Bandwidth Exchange as an Incentive for Relaying

Dan Zhang, Omer Ileri, Narayan Mandayam
WINLAB
Rutgers, The State University of New Jersey
www.winlab.rutgers.edu
{bacholic, narayan}@winlab.rutgers.edu
omer.ileri@radio.kth.se

An Overview of Relay Cooperation

- Enhance potential connectivity and throughput
- Essential to ad hoc networks
- Incurs costs of power, delay, ...
- Not guaranteed without sufficient incentive

Existing Incentive Mechanisms

- Reputation-based mechanism
- Credit-based mechanism
- Network-assisted pricing

Some difficulties of these approaches:

- Stable currency
- Central system of credit
- Shared understanding of worth
- A good deal of record keeping

Bandwidth Exchange

- Delegate bandwidth for cooperation
- Simple bartering of basic wireless resource
- Avoid unwarranted complexity
- Different implementations
  - Delegate time slots
  - Use spreading codes of different length
  - Use different maximum backoff window
- We study exchanging orthogonal frequency bands

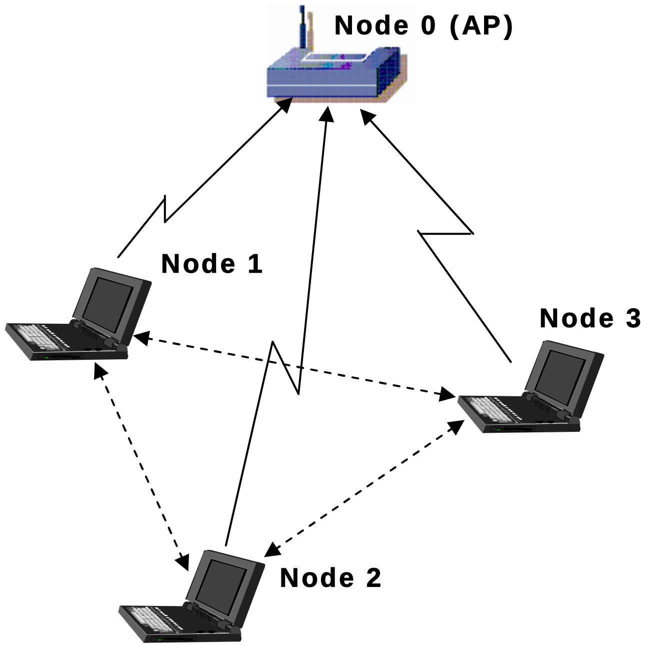
System Model

- N nodes communicating to AP under Rayleigh fading:

$$p(\rho_{ij}) = \frac{1}{\bar{\rho}_{ij}} \exp\left(-\frac{\rho_{ij}}{\bar{\rho}_{ij}}\right)$$

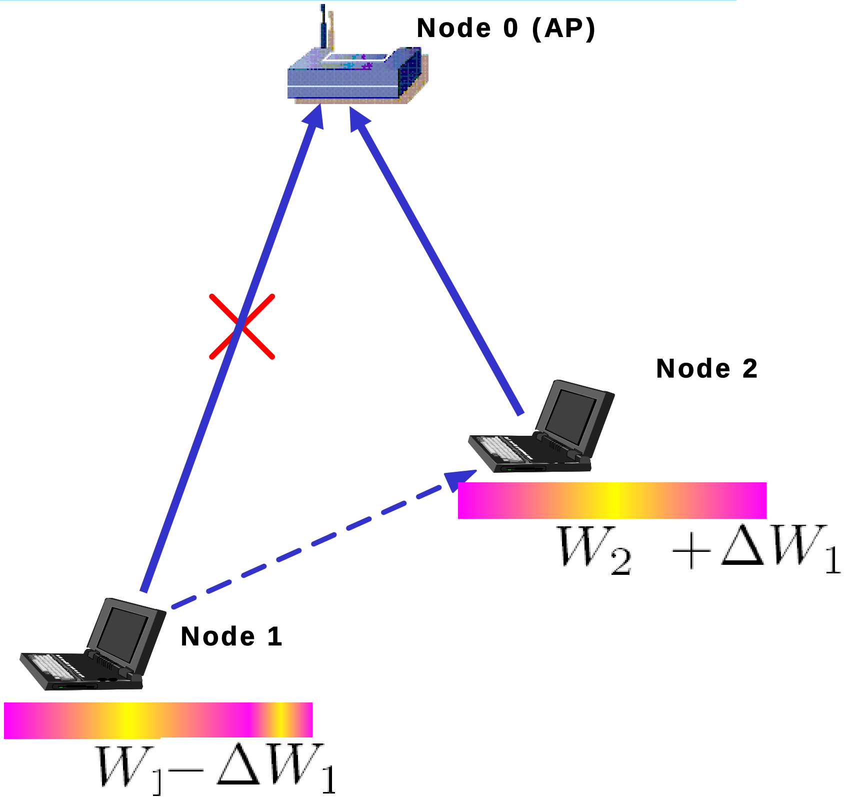
- Each node has its exclusive bandwidth W_i and minimum required rate $R_i^{\min}$
- Instantaneous rate on link ij is modeled as

$$R_{ij} = W_i \log_2\left(1 + \frac{\rho_{ij} P_i^T}{W_i}\right) \text{ (bits/second)}$$



Bandwidth Exchange for Two nodes

- When $R_{10} < R_1^{min}$ and $R_{12} \geq R_1^{min}$ node 1 requests relay from node 2



- Delegating part of its bandwidth ΔW_1 to node 2 such that

$$R_1^{min} = (W_1 - \Delta W_1) \log\left(1 + \frac{\rho_{12} P_1^T}{W_1 - \Delta W_1}\right)$$

