Optimal Resource Allocation for OFDM Uplink Communication: A Primal-Dual Approach

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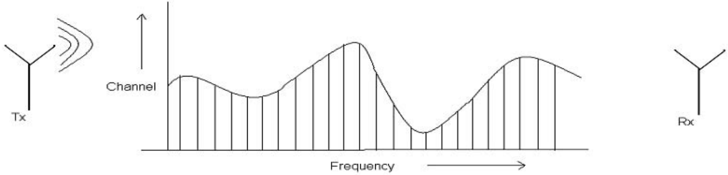
OFDM Systems



- Frequency band divided into several parallel orthogonal carriers/tones.
- High spectrum efficiency.
- Eliminate inter-symbol-interference (ICI) due to multi-path fading.
- Applications: WiMAX (802.16), Wi-Fi (802.11a/g), DSL, etc.

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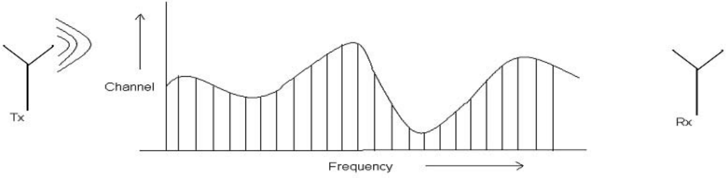

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- Applications: WiMAX (802.16), Wi-Fi (802.11a/g), DSL, etc.
- How to perform distributed and efficient resource allocation?

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- Resource allocation = **weighted rate maximization**
  - "Network": assigning carriers to users
  - "User": assign power over carriers

Achievable Rate Per Carrier

- If a carrier j is allocated to a single user i:
  - $r_{ij} = \log(1 + SNR) = \log(1 + p_{ij} e_{ij})$
  - p_{ij} = power user i allocates to carrier j.
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- Why "time-sharing"?
  - Convexify the problem.
  - Can be achieved in practice.
  - Not affecting the optimal objective value with large number of carriers.

OFDMA rate region

- **Rate region**:

$$\mathcal{R}(\mathbf{e}) = \left\{ \mathbf{r} : \ r_i = \sum_j x_{ij} \log\left(1 + \frac{p_{ij} e_{ij}}{x_{ij}}\right), \ \forall i \right\}$$

where $(\mathbf{x}, \mathbf{p}) \in \mathcal{X}$ such that

- $(\mathbf{x}, \mathbf{p}) \geq \mathbf{0}$
- Each user i: $\sum_j p_{ij} \leq P_i$ (uplink transmission power constraint)
- Each carrier j: $\sum_i x_{ij} \leq 1$ (channel allocation constraint)

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- Variation: **sub-channelization** (bundle carriers to reduce overhead)
  - Interleaved (802.16 standard mode)
  - Adjacent (Band AMC mode)
  - Random (e.g. frequency hopped)

Weighted Rate Maximization Problem

$$\max_{\mathbf{r}\in\mathcal{R}(\mathbf{e})} \mathbf{w}\cdot\mathbf{r} = \max_{(\mathbf{x},\mathbf{p})\in\mathcal{X}} \sum_i w_i \sum_j x_{ij} \log\left(1 + \frac{p_{ij} e_{ij}}{x_{ij}}\right) \qquad \text{(OPT)}$$

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- Concave maximization problem
- Non-strictly concave: typically many local/global optimal allocations
- What is known: centralized optimal and (very good) heuristic algorithms [HSBA'07]
- What we will show: distributed optimal algorithm [This talk]

Centralized Optimal Algorithm [HSBA'07]

- Problem (OPT) is convex and satisfies Slater's condition
  $\Rightarrow$ No duality gap.
- Consider Lagrangian:

$$L(\mathbf{x}, \mathbf{p}, \boldsymbol{\lambda}, \boldsymbol{\mu}) := \sum_i w_i \sum_j x_{ij} \log\left(1 + \frac{p_{ij} e_{ij}}{x_{ij}}\right) + \sum_i \lambda_i \left(P_i - \sum_j p_{ij}\right) + \sum_j \mu_j \left(1 - \sum_i x_{ij}\right).$$

- Dual function:

$$L(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \max_{(\mathbf{x},\mathbf{p}) \in \mathcal{X}} L(\mathbf{x}, \mathbf{p}, \boldsymbol{\lambda}, \boldsymbol{\mu})$$

- Optimal objective value to Problem (OPT):

$$V^* = \min_{(\boldsymbol{\lambda},\boldsymbol{\mu}) \geq \mathbf{0}} L(\boldsymbol{\lambda}, \boldsymbol{\mu})$$

A Dual-Based Centralized Algorithm [HSBA'07]

1. For fixed $\boldsymbol{\lambda}$, analytically solve for $\boldsymbol{\mu}(\boldsymbol{\lambda})$, $\mathbf{x}(\boldsymbol{\lambda})$ and $\mathbf{p}(\boldsymbol{\lambda})$.
2. Multi-dimensional subgradient search of optimal $\boldsymbol{\lambda}^*$.
3. Find optimal $\mathbf{x}^*$ and $\mathbf{p}^*$ by solving a system of linear equations (multiple solutions).

