Threshold Structure of Channel Aware Distributed Scheduling in Ad-Hoc Networks: An Optimal Stopping View

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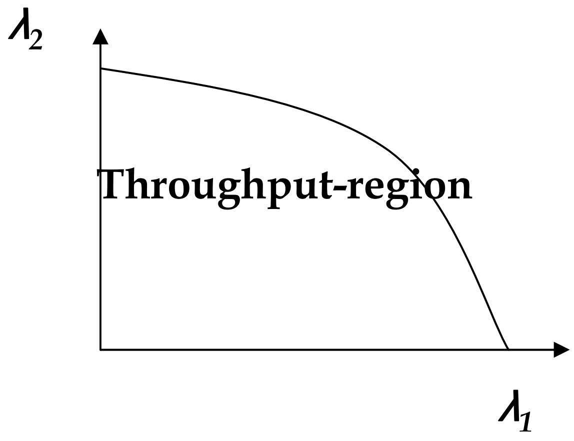
MAC Scheduling

- Basic setting: wireless ad-hoc networks
- Question: which links to activate and when?
- Popular model: constrained queuing network model
  - "Max-Weight" Scheduling is throughput-optimal scheduling (i.e., largest stability region)
  - seminal work [Tassiulas-Ephremides92]



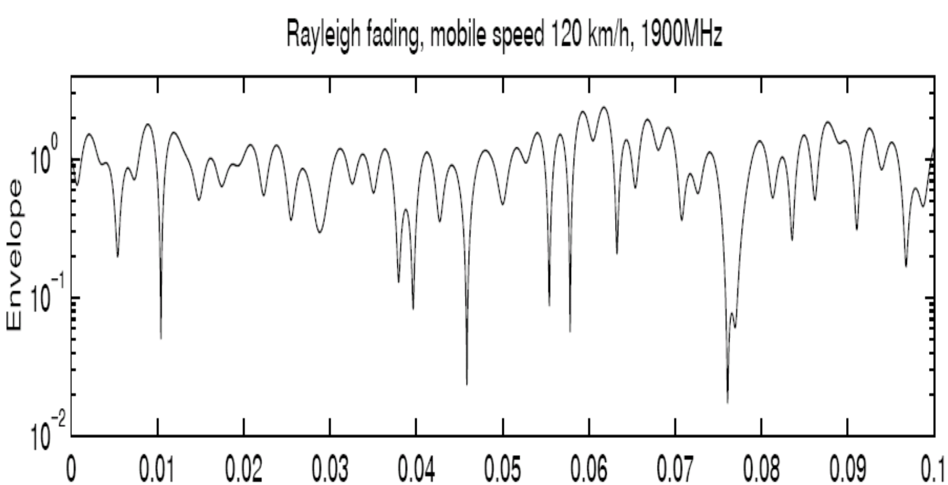
Wireless Scheduling

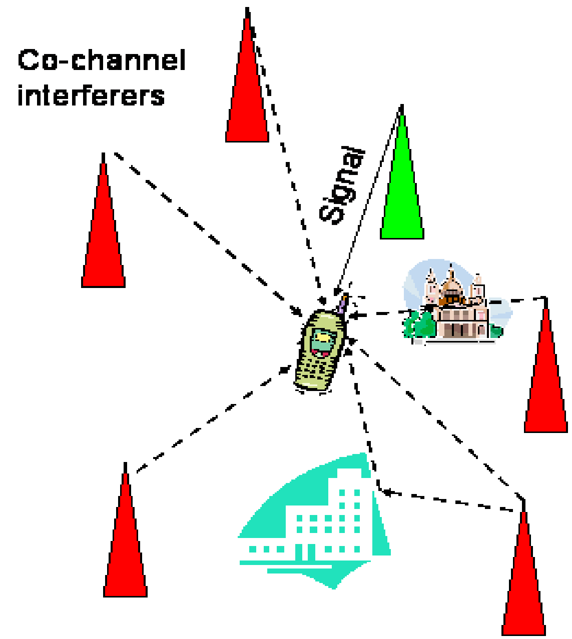
Based on interference models:

- Maximal/greedy scheduling
  - [Lin-Shroff05] [Charpokar-Saswati05,06], [Wu-Srikant05,06], [Sharma-Shroff-Mazumdar05,06]
- Distributed T-O scheme with polynomial complexity based on random mixing
  - [Modiano-Shah-Zussman 06], [Eryilmaz-Ozdaglar-Modiano 07], [Yi-de Veciana-Shakkottai 07]
- Distributed scheduling with constant time complexity
  - [Lin-Rasool 06], [Sanghavi-Bui-Srikant 07]