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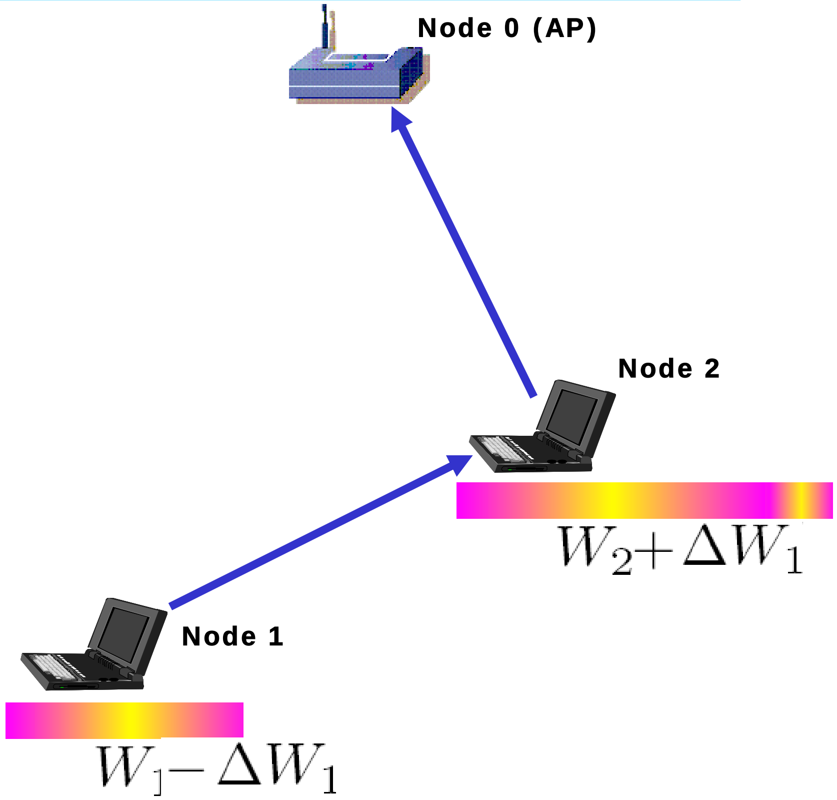
Bandwidth Exchange for Two nodes

- If node 2 cooperates, it allocates R_1^{min} from the total capacity $$(W_2+\Delta W_1)\log\left(1+\frac{\rho_{20}P_2^T}{W_2+\Delta W_1}\right)$$ for node 1

- Node 2 will not grant cooperation if the total capacity is less than $R_1^{min}+R_2^{min}$



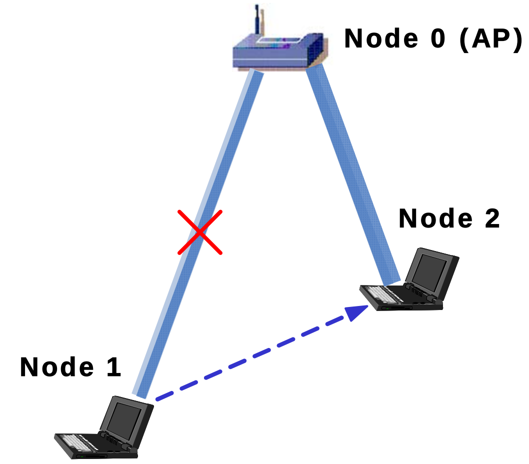
Two Node Relay Game

Given a snapshot of the channel

- Reserved rate U_i^{NC} :rate achieved without cooperation
- Actual rate U_i : rate achieved with or without cooperation
- Define payoff ("gain"): $-u_i = U_i - U_i^{NC}$
  - Reflects the effect of cooperation on rates
  - Without cooperation payoff is zero
  - With cooperation, payoff for source i is R_i^{min}

Static Channel

- Node 1 and node 2 take a constant strategy
- With BE potential relay cooperates if payoff is positive, does not cooperate if negative
- Without BE there is no cooperation since relay always incurs loss of rate
  - Payoff is negative
  - Loss = min required source rate



