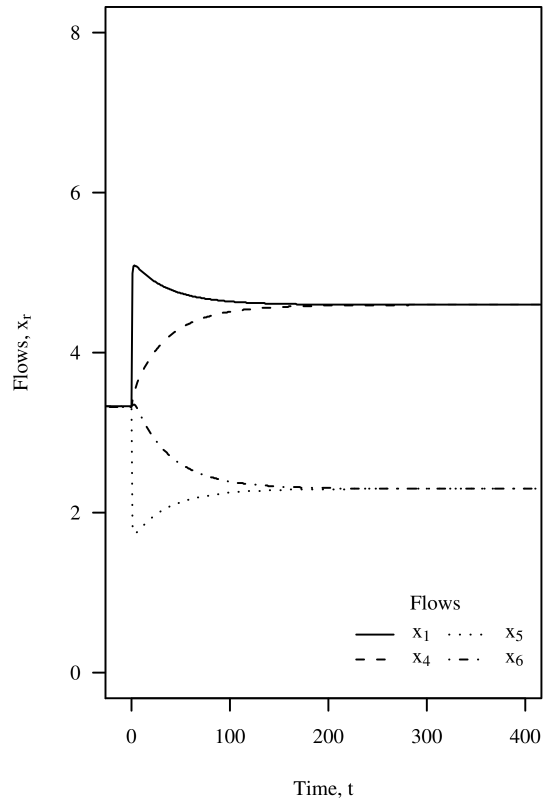

Fig. 2. Flows over time under the operation of the primal algorithm.

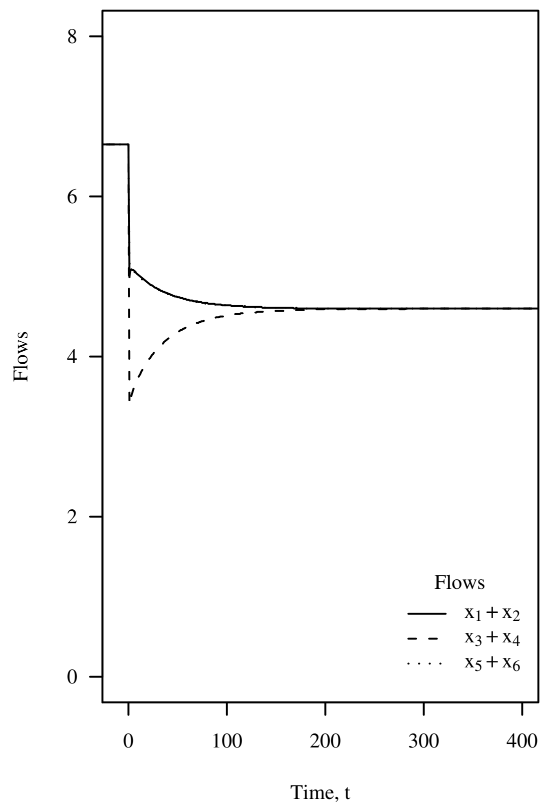

Fig. 3. Aggregate flows for the different sources with the primal algorithm.

illustrates how at first x_{r_6} increases slightly before declining to its new equilibrium value.

V. Conclusion

In this brief study of a simple three resource network we have examined the behaviour of both primal and dual multi-path algorithms. A notable feature in our results is the different timescales over which sources will attempt to re-balance their flows across the available paths.

Acknowledgment

The author would like to thank Frank Kelly, Peter Key, Don Towsley and Tom Voice for useful discussions on multi-path problems. The author also wishes to acknowledge the financial support of the EPSRC (research grant GR/S86266/01).

References

[1] F. Kelly and T. Voice, "Stability of end-to-end algorithms for joint routing and rate control," *ACM SIGCOMM Computer Communication Review*, vol. 35, no. 2, pp. 5–12, Apr. 2005.

[2] T. Voice, "Stability of mulit-path dual congestion control algorithms," *IEEE/ACM Trans. Netw.*, vol. 15, no. 6, pp. 1231–1239, Dec. 2007.

[3] H. Han, S. Shakkottai, C. Hollot, R. Srikant, and D. Towsley, "Overlay TCP for multi-path routing and congestion control," in *ENS-INRIA ARC-TCP Workshop*, 2003, Paris, France.

[4] T. Kelly, "Scalable TCP: improving performance in highspeed wide area networks," *Computer Communication Review*, vol. 32, no. 2, pp. 83–91, 2003.

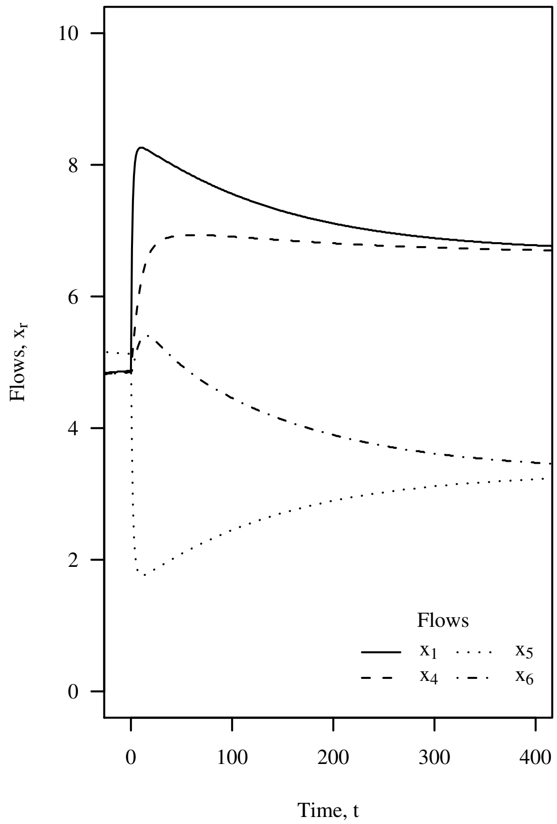

Fig. 4. Flows over time under the operation of the dual algorithm.

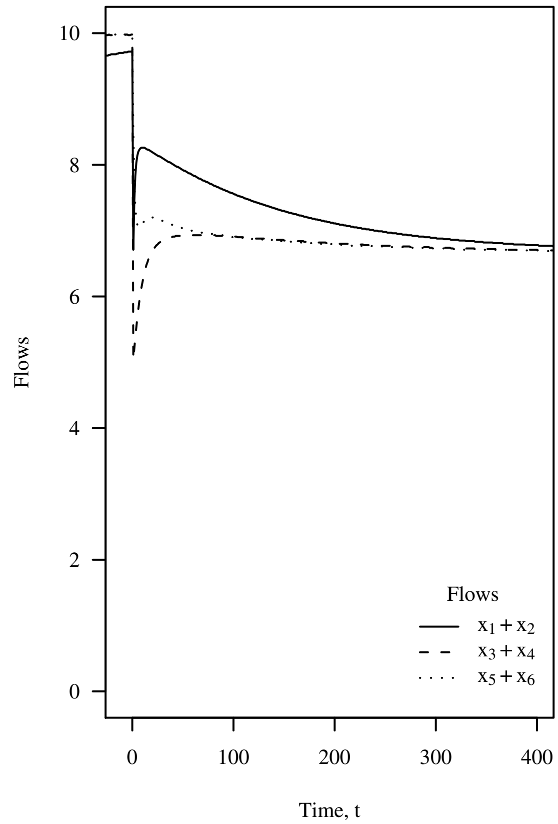

Fig. 5. Aggregate flows for the different sources with the dual algorithm.