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Properties:

- Centralized computation (step 3)
- Slow convergence (step 2)
- High signaling overhead (steps 1, 2 and 3): all channel information and power constraints

Distributed Primal-Dual Algorithm

- Try to reach a saddle point of Lagrangian $L(\mathbf{x}, \mathbf{p}, \boldsymbol{\lambda}, \boldsymbol{\mu})$:

$$\begin{aligned}
\dot{x}_{ij} &= k_{ij}^{x} \cdot \partial L / \partial x_{ij} = k_{ij}^{x} \left(f_{ij}(x_{ij}, p_{ij}) - \mu_j \right)_{x_{ij}}^{+}, \text{(mobile)} \\
\dot{p}_{ij} &= k_{ij}^{p} \cdot \partial L / \partial p_{ij} = k_{ij}^{p} \left(g_{ij}(x_{ij}, p_{ij}) - \lambda_i \right)_{p_{ij}}^{+}, \text{(mobile)} \\
\dot{\lambda}_{i} &= k_{i}^{\lambda} \cdot \partial L / \partial \lambda_{i} = k_{i}^{\lambda} \left(\sum_j p_{ij} - P_i \right)_{\lambda_i}^{+}, \text{(mobile)} \\
\dot{\mu}_{j} &= k_{j}^{\mu} \cdot \partial L / \partial \mu_{j} = k_{j}^{\mu} \left(\sum_i x_{ij} - 1 \right)_{\mu_j}^{+}, \text{(base station)}
\end{aligned}$$

Distributed Primal-Dual Algorithm

Advantages:

- Distributed and simple updates by mobiles and base station
- Low communication overhead: user feedback x_{ij}'s and base station announces μ_j

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Challenge:

- How to achieve global convergence with non-strictly concave objective functions
  - Difficult in general
  - Example: multi-path routing [Voice'06]

Convergence Result: Part I

Theorem (Convergence to an Invariant Set)

- *All trajectories of the primal-dual system converge to an invariant set V_0 globally and asymptotically.*
- *All optimal solutions of Problem (OPT) are contained in set V_0.*

Convergence Result: Part I

Theorem (Convergence to an Invariant Set)

- *All trajectories of the primal-dual system converge to an invariant set V_0 globally and asymptotically.*
- *All optimal solutions of Problem (OPT) are contained in set V_0.*

- **Proof**: constructing a proper Lyapunov function.
- **Question**: Will set V_0 contain non-optimal solutions?

Convergence Result: Part II

Theorem (Convergence to a Global Optimal Solution)

All trajectories of the primal-dual system globally converge to optimal solutions of Problem (OPT) under properly chosen stepsizes.

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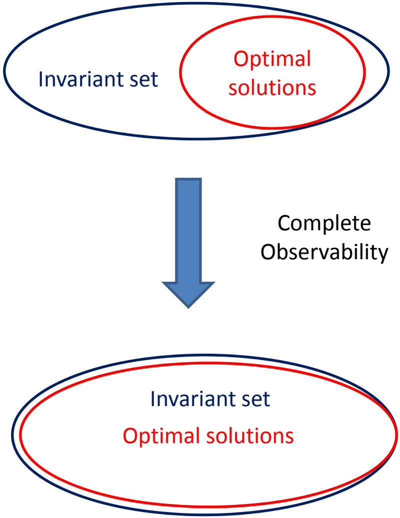
Theorem (Convergence to a Global Optimal Solution)

All trajectories of the primal-dual system globally converge to optimal solutions of Problem (OPT) under properly chosen stepsizes.

- Set V_0 only contains optimal solutions.
- Stepsize choice is easy: e.g., not all k_j^{μ}'s are the same.
- **Proof**:
  - Over set V_0, the nonlinear system reduces to a linear one.
  - The linear system in marginal stable.
  - Set V_0 contains only the optimal solution if (λ, μ) is completely observable from $B[x^T, p^T]^T$.

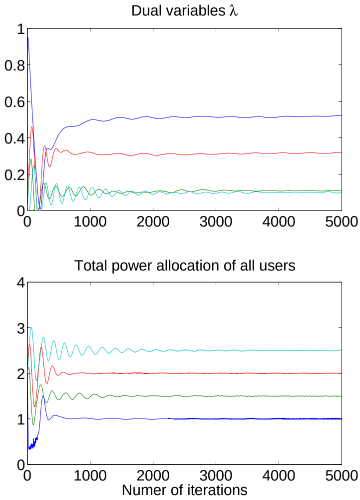
Relationship



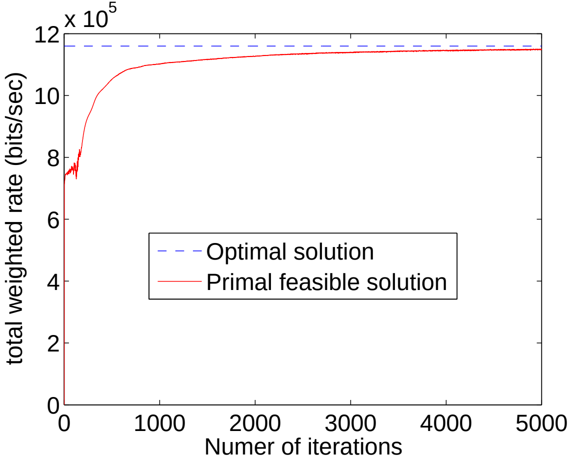
Simulation Set-up

- Single cell, $M = 4$ users.
- e_{ij} = (fixed location-based term)$\times$(frequency selective fast fading)
  - Fixed term = empirical distribution.
  - frequency selective term = block fading in time (2*msec* coh. time $\approx$ 250Hz Doppler); standard ref. mobile delay spread (6 taps, 1 μsec).
- 5 MHz BW, 512 carriers.
- Adjacent channelization, 8 carriers/subchannel.
- Resource allocation over 20 OFDM symbols.
- Randomly generated weights in $[0, 1]$.

Convergence of Primal and Dual Variables



Convergence of Total Weighted Rate



Achieve 90% within 500 iterations

Conclusions

- **Topic**: optimal resource allocation for uplink OFDM systems
- **Algorithm**: Primal-Dual algorithm
- **Properties**
  - Distributed and simple updates
  - Low communication overhead per iteration
- **Future Work**:
  - Convergence with delay and asynchronous updates
  - Faster convergence

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