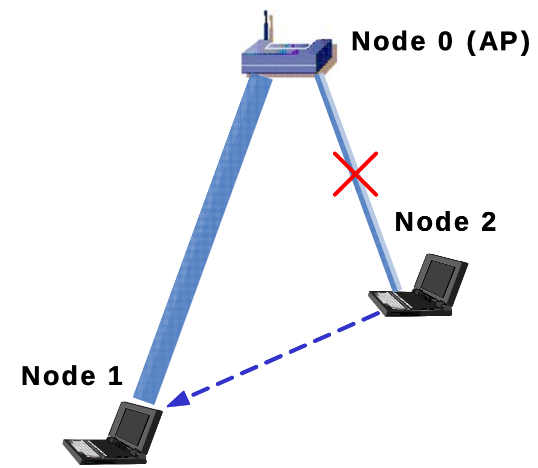


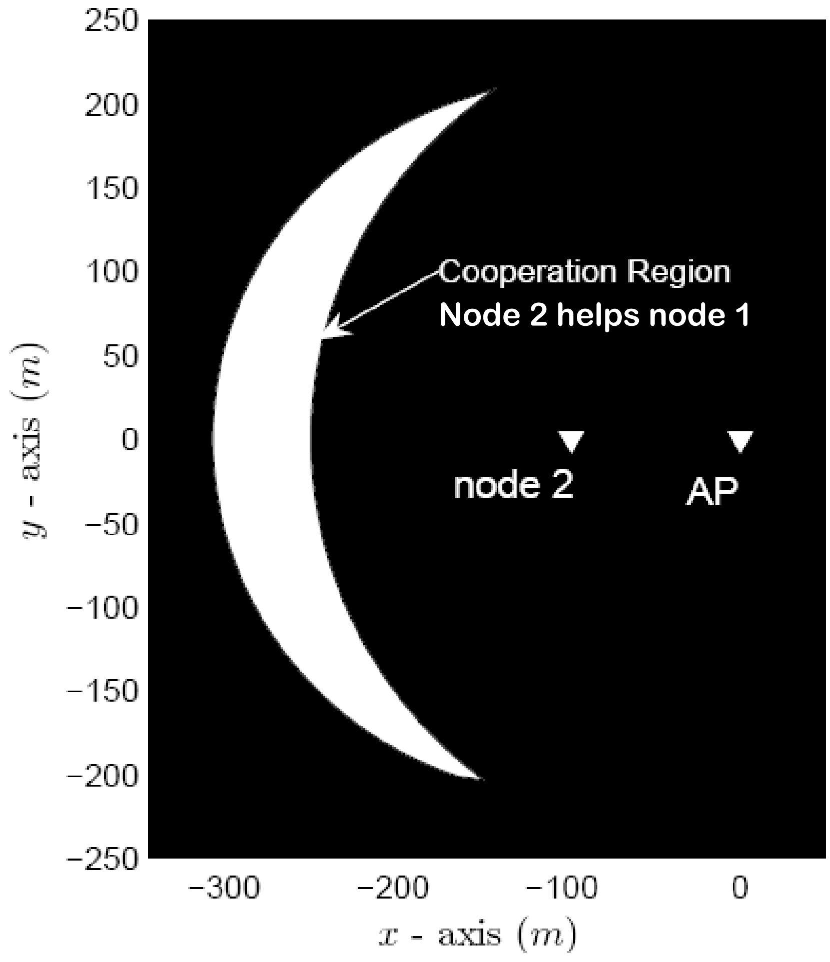
Illustration of Static Channels with BE

- Node 2 at $(-100\text{m}, 0)$
  AP at origin

$$P_1^T = P_2^T = 100\text{mW}$$
$$W_1 = W_2 = 20\text{MHz}$$
$$R_1^{min} = R_2^{min} = 30\text{Mbps}$$

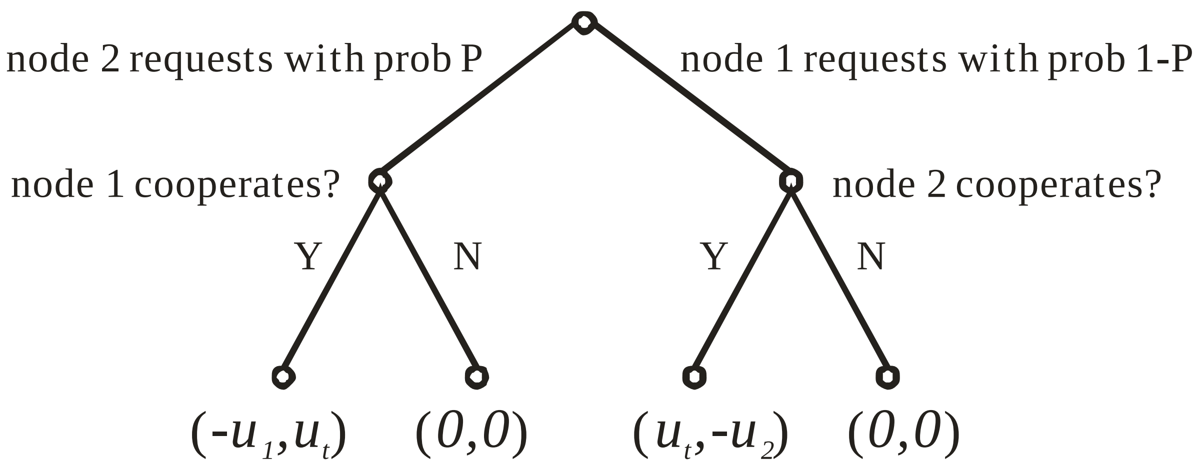
- cooperation region: locations (x_1, y_1) of node 1 such that

$$R_{10} < R_1^{min}$$
$$R_{12} \geq R_1^{min}$$
$$U_2^C \geq U_2^{NC}$$



Two Node Relay Game in a Fading Channel

- Link gains change from one snapshot to another
- Consider a snapshot with a relay request generated
- WLOG assume $R_1^{min} = R_2^{min} = u_t$
- Request from node 2 to node 1 with probability P from node 1 to node 2 with probability $1 - P$



Conditional Request Probability

- P is the conditional probability of node 2 sending a relay request to node 1 given there is a request generated in the snapshot

$$P = P_2/(P_1 + P_2)$$

Unconditional probability that node 1 sends node 2 a request

$$P_1 = \iiint_{\substack{R_{10} < R_1^{min} \\ R_{12} \geq R_1^{min} \\ U_2^C \geq R_2^{min}}} p(\rho_{10}, \rho_{20}, \rho_{12}) \mathrm{d}\rho_{10} \mathrm{d}\rho_{20} \mathrm{d}\rho_{12}$$

Unconditional probability that node 2 sends node 1 a request

$$P_2 = \iiint_{\substack{R_{20} < R_2^{min} \\ R_{21} \geq R_2^{min} \\ U_1^C \geq R_1^{min}}} p(\rho_{10}, \rho_{20}, \rho_{12}) \mathrm{d}\rho_{10} \mathrm{d}\rho_{20} \mathrm{d}\rho_{12}$$

Two Node Relay Game in a Fading Channel

- Normal form with average payoffs

1 \ 2	Cooperation (C)	Noncooperation (N)
C	$(-Pu_1+(1-P)u_t, Pu_t-(1-P)u_2)$	$(-Pu_1, Pu_t)$
N	$((1-P)u_t, -(1-P)u_2)$	$(0,0)$

- *Proposition*: Without BE (payoff for cooperation is $-u_t$) nodes do not cooperate

1 \ 2	Cooperation (C)	Noncooperation (N)
C	$((1-2P)u_t, -(1-2P)u_t)$	$(-Pu_t, Pu_t)$
N	$((1-P)u_t, -(1-P)u_t)$	$(0,0)$

efficient NE

How Bandwidth Exchange Helps?

- With bandwidth exchange, the payoff loss is compensated to a certain extent
  - Payoff loss is less than u_t
- In spite of the loss there can still be sufficient incentive for cooperation
  - $\langle C, C\rangle$, $\langle N, C\rangle$, $\langle C, N\rangle$ can Pareto dominate $\langle N, N\rangle$

1 \ 2	Cooperation (C)	Noncooperation (N)
C	$(-Pu_1+(1-P)u_t, Pu_t-(1-P)u_2)$	$(-Pu_1, Pu_t)$
N	$((1-P)u_t, -(1-P)u_2)$	$(0, 0)$

Over-Compensation

- *Proposition*: If $u_1 < 0$ (node 1 benefits from the extra bandwidth received) node 1 always cooperates

1 \ 2	Cooperation (C)	Noncooperation (N)
C	$(-Pu_1+(1-P)u_t, Pu_t-(1-P)u_2)$	$(-Pu_1, Pu_t)$
N	$((1-P)u_t, -(1-P)u_2)$	$(0, 0)$

