# Pricing in the Presence of Peering

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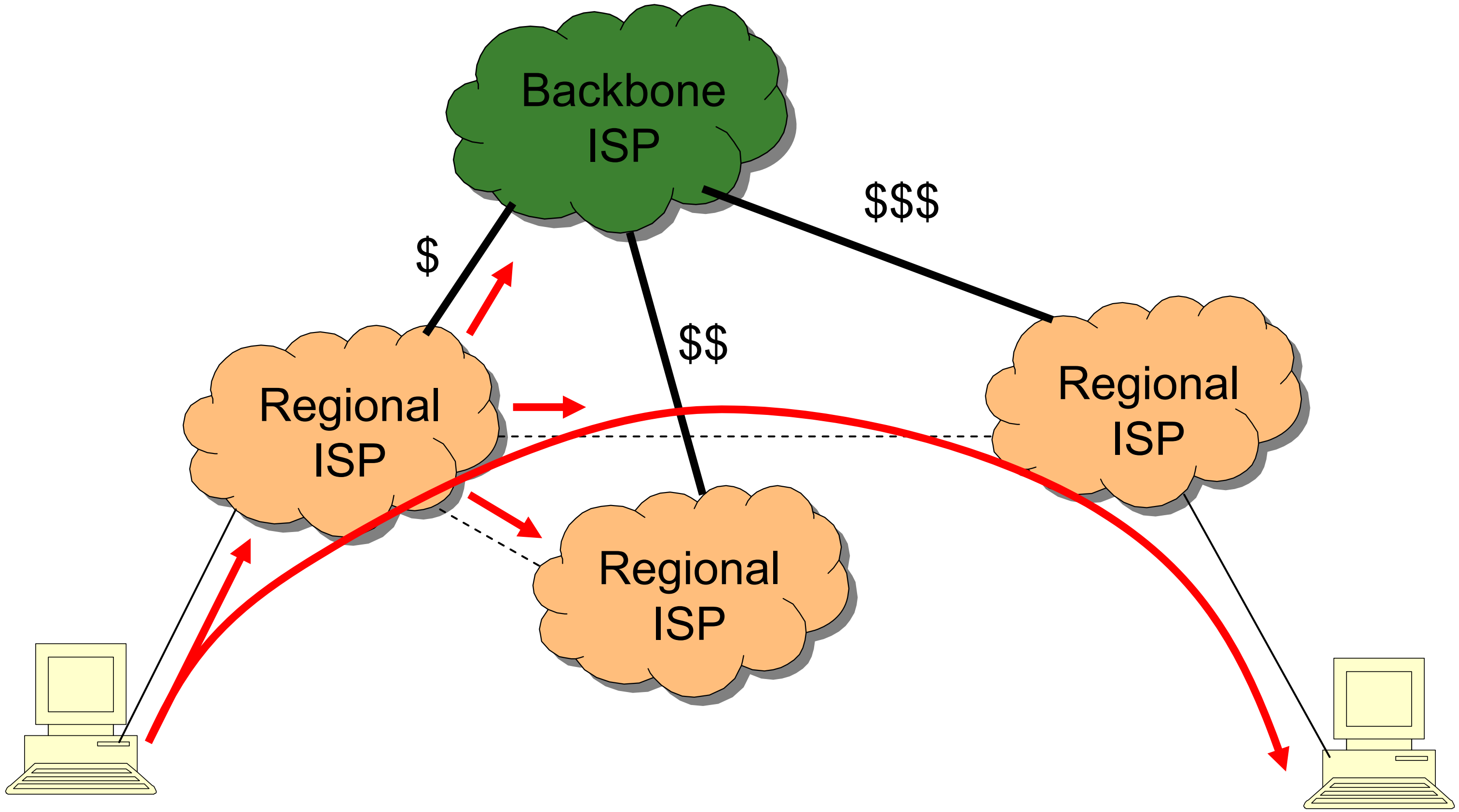
# Outline