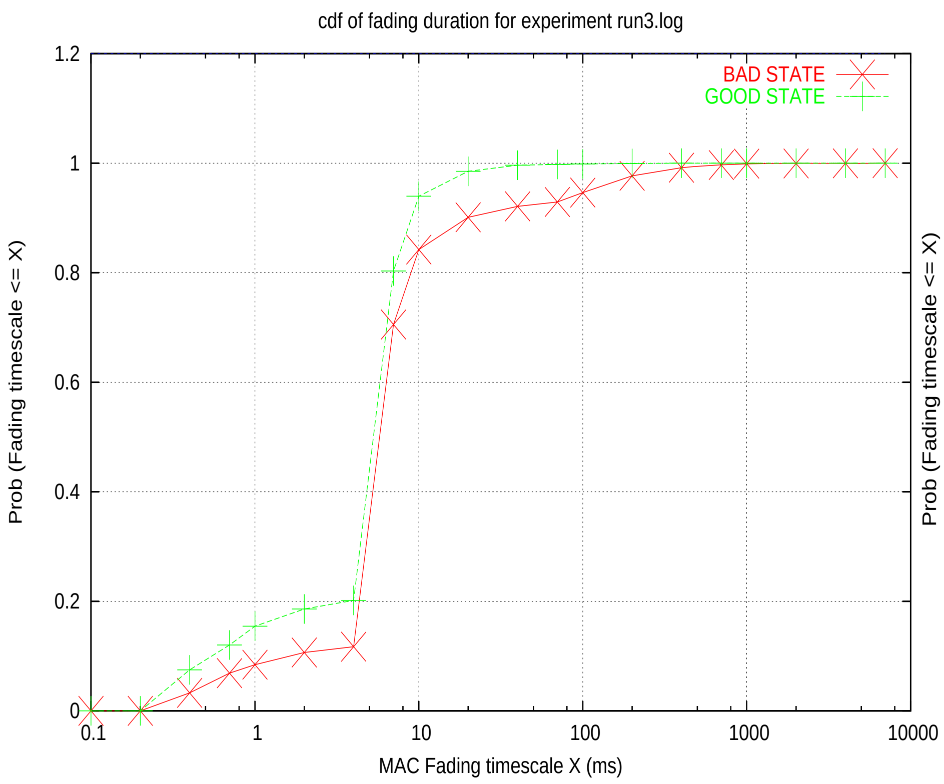
Unique Challenges in Wireless Networks

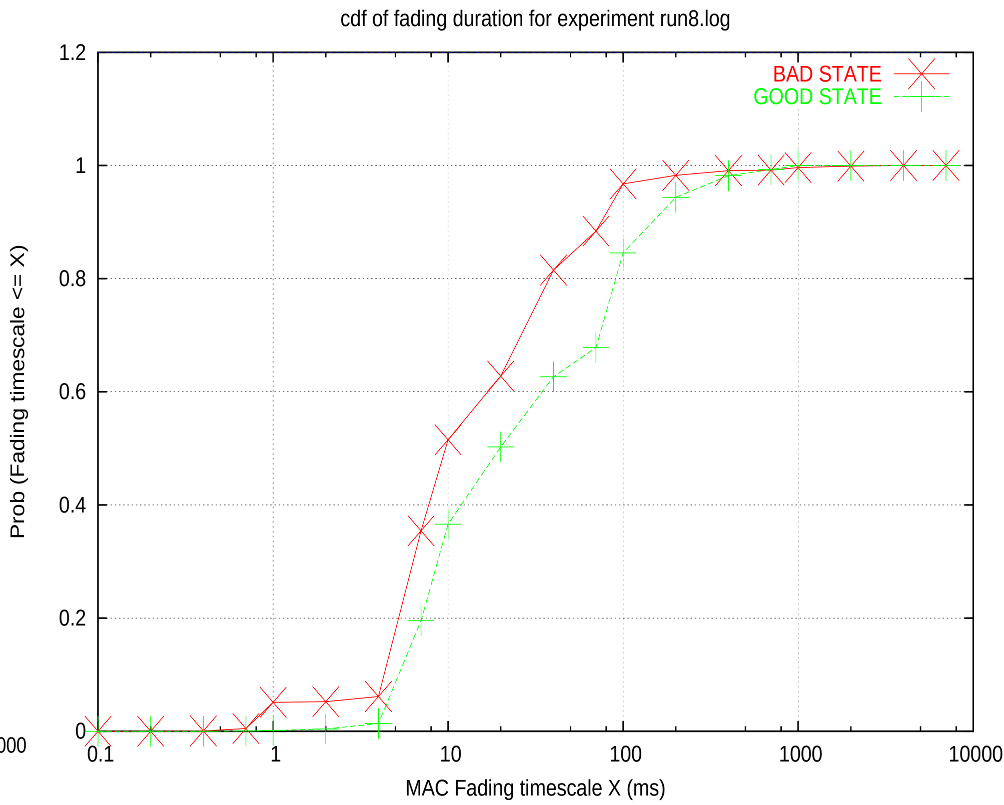
- Two main challenges:
  - Channel fading; co-channel interference
  - Interference depends on fading too.





Time Scales of Channel Variation and MAC (interference) Variation





- Measurement data [Aguayo-Bicket-Biswas-Judd-Morris 04] [Cao-Raghunagthan-Kumar 06]