- Similarly, if $u_2 < 0$ node 2 always cooperates

Under-Compensation

- What if $0 \leq u_1, u_2 \leq u_t$?
- NE of the stage game is $\langle N, N \rangle$
- Can be quite inefficient

1 \ 2	Cooperation (C)	Noncooperation (N)
C	$(-Pu_1+(1-P)u_t, Pu_t-(1-P)u_2)$	$(-Pu_1, Pu_t)$
N	$((1-P)u_t, -(1-P)u_2)$	$(0, 0)$

NE

- $\langle C, C \rangle$ can dominate $\langle N, N \rangle$
  - But not a NE, thus unattainable

Repeated Game and Nash Bargaining

- Repeated game in every snapshot
  - Inefficient NE can be avoided
  - Many subgame perfect equilibria (Folk Theorem)

- Need a criterion to settle down to the "best" subgame perfect equilibrium
  - Implemented by a Nash Bargaining Solution with mixed strategies

Nash Bargaining Solution (NBS)

- Exists and unique
- Pareto optimal
- Proportional fair
- With $v_1, v_2, \ldots, v_n$ being the correlated payoff profile, it boils down to solving

$$\underset{v_i, i=1,2,\ldots,N}{\text{maximize}} \quad \prod_{i=1}^{N} v_i$$
$$\text{subject to} \quad (v_1, v_2, \ldots, v_N) \text{ feasible}$$

NBS for the Two Node Relay Game

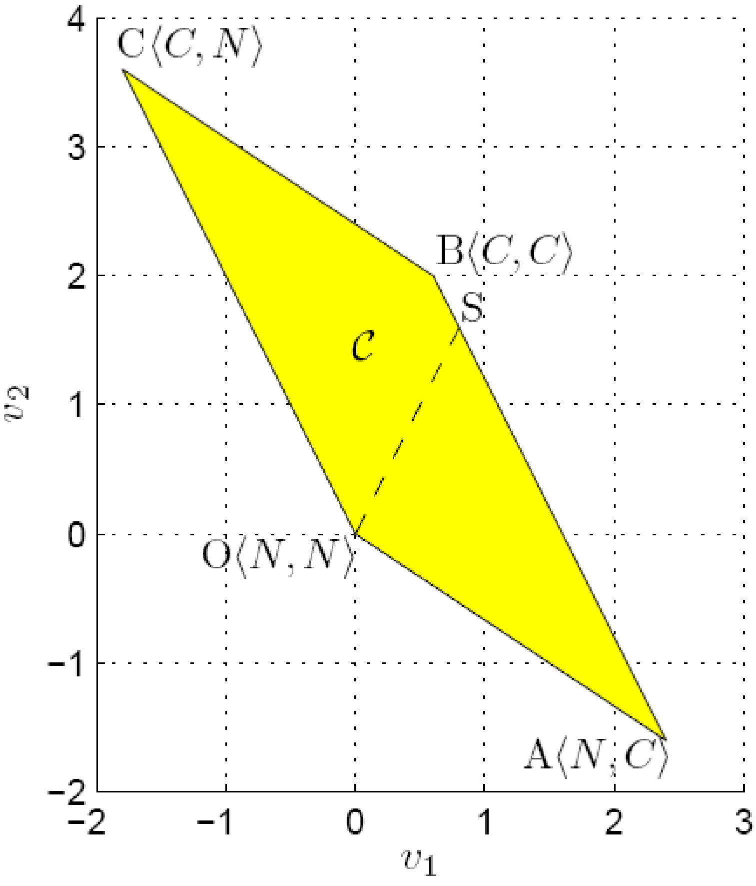
Strategy	Payoff
$\langle N, N\rangle$	$(\tilde{v}_1^1, \tilde{v}_2^1) = (0, 0)$
$\langle N, C\rangle$	$(\tilde{v}_1^2, \tilde{v}_2^2) = ((1-P)u_t, -(1-P)u_2)$
$\langle C, C\rangle$	$(\tilde{v}_1^3, \tilde{v}_2^3) = (-Pu_1 + (1-P)u_t, Pu_t - (1-P)u_2)$
$\langle C, N\rangle$	$(\tilde{v}_1^4, \tilde{v}_2^4) = (-Pu_1, Pu_t)$

NBS: a set of mixing probs $\lambda_i \geq 0, i = 1, 2, 3, 4,$ solving

$$\begin{aligned} \underset{\lambda_i, i=1,2,3,4}{\text{maximize}} \quad & v_1 v_2 \\ \text{subject to} \quad & v_1 = \lambda_1 \tilde{v}_1^1 + \lambda_2 \tilde{v}_1^2 + \lambda_3 \tilde{v}_1^3 + \lambda_4 \tilde{v}_1^4 \\ & v_2 = \lambda_1 \tilde{v}_2^1 + \lambda_2 \tilde{v}_2^2 + \lambda_3 \tilde{v}_2^3 + \lambda_4 \tilde{v}_2^4 \end{aligned}$$

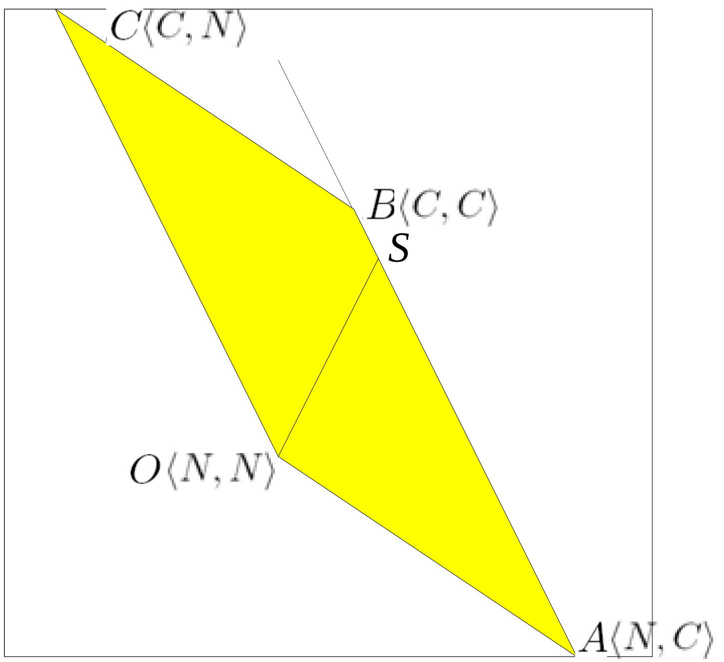
Two Node Nash Bargaining

- A simple geometric interpretation exists
- Feasible set $\mathcal{C}$ for the two node relay game is a parallelogram
- NBS S lies on Pareto boundary
  - Negative gradient of OS is a subgradient of the boundary at S