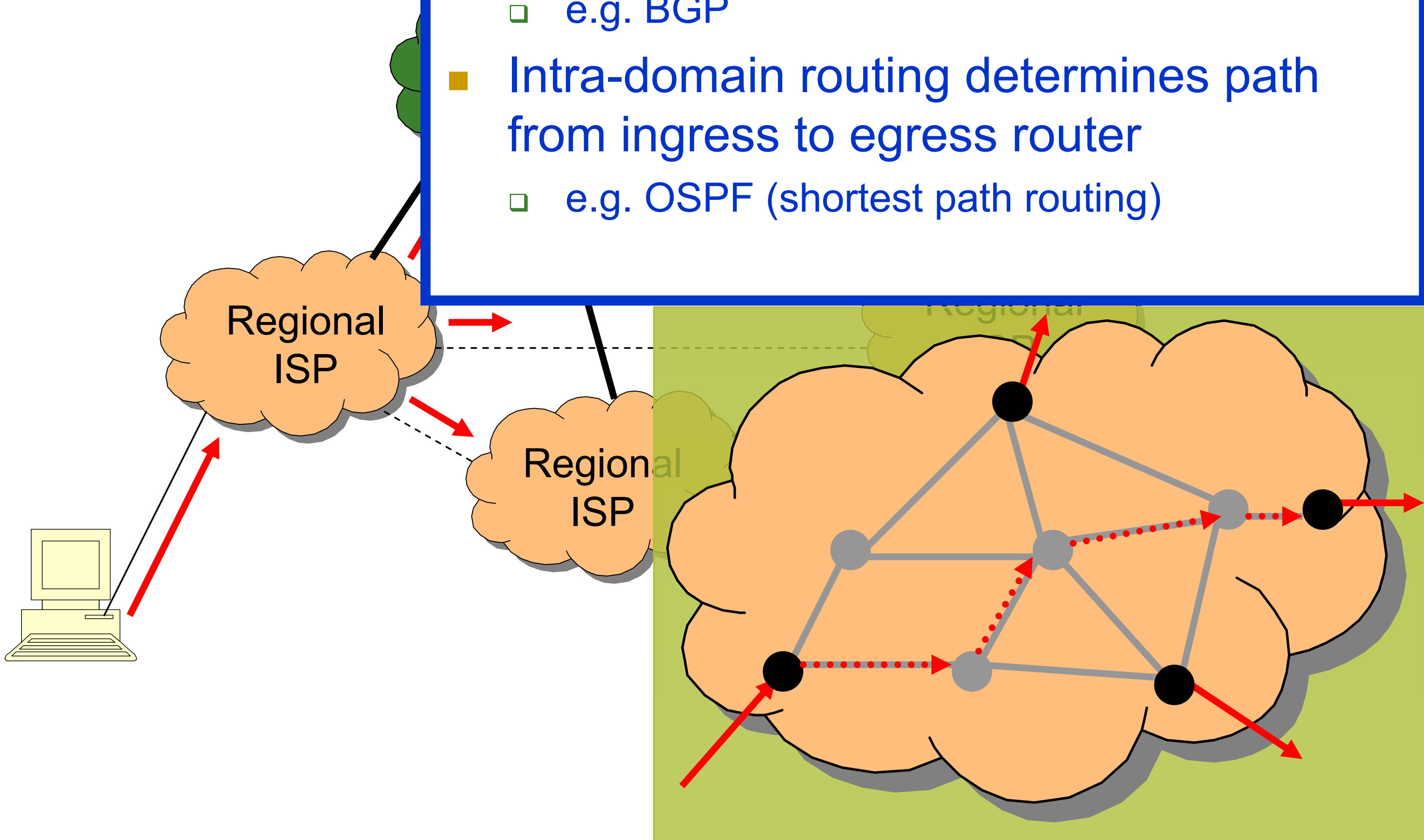
- Inter-domain routing on Internet
- Pricing to maximize revenue
  - Model
  - NP-hardness
- Approximation algorithm
  - Local prices
  - Max-cut based algorithm