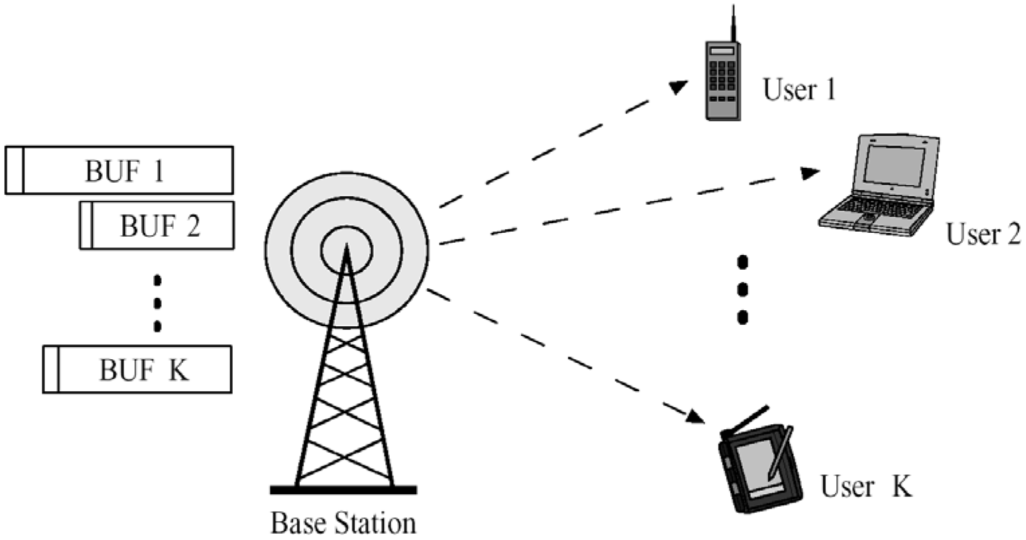
Unified PHY/MAC Optimization

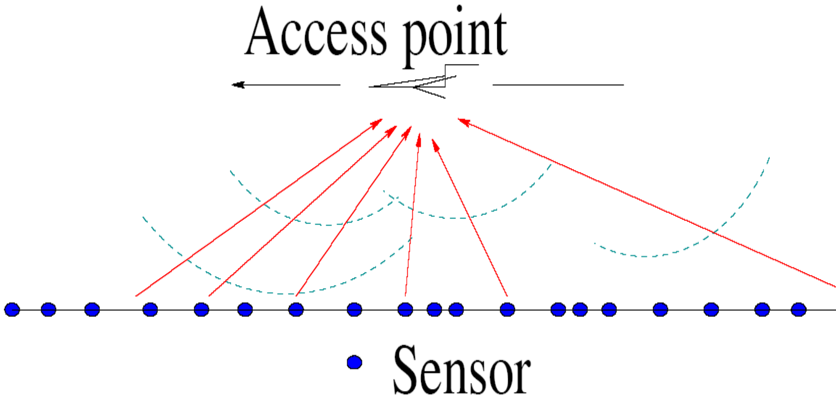
- Traditional wisdom treats link losses due to fading separately from those incurred by interference;
  - MAC layer: scheduling used to resolve interference
  - PHY layer: coding/modulation, diversity schemes
- However, fading can often adversely affect MAC layer!
- Indeed, time scales of channel variation and MAC variation are of the same order.

➡ This calls for channel-aware scheduling!

Centralized Opportunistic Scheduling and Channel-Aware Aloha

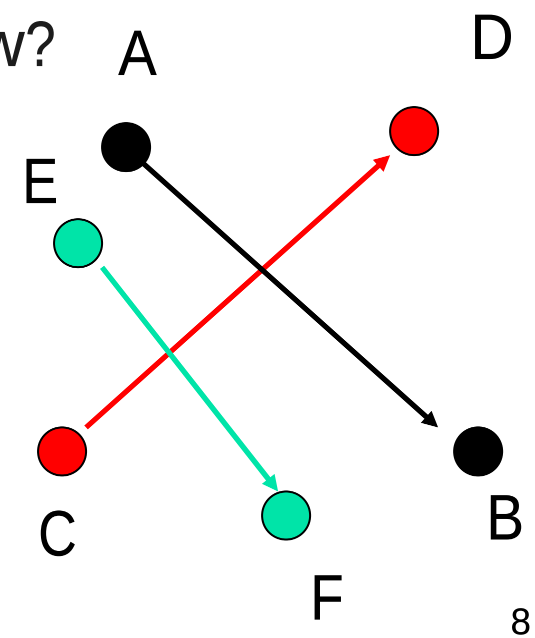
- Downlink scheduling: [Tse00], [Liu-Chong-Shroff01], [Borst01], [Andrews et al 01],
- Channel-aware Aloha: [Qin-Berry03] [Adireddy-Tong05]





Peer-to-Peer Communications in Ad-hoc Networks

- Challenges in devising channel-aware scheduling for ad-hoc communications:
  - Links have no knowledge of others' channel conditions; even their own channel conditions are unknown before probing.
  - Q) which link to schedule, and how?



Model: consider contention based ad-hoc networks (e.g., CSMA-type)

Talk Outline

Theme: Channel aware distributed scheduling for exploiting PHY/MAC diversities

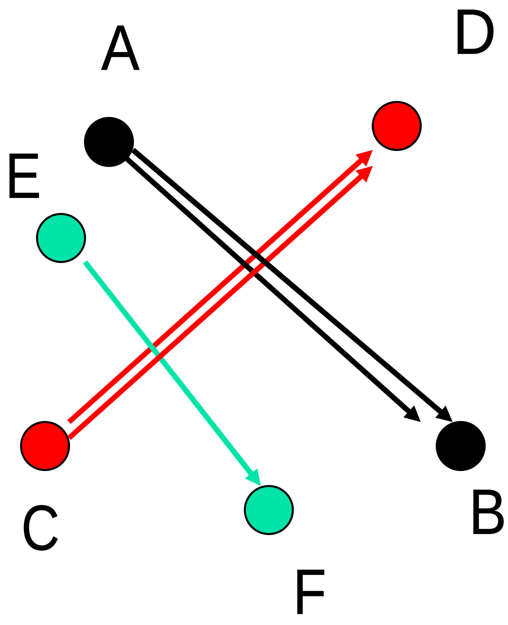
- Single-hop ad-hoc networks:
  - Threshold-based distributed scheduling for the single-receiver model
  - Threshold-based distributed scheduling for the multi-receiver model
  - Threshold-based distributed scheduling for the PHY-interference model
  - Threshold-based distributed scheduling with imperfect channel information
- Multi-hop ad-hoc networks:
  - Threshold-based distributed scheduling
  - Open issue: delay performance