Possible Locations of NBS

- *Proposition*: if $u_1, u_2 > 0$ NBS can be determined by checking gradients

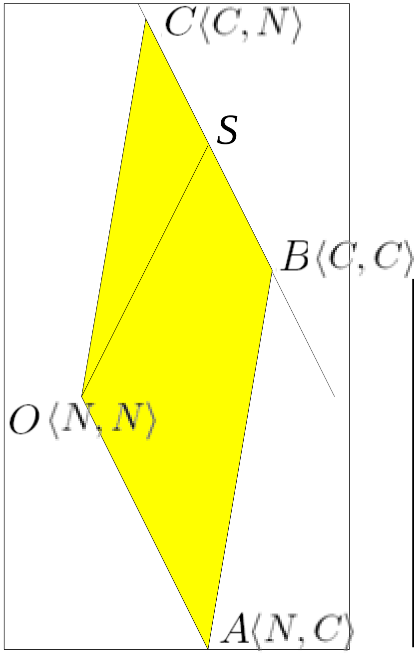


1. $\tan B < -\frac{u_t}{u_1}$ or $\tan B > \frac{u_t}{u_1}$

$\iff$ S on AB

Node 2 always cooperates, node 1 cooperates with a probability

Possible Locations of NBS

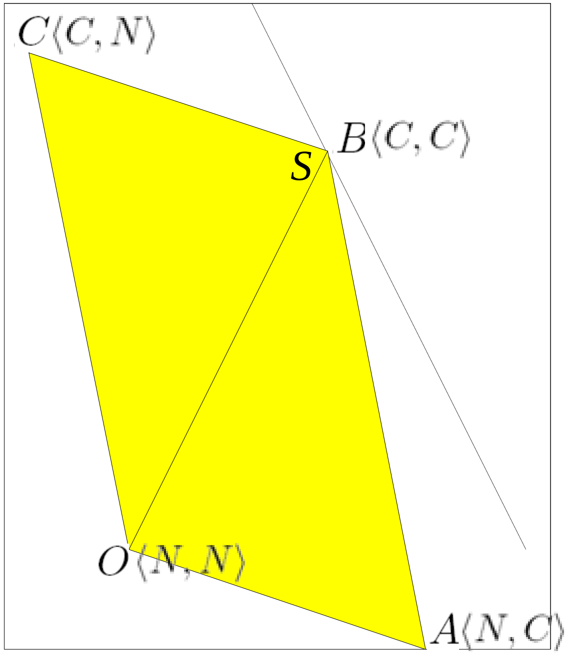


2. $-\frac{u_2}{u_t} < \tan B < \frac{u_2}{u_t}$

$\iff S$ on BC

Node 1 always cooperates, node 2 cooperates with a probability

Possible Locations of NBS



3. $\frac{u_2}{u_t} \leq \tan B \leq \frac{u_t}{u_1}$

$\Longleftrightarrow \; S \text{ at } B$

Both node 1 and node 2 always cooperate

Special Case I – Complete Symmetry

- Identical bargaining power – equal outage probabilities and rate loss

- Typical in an ad hoc network with
  - homogeneous nodes
  - i.i.d. channels

- *Proposition*: if $P = 0.5, u_t \geq u_1 = u_2 \geq 0$ the best strategy for each node is to always cooperate

Special Case II – Symmetric Outage

- Different bargaining power due to different rate loss; but identical outage probabilities
- *Proposition*: If $P = 0.5, u_t \geq u_1 > u_2 \geq 0$ node 2 always cooperates; node 1 cooperates with probability

$$P_C = \begin{cases} \frac{1}{2}\left(\frac{u_t}{u_1} + \frac{u_2}{u_t}\right), & u_1 \geq \frac{u_t^2}{2u_t - u_2} \\ 1, & u_1 < \frac{u_t^2}{2u_t - u_2} \end{cases}$$

- Node suffering less rate loss always cooperates
- Other node starts to mix in noncooperation when loss becomes too high

Special Case III – Symmetric Rate Loss

- Different bargaining power due to different outage probabilities; but identical rate loss
- *Proposition*: If $P > 0.5, u_t \geq u_1 = u_2 \geq 0$ node 2 cooperates; node 1 cooperates with probability

$$P_C = \begin{cases} \frac{1-P}{2P}\left(\frac{u_t}{u} + \frac{u}{u_t}\right), & P \geq \frac{u^2 + u_t^2}{(u+u_t)^2} \\ 1, & P < \frac{u^2 + u_t^2}{(u+u_t)^2} \end{cases}$$

- Node sending more requests always cooperates
- The other node starts to mix in noncooperation when the incoming requests becomes too fast