# Inter-domain routing



# Inter-domain routing

- Inter-domain routing determines next ISP & thus egress router
  - e.g. BGP
- Intra-domain routing determines path from ingress to egress router
  - e.g. OSPF (shortest path routing)



# Inter-domain routing

- Inter-domain routing policy determines the macroscopic structure of Internet
  - AS-level routing & peering structure
  - Not just physical connectivity

- Inter-domain routing policy optimizes
  - Bandwidth revenue/cost
  - Performance
  - Other policy objectives (that may be hard to quantify)
- Generally not shortest-path algorithm
  - Dominate implementation: BGP

# Our goal

- Understand routing and peering structure of Internet
- … that results from interaction of economics and routing protocol

- This talk
  - Tier-1 ISP prices tier-2 ISPs to max revenue
  - Tier-2 ISPs can peer if price too high
  - How to price?
  - What is the resulting peering structure?

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# Model

-Tier-1 ISP charges each Tier-2 ISP by data carried

-Tier-2 ISP pairs can build links ("peer") to bypass Tier-1

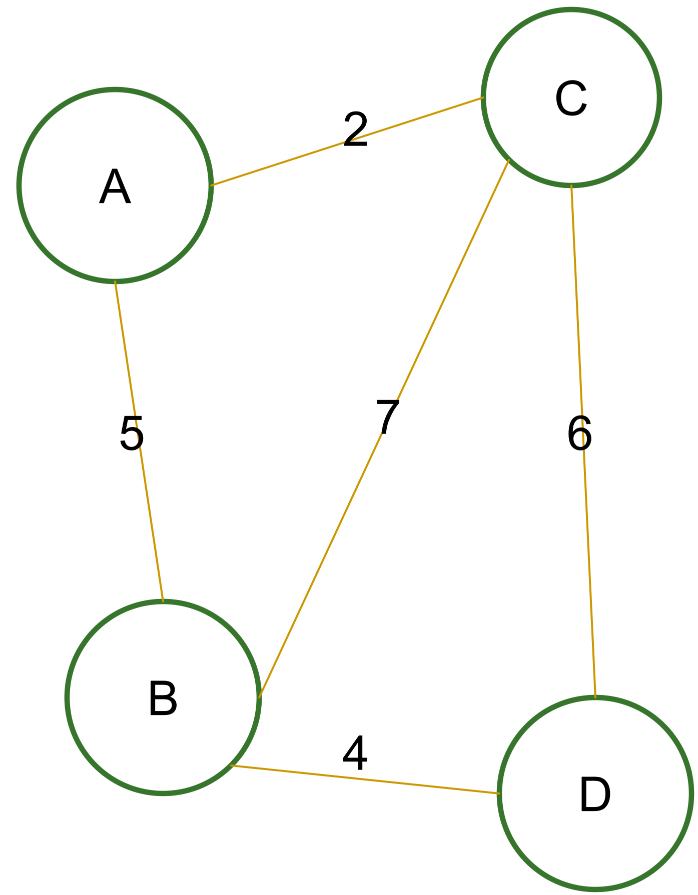
-Revenue to Tier-1 ISP:

If $\phi$(price to A, price to B) >"peering" cost

0

else

(price to A + price to B)*traffic




Goal: decide price for each ISP to maximize revenue

# Modeling assumption

- No competition – only one Tier-1 provider
- "Peering cost" is recurring cost, not one-off
- Traffic between Tier-2 ISPs = W(*sum* of prices)

- Peering decision based on Tier-1 charge

# Willingness to peer

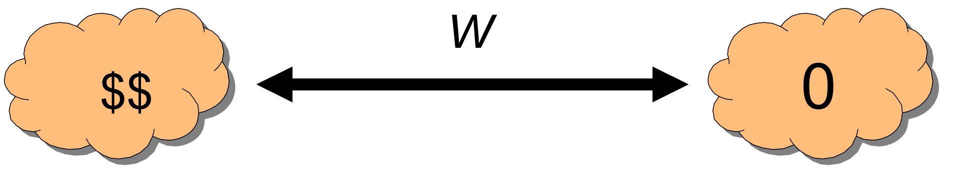
- Earlier model: ISPs peer if cheaper *overall*
  - $\phi(a,b)=a+b$
- ISPs peer if it is cheaper for *both*
  - $\phi(a,b)=\min(a,b)$

$$\phi(u,v) \le c$$

- Symmetric: $\phi(a,b) = \phi(b,a)$
- Increasing: $b \le c \;\Rightarrow\; \phi(a,b) \le \phi(a,c)$
- Left continuous, nonnegative