The Single-receiver Model (1)

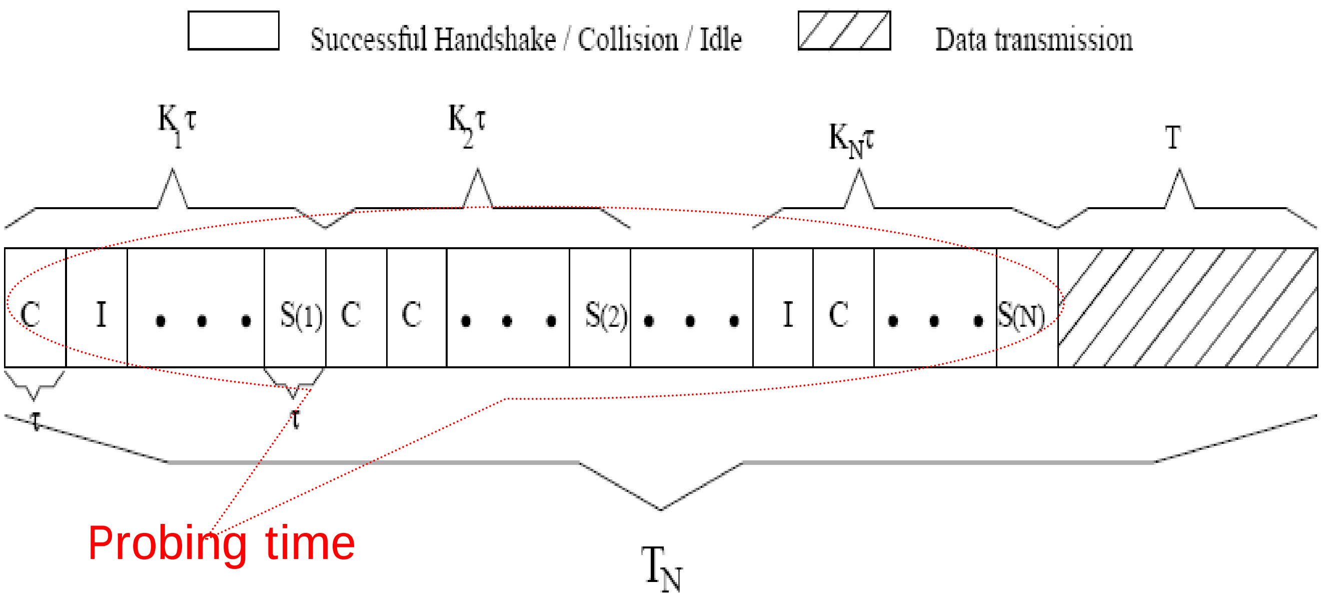
Suppose after one successful contention, channel condition is poor. Two options available:

- Continue data transmission;
- Or, alternatively, let this link give up this opportunity, and let all links re-contend.

- Intuition: At additional cost, further channel probing can lead to data transmission with better channel conditions.
- In this way, multiuser diversity and time diversity can be exploited in a distributed and opportunistic manner.



The Single-Receiver Model (2)



- s(n) denote the successful link in n-th round of channel probing.
- Clearly, there is a tradeoff between throughput gain from better channel conditions and the cost for further channel probing.
- Using optimal stopping theory, we characterize this tradeoff for distributed scheduling.

The Single-Receiver Model (3) : Maximizing Rate of Return

- Objective: to maximize average network throughput

The rate of return

$$\mathrm{x}_L \longrightarrow \frac{E[R_{(N)}T]}{E[T_N]} \quad a.s.$$

- Problem: find optimal stopping policy for maximizing average network throughput:

$$N^* \triangleq \mathrm{argmax}_{N\in Q}\frac{E[R_{(N)}T]}{E[T_N]}, \quad x^* \triangleq \sup_{N\in Q} \frac{E[R_{(N)}T]}{E[T_N]},$$

where

$$\mathrm{Q} \triangleq \{N : N \geq 1, E[T_N] < \infty\}.$$

The Single-Receiver Model (4): Homogeneous Link Case

Proposition: *a) The optimal stopping rule N^* exists, and is given by*

$$N^* = \min\{n \geq 1 : R_{(n)} \geq x^*\}.$$

A pure threshold strategy

b) The maximum throughput x^* is an optimal threshold, and is the unique solution to

$$E(R_{(n)} - x)^+ = \frac{x\tau}{p_s T},$$

Threshold can be pre-computed

The Single-Receiver Model (5): Heterogeneous Link Case

- Different links have different channel statistics $\{\mathrm{F}_m(r)\}$;
- $\mathrm{R}_{n,s(n)}$ and $\mathrm{R}_{n+1,s(n+1)}$ may follow different distributions.
- Nevertheless, we can treat $\mathrm{R}_{n,s(n)}$ as a compound r.v.

$$\mathrm{P}(\mathrm{R}_{(n)} \leq r) = P(R_{n,s(n)} \leq r) = E\left[P(R_{n,m} \leq r) | s(n) = m\right]$$