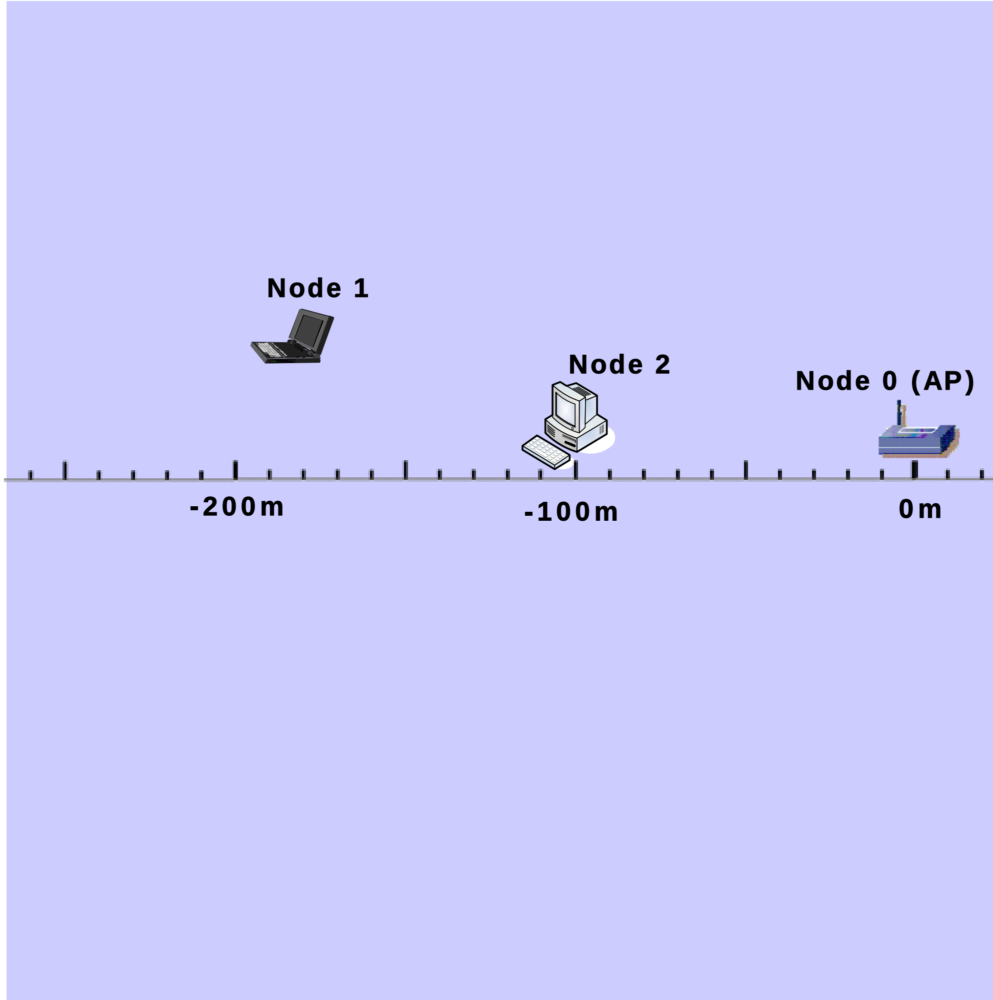
Numerical Results

- Node 2 fixed at $(-100\text{m}, 0)$
- AP fixed at origin
- Node 1 in a $400 \times 600\text{m}^2$ rectangular area