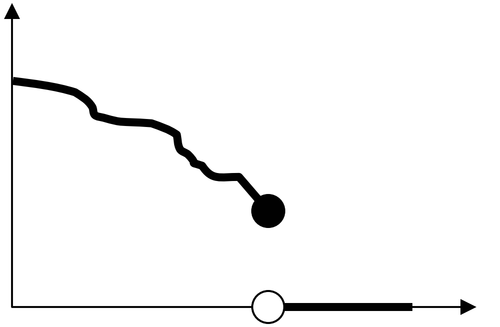
# Demand weight

- Previous work: Demand constant
- If provider too expensive, ISPs throttle demand

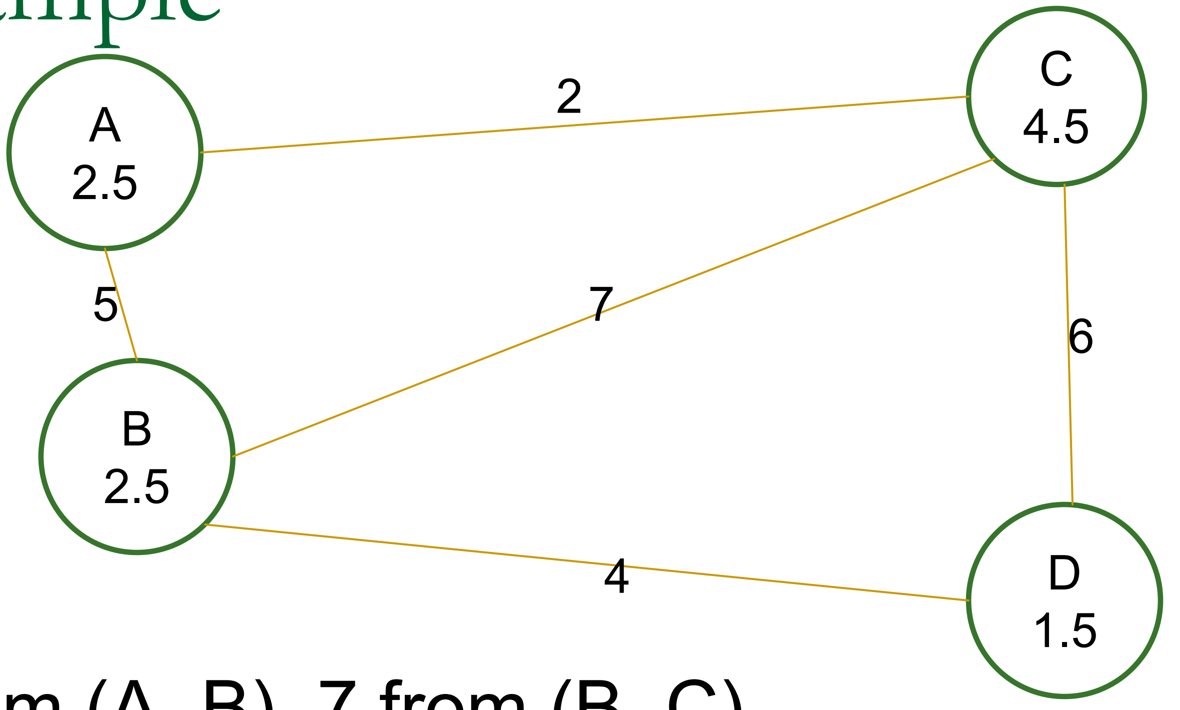



- $W$ and $\phi$ determine whether *finite* prices optimal

- Left continuous, non-negative
- Finitely many local max $xW(x)$



# Example




5 from (A, B), 7 from (B, C),
0 from (A, C)
4 from (B, D), 6 from (C, D) ➔ 22 is the optimum