Optimal DOS policy:

The maximum network throughput x^* in the heterogeneous case is optimal threshold and is the unique solution to the following fixed point equation

$$x = \frac{\sum_{m=1}^{M} p_{s,m} \int_x^\infty r dF_m(r)}{\delta + \sum_{m=1}^{M} p_{s,m} \left(1 - F_m(x)\right)}.$$

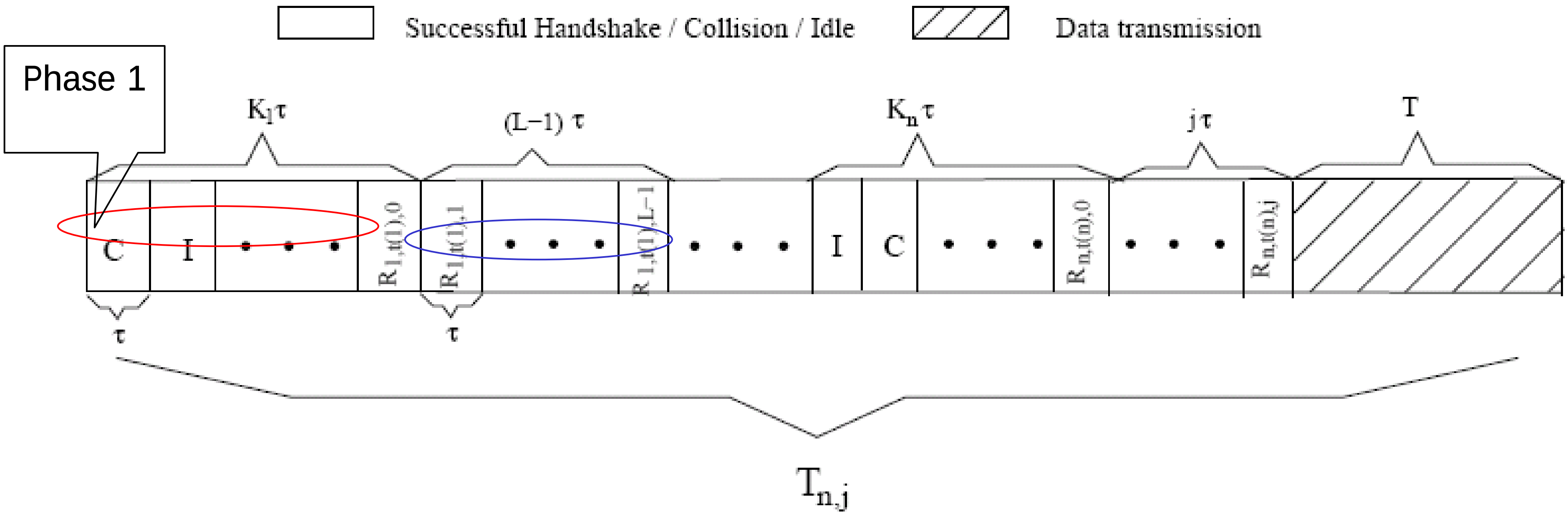
Somewhat surprising, threshold is the same for all links!

The Multi-receiver Model (1)

- Consider a network with M transmitters; each transmitter has L receivers, and contends with some probability p for data transmission.
- Let t(n) denote the successful transmitter in the n-th round of channel contention, and $R_{n,t(n),j}$denote the corresponding rate for receiver j, j=0,1,..., L-1.
- Channel probing takes place in two phases:
  - In phase 1, initial channel probing runs until a transmitter node has a successful channel contention; probing cost is random;
  - In phase 2, subsequent probings from t(n) to its receivers are performed according to specific strategies; probing cost is constant.
- Consider Sequential Probing Without Recall (SPWOR)

The Multi-receiver Model (2)

- SPWOR: The transmitter probes its receivers sequentially, and stops probing once it finds a ``good'' receiver, followed by data transmission to the current receiver.



The Multi-receiver Model (3)

Proposition *a) Suppose that sequential probing without recall (SPWOR) is used for channel probing. Then the optimal stopping rule for distributed scheduling is given as follows:*

$$N^*_{SPWOR} = \min\{\kappa \geq 1 : R_{n,t(n),j} \geq \theta^*_j,\ \textit{where}\ n = \lceil \frac{\kappa}{L} \rceil, j = mod(\kappa - 1, L$$

and the thresholds $\{\theta^*_j\}$ *are determined by*

$$\theta^*_j = x^*_{SPWOR} + v^*_{j+1}, \forall\ j = 0, 1, \ldots, L-1;$$

Corollary *The optimal thresholds* $\{\theta^*_j, \forall\ j = 0, 1, \ldots, L-1\}$ *monotonically decrease, i.e.,*

$$\theta^*_0 \geq \theta^*_1 \geq \cdots \geq \theta^*_{L-1}.$$

The Multi-receiver Model (4)

Proposition *b) The maximum network throughput* x^*_{SPWOR} *is the unique solution to the following fixed point equation:*

$$E[\max(R - x, V_1^*(x))] - \frac{x\delta}{p_s} = 0,$$

where R *is a random variable with distribution* $F(r)$, $v_j^* \triangleq V_j^*(x^*_{SPWOR}), \forall\ j = 1, 2, \ldots, L,$ *and* $\{V_j^*(x)\}$ *are defined (in a backward order) as follows:*

$$\begin{aligned} V_L^*(x) &\triangleq 0, \\ V_j^*(x) &\triangleq E[\max(R - x, V_{j+1}^*(x))] - x\delta, \forall\ j = L-1, L-2, \ldots, 1. \end{aligned}$$

The PHY-Interference Model (1)

- PHY-interference (SINR) model: multiple packet reception
  - Probabilistic reception
  - Can decode multiple packets simultaneously
- Challenges beyond collision model
  - Multiple links can simultaneously transmit successfully
  - The number of transmission links is random, and not known to each link
  - Each link has no knowledge of the instantaneous rates of other links
  - Decision is made in a distributed manner at each link with local information only
- Q) How to do scheduling to improve throughput?