$$P_1^T = P_2^T = 100\text{mW}$$

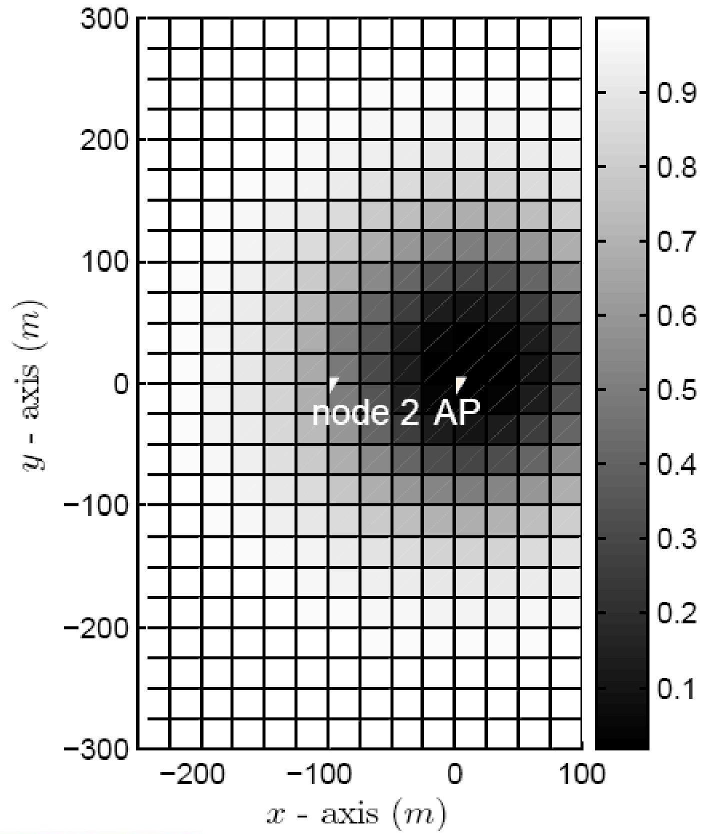
$$W_1 = W_2 = 20\text{MHz}$$

$$R_1^{min} = R_2^{min} = 30\text{Mbps}$$

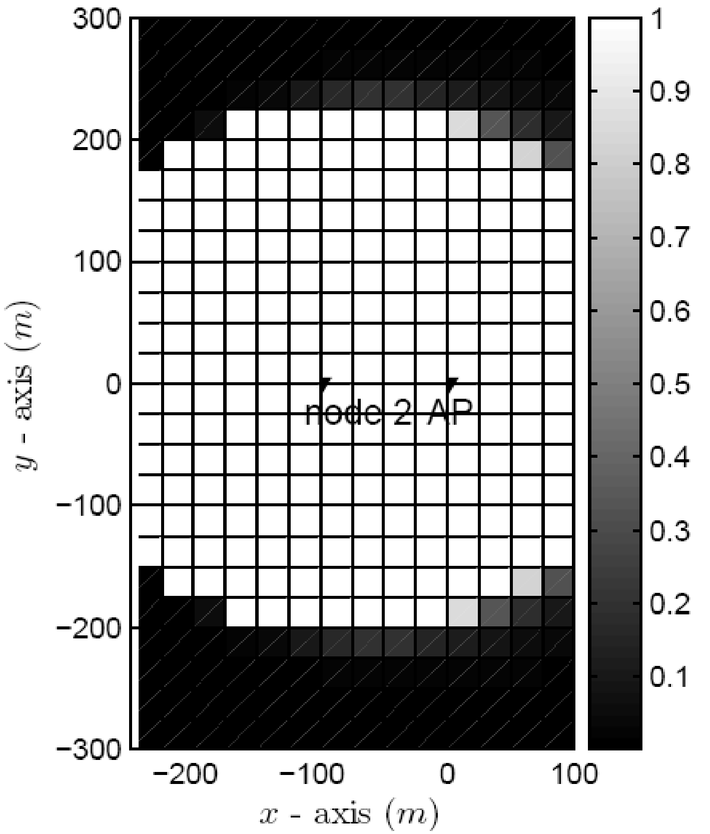


Request and Cooperation Probabilities

probability node 1 requests relay cooperation from node 2

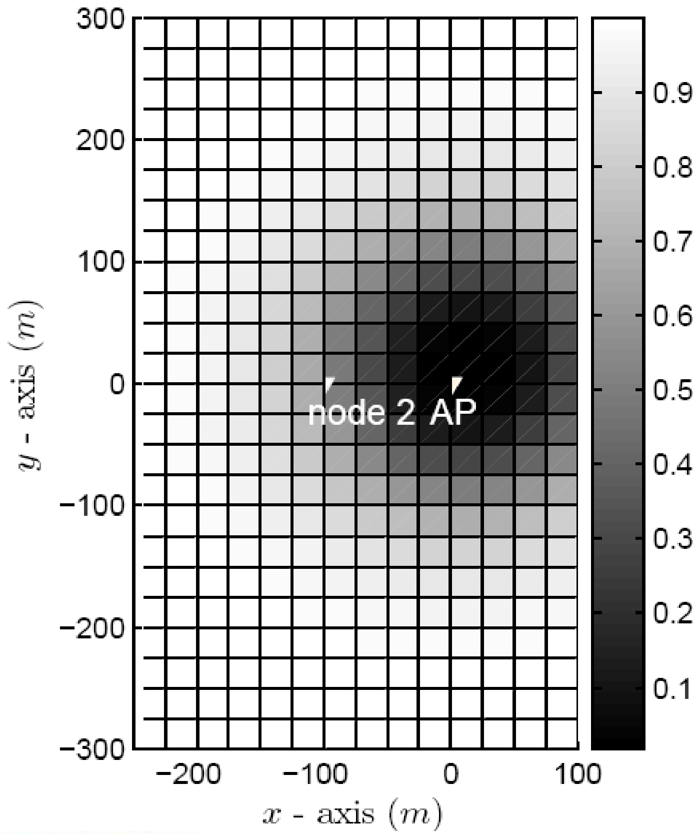


probability node 2 grants a relay request from node 1