The peering arrangement of the optimum
= (A, C)

# Problem formulation

Given *graph* G = (V,E)

*cost function* $c : E \to R^+$

*willingness to peer* $\phi : R^2 \to R^+$

*demand weight* $W : R^+ \cup \{\infty\} \to R^+ \cup \{0\}$

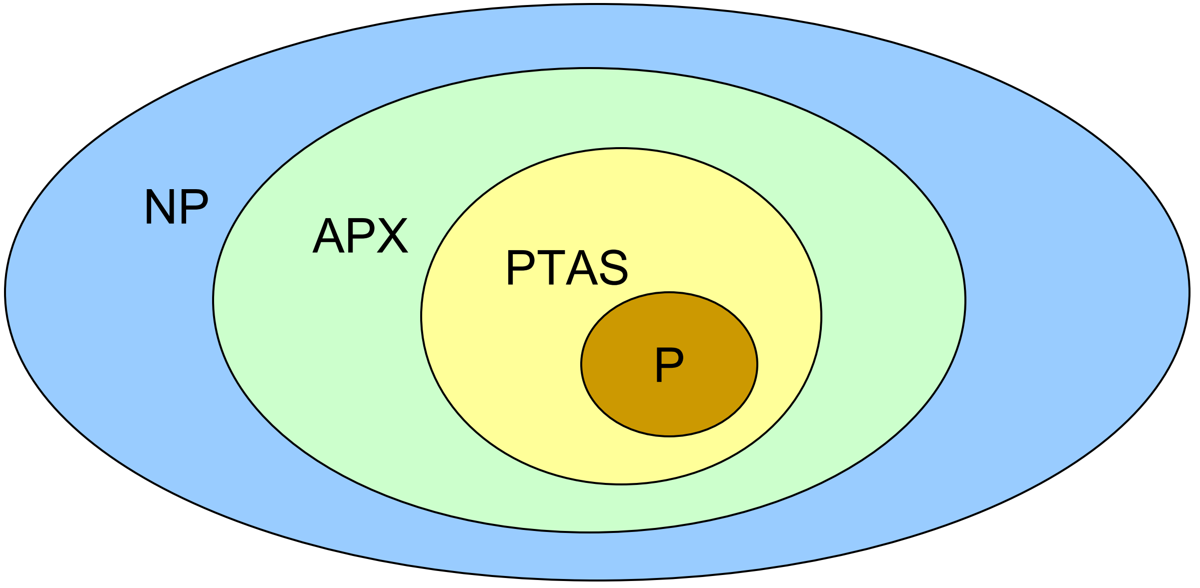
For pricing function $\mu : V \to R^+ \cup \{0\}$

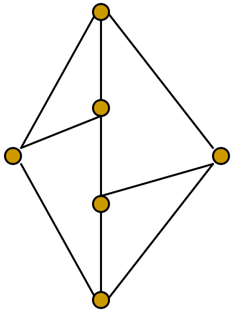
$$R(\mu) \coloneqq \sum_{\substack{(u,v)\in E \\ \phi(\mu(u),\mu(v))\le c(u,v)}} (\mu(u)+\mu(v))\mathrm{W}(\mu(u)+\mu(v))$$

$$\boxed{\text{P1:} \quad \max_{\mu} R(\mu) \eqqcolon \operatorname{opt}(G,c)}$$

# NP- and APX-completeness

- APX – approximable to a constant factor
- PTAS – approximable to *any* constant factor
- **<u>Theorem</u>** P1 is APX-complete
- Proof: Max-cut-3 reduces to P1 polynomially