The PHY-Interference Model (2)

- Approach: optimal stopping
- The average network throughput (rate of return)

$$x = \frac{E[\sum_{j \in K_n} R_{j,n} T]}{E[T_n]}.$$

The number of transmission links is random

Random due to channel fading and interference

The duration of channel probings is random

- Problem: find optimal policy for maximizing rate of return

$$N_i^* = \arg\max_{N_i \in Q} \frac{E[\sum_{j \in K_{N_i}} R_{j,N_i} T]}{E[T_{N_i}]}.$$

- Different links may have different stopping rules
- Accordingly, the optimal stopping rule of the network would be the earliest one among all links, i.e.,

$$N^* = \min_i \{N_i^*, \ i = 1, 2, \cdots, M\}.$$

The PHY-Interference Model (3)

Proposition *The optimal stopping rule* N^* *for PHY-aware distributed scheduling exists, and is given by*

$$N^* = \min\left\{n \geq 1 : \max_i\{R_{i,n}\} \geq \alpha^* - E[Y_n]\right\}.$$

where $Y_n = \sum_{i \in K_n} R_{i,n} - \max_i\{R_{i,n}\}$, *and* α^* *is the unique solution to*

$$E(\sum_{i \in K_n} R_{i,n} - \alpha)^+ = \frac{\alpha\tau}{T}.$$

Noisy Probing Model (1)

- In the above, channel state information (CSI) is assumed to be perfectly known at receiver/transmitter after channel probing.

- In practical scenarios, channel conditions are often estimated using noisy observations, and CSI is imperfect.

- Next, we explore channel-aware distributed scheduling with noisy channel estimation.

Noisy Probing Model (2)

Using MMSE estimator, we have that

$$h_n = \hat{h}_n + \tilde{h}_n,$$

where $\hat{h}_n$ is channel estimator, and $\tilde{h}_n$ estimation error. Both are zero-mean complex Gaussian random variables, and satisfy

$$E[|h_n|^2] = E[|\hat{h}_n|^2] + E[|\tilde{h}_n|^2].$$

Treating the estimation error as noise, the actual SNR is

$$\lambda_n = \frac{\rho|\hat{h}_n|^2}{1 + \rho|\tilde{h}_n|^2}.$$

Estimated SNR

Noisy Probing Model (3)

Two major differences between perfect and noisy channel estimation cases:

1. The stopping rule N in noisy channel case is defined over the filtration $\{\mathcal{F}'_n\}$ generated by $\{(\rho|\hat{h}_j|^2, K_j), j = 1, 2, \ldots, n\}$.
2. The instantaneous rate, R_n, is now a r.v., and is not perfectly known.

Using the tool of Statistical Version of Prophets Inequality, can show structure of optimal scheduling strategy remains same, except that random "reward" R_n is replaced with its conditional expectation.

Proposition

$$\sup_{N \in Q'} \frac{E[R_N T]}{E[T_N]} = \sup_{N \in Q''} \frac{E[\bar{R}_N T]}{E[T_N]}.$$

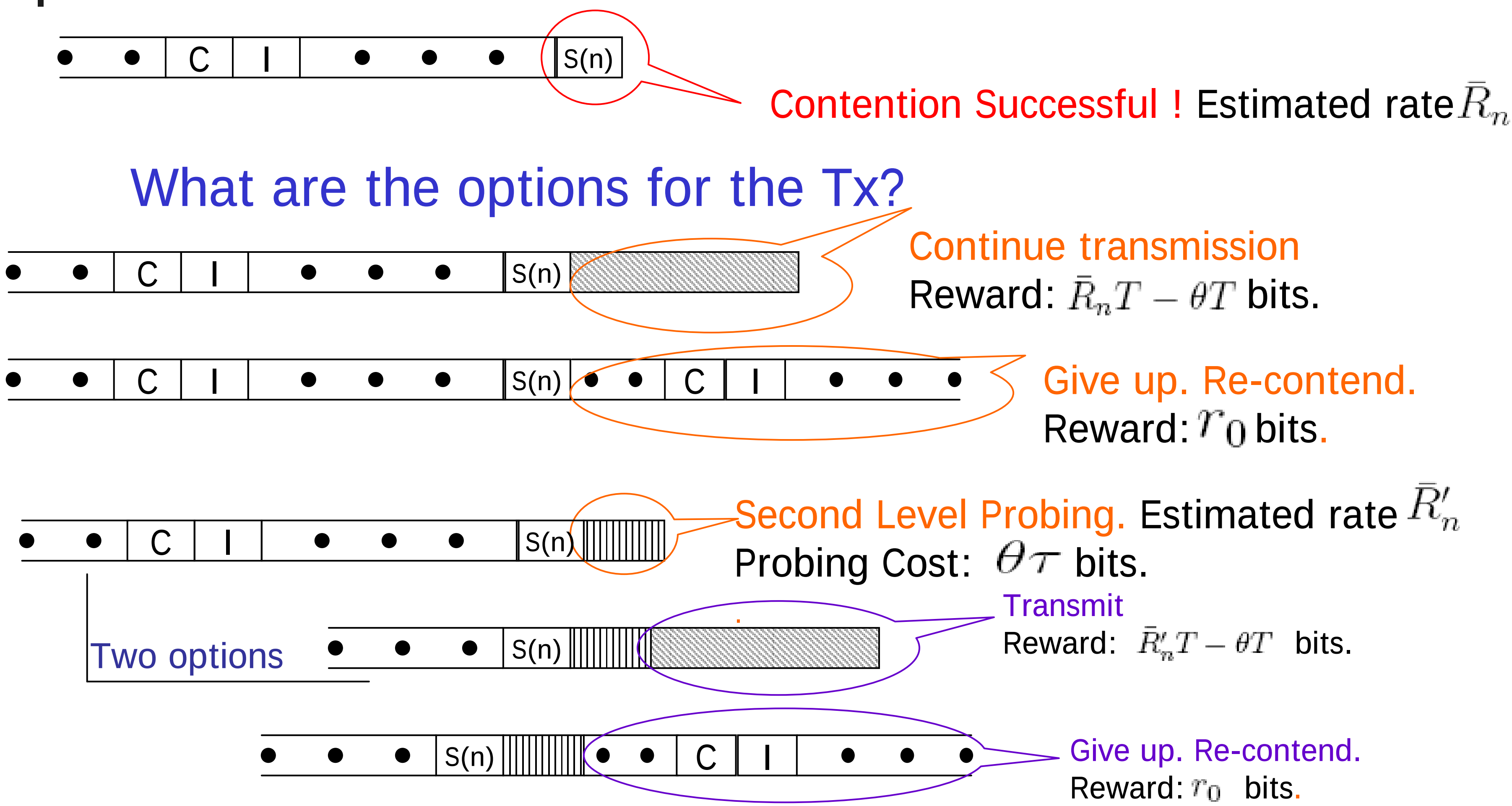
Two Level Noisy Probing Model (1)

- Channel state information available is noisy.
- Further channel estimation may be helpful to improve the quality of channel estimation and hence the throughput.
- Particularly interested in the “wideband low SNR” regime, i.e., $\rho \to 0$ and $W = \Theta(\frac{1}{\rho})$.
- Rate of transmission is $R = \Theta(1)$
- Trade-off between the enhanced rate due to improved channel estimate and further probing cost.

Two Level Noisy Probing Model (2)

- Suppose after one successful contention, estimated transmission rate is $\bar{R}_n$. Three options available:
  - Continue data transmission at rate $\bar{R}_n$;
  - Or, let this link give up this opportunity, and let all links re-contend.
  - Alternatively, obtain refined rate estimate $\bar{R}'_n$ with additional probing cost and then
    - Transmit at rate $\bar{R}'_n$
    - Or, let this link give up this opportunity, and let all links re-contend.

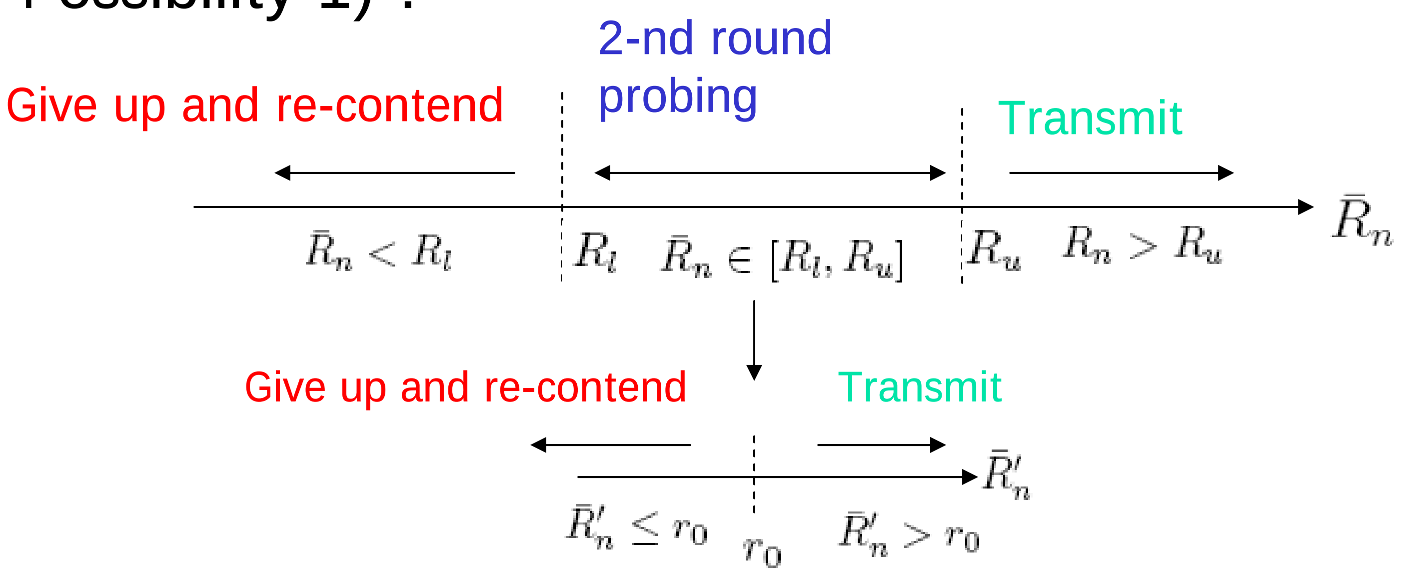
Two Level Noisy Probing Model (3)