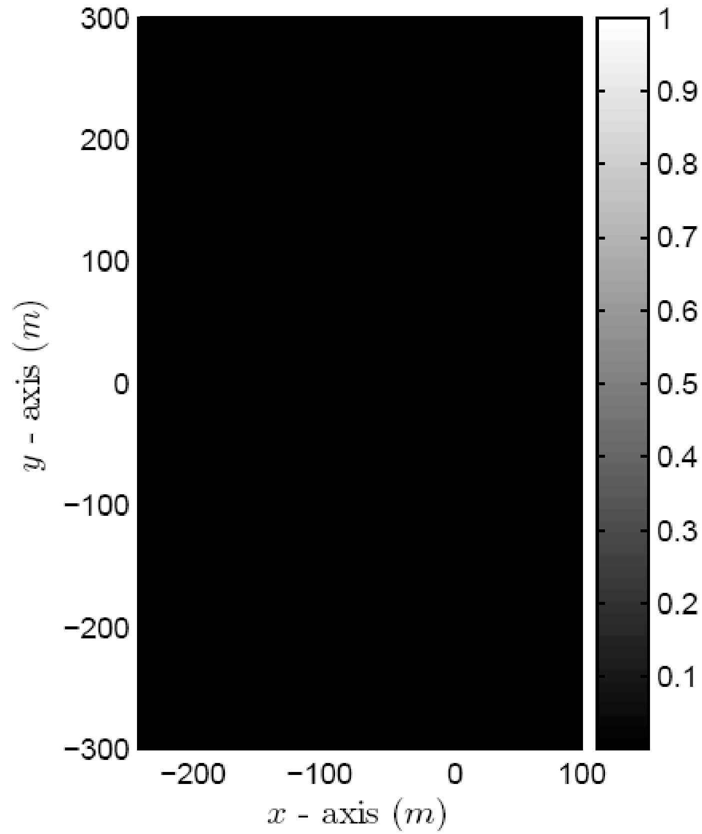


Without Bandwidth Exchange…

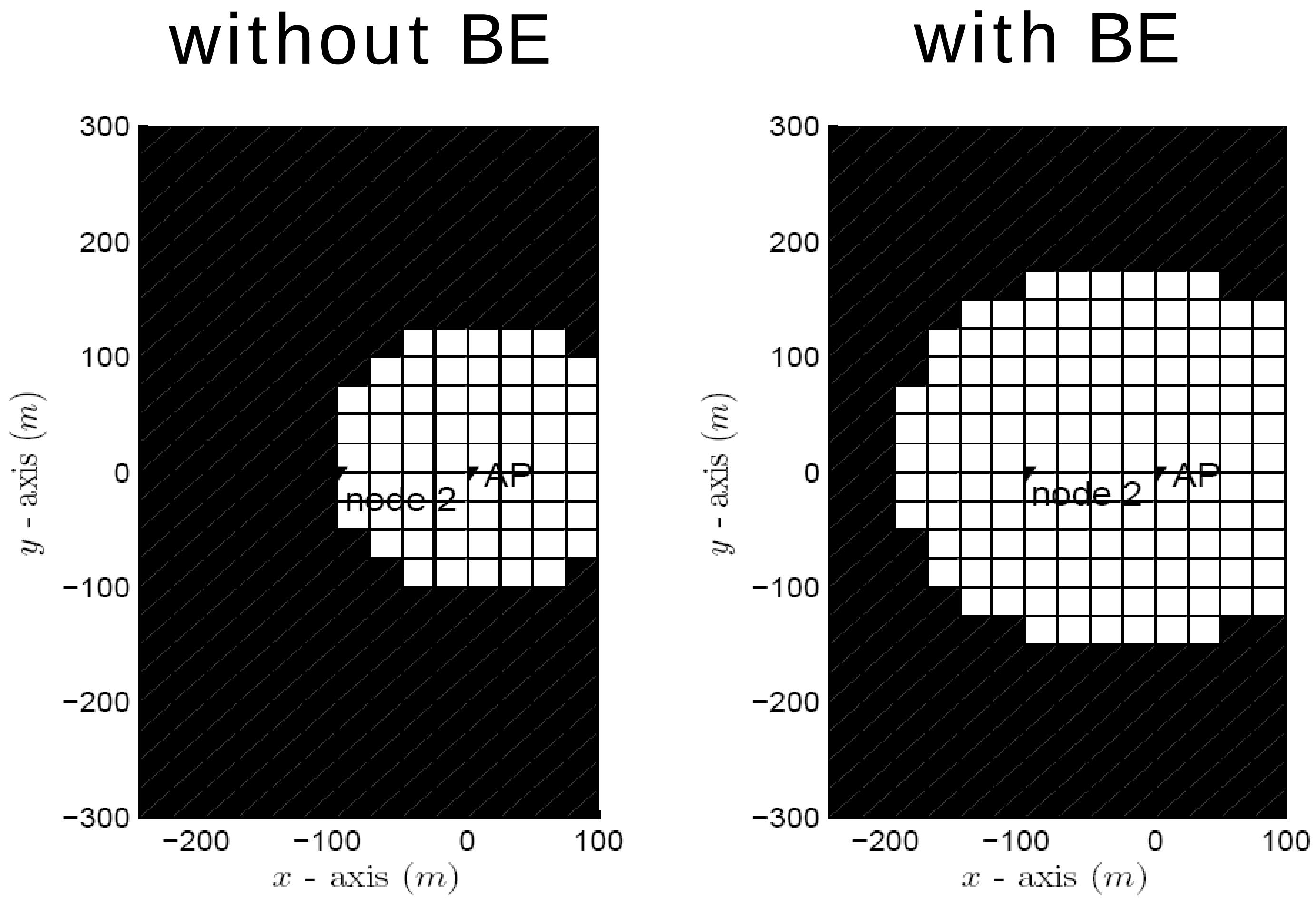
probability node 1 requests relay cooperation from node 2



probability node 2 grants a relay request from node 1



Coverage Area - Outage Prob < 0.1



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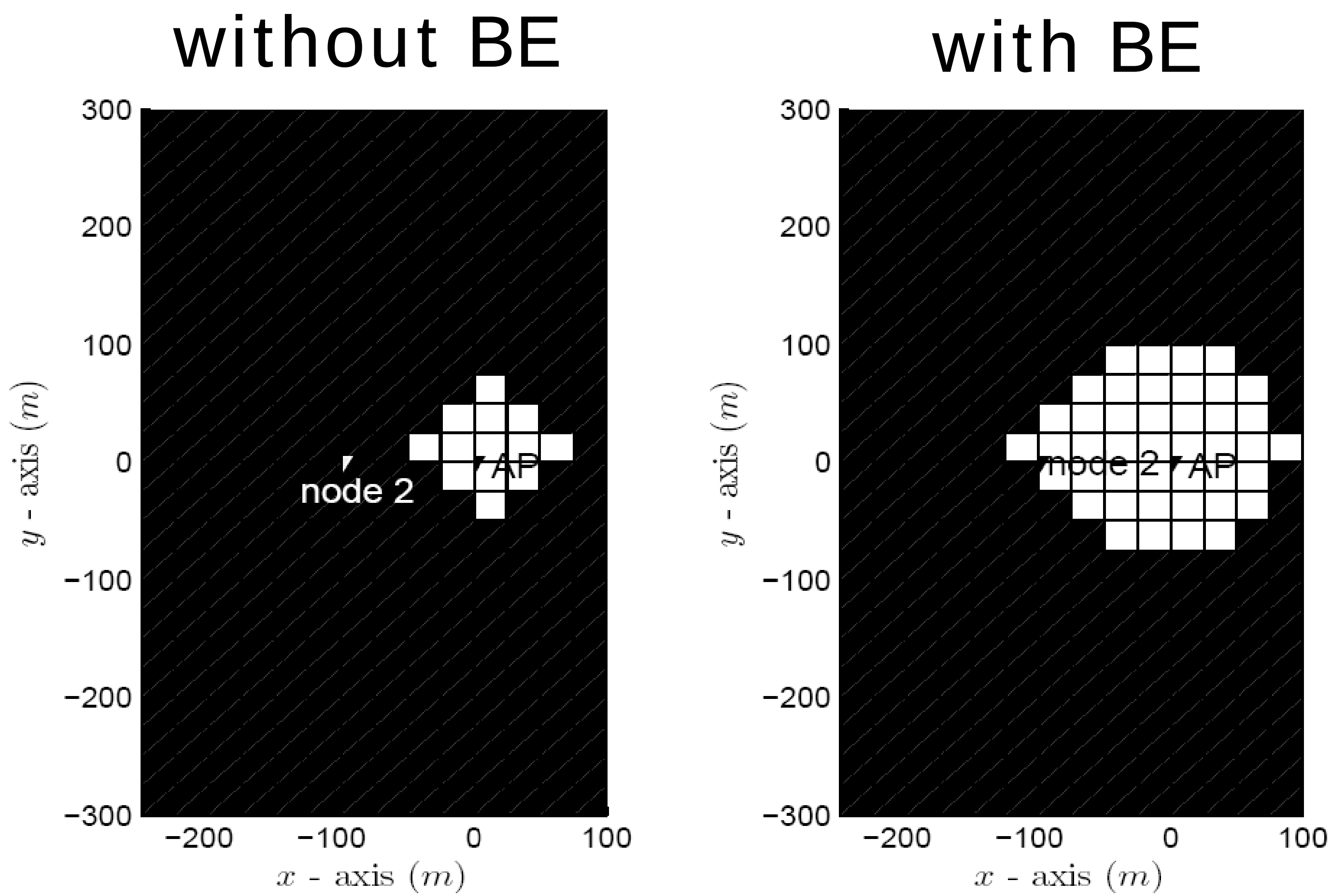