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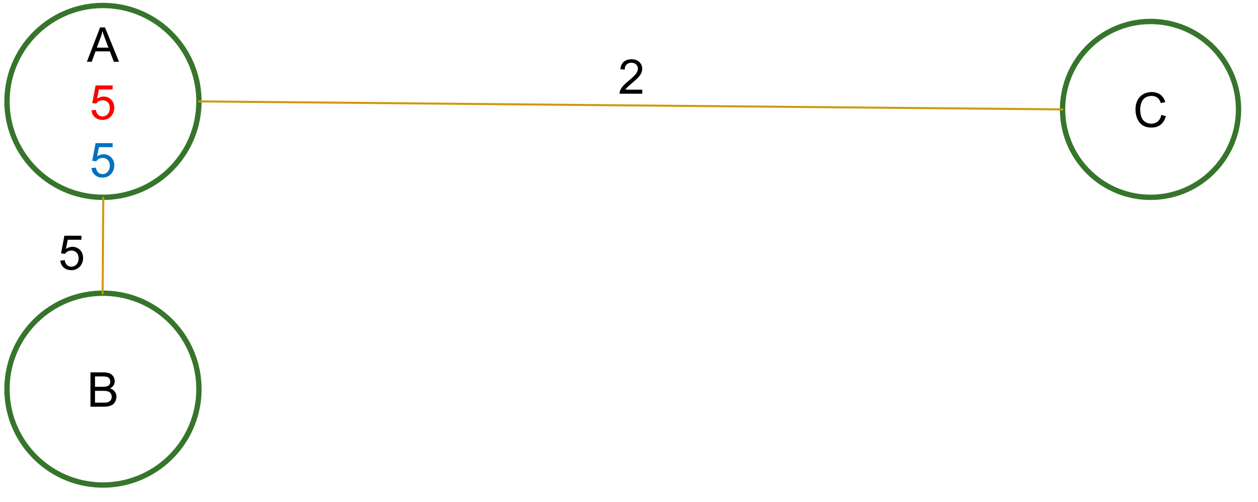
# Observation: local revenue

- For each vertex v, (assume that we charge 0 to others)

  f(v) : max possible revenue from v

  g(v) : charge to v to get f(v)



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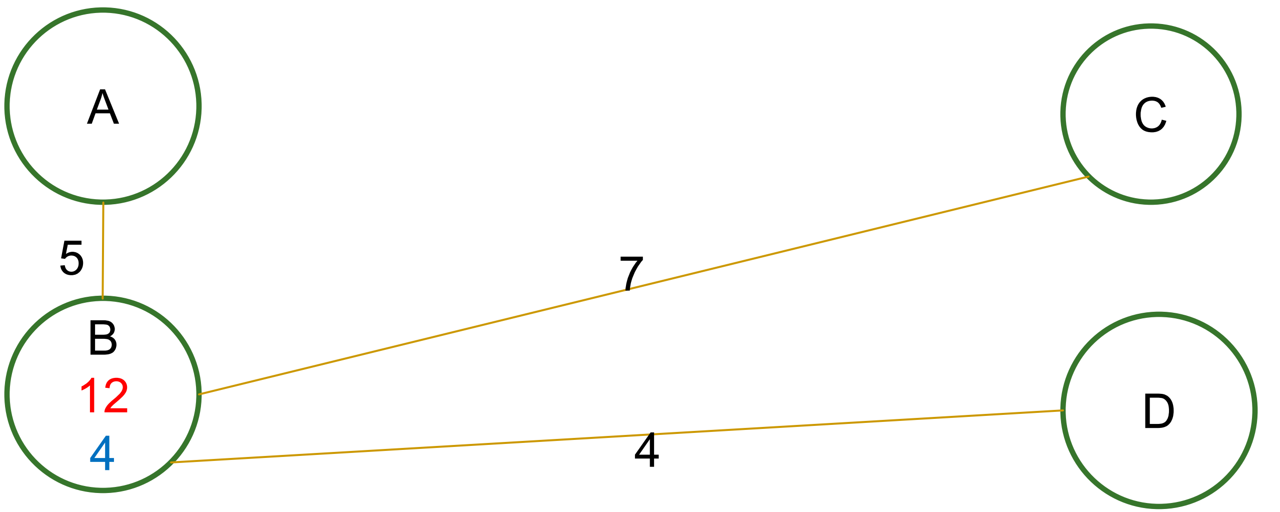