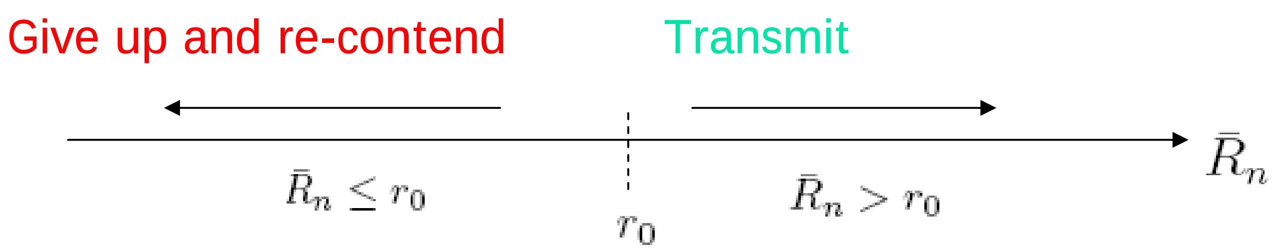
What is the optimal strategy?

Two Level Noisy Probing Model (4)

Possibility 1) :



Possibility 2) :



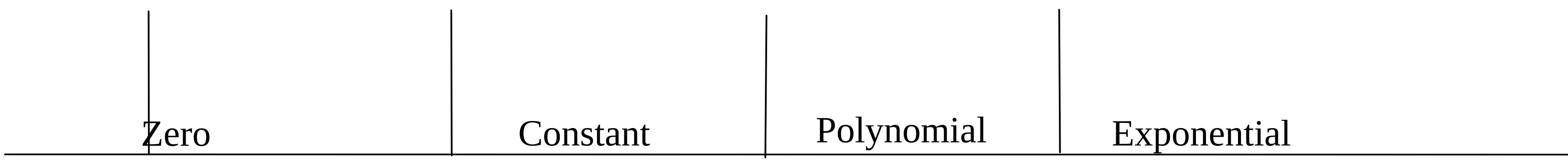
Two Level Noisy Probing Model (5)

Proposition: *Let r_0 denote the expected throughput. After the n-th successful channel contention, one of the following two statement holds:*

1) *There exist two real numbers $R_l \leq R_u$ such that it is optimal*
   a) *to transmit immediately after the first-level probing if $\bar{R}_n > R_u$*
   b) *to give up the transmission and let all the nodes re-contend if $\bar{R}_n < R_l$*
   c) *to engage in the second-level probing if $\bar{R}_n \in [R_l, R_u]$ and then to transmit at the rate $\bar{R}'_n$ if $\bar{R}'_n > r_0$ and to give up the transmission if $\bar{R}'_n < r_0$.*
2) *It is never optimal to demand additional information. It is optimal to transmit at a rate $\bar{R}_n$ immediately after the first-level probing if $\bar{R}_n > r_0$ and to defer transmission and re-contend if $\bar{R}_n < r_0$.*

Threshold-based Distributed Scheduling in Multi-hop Wireless Networks (1)

- Developing channel-aware distributed scheduling is much more challenging in multi-hop wireless networks.



- Suppose all nodes in the network have infinite buffers, and can store a sequence of pre-defined thresholds.

Motivated by max-weight scheduling, one can carry out quantization of the product of queue length and channel rate, μQ, over its full range, and form a sequence of thresholds $\{x_k, k = 1, 2, \ldots\}$. (This is in analogous to universal quantization.)

Threshold-based Distributed Scheduling in Multi-hop Wireless Networks (2)

- Approximation algorithm for max-weight scheduling:
  - For each link l, whenever its local weight μQ crosses (either up or down) a threshold x_k, it notifies its neighboring links; and the "latest" max-scheduling is re-computed.
  - One main difference between this algorithm and max-weight scheduling is that search for max-weight matching is triggered only when the thresholds are crossed and the scheduling is updated less frequently.
- Can show that this algorithm is asymptotically throughput-optimal as long as the maximum difference between two thresholds is bounded.
- It remains open to characterize the optimal quantization of μQ.

Delay Performance of Distributed Scheduling in Multi-hop Wireless Networks

- Delay performance corresponding to wireless scheduling is an under-explored area.
- One main challenge due to coupling between arrival rate and service rate: service rates depend on queue sizes, which in turn depend on the arrival rates and channel conditions.
- Possible approaches:
  - Standard queueing analysis - Mission impossible?
  - Large deviation approach: limited by dimensionality
  - Need some elegant simplification and abstraction (e.g., state space collapse) → Heavy traffic analysis?

Conclusions

- We explore channel aware distributed scheduling for exploiting PHY/MAC diversities
- Single-hop ad-hoc networks: threshold-based distributed scheduling for a variety of models: single-receiver model; multi-receiver model; PHY-interference model; noisy probing model.
- Multi-hop ad-hoc networks:
  - Threshold-based distributed scheduling
  - Open issue: delay performance