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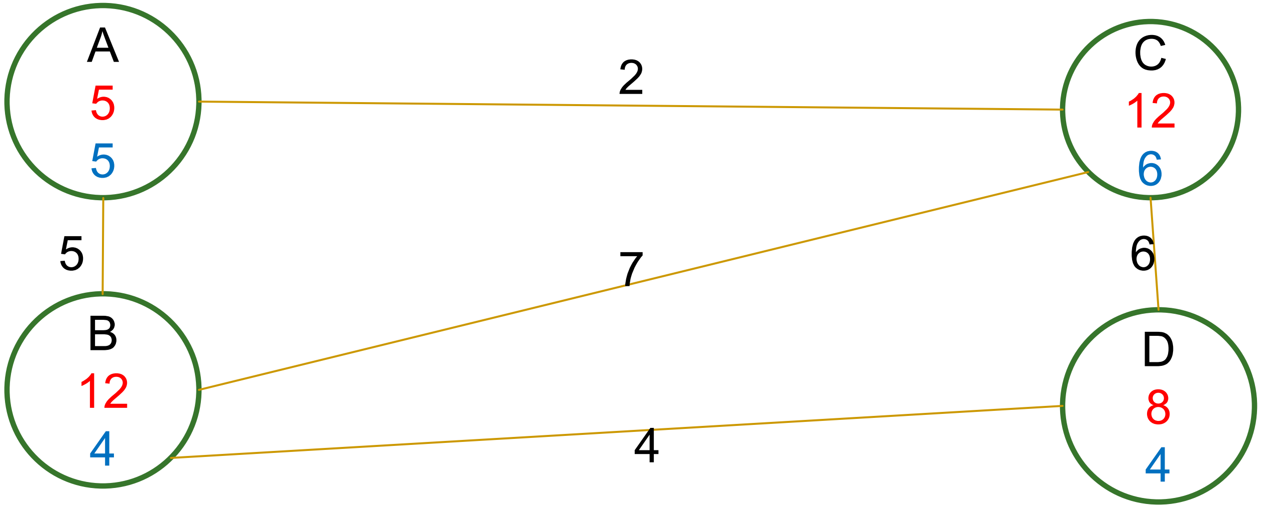



$$F(V) := \sum_{v \in V} f(v) = 5 + 12 + 12 + 8 = 37 > opt(G, c)$$

# Max-cut: 1/4 Approximation

1. Compute link weights

$$d(e) := \sum_{v \in e} g(v)\,\mathbf{1}\big(g(v) \le c(e)\big)$$

2. Appr. max-cut algorithm: $\left(X, \overline{X}\right)$
3. (WLOG)

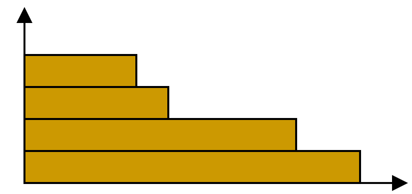
$$\mu(v) = \begin{cases} g(v) & \text{if } \; v \in X \\ 0 & \text{if } \; v \in \overline{X} \end{cases}$$

**Theorem**

$$R(\mu) \ge \frac{1}{4}\,\text{opt}(G, c)$$

# Computing $f$ and $g$

- Non-convex 1-D optimization
- Raising price reduces revenue-bearing edges
  - Optimize for each subset of edges




- Optimal price $g$ may be infinite
  - If so, optimal revenue $f$ may or may not be

# Extensions and open questions

- Better than 1/4 possible? How?
  - At forefront of theoretical computer science
- Costs of carrying traffic
- Peering decisions not based purely on price
  - Re-interpret $W$ as probability of not peering?

# Conclusion

- Peering/pricing affects Internet routes
- Maximum provider revenue hard to calculate
- 1/4 approximation based on Max-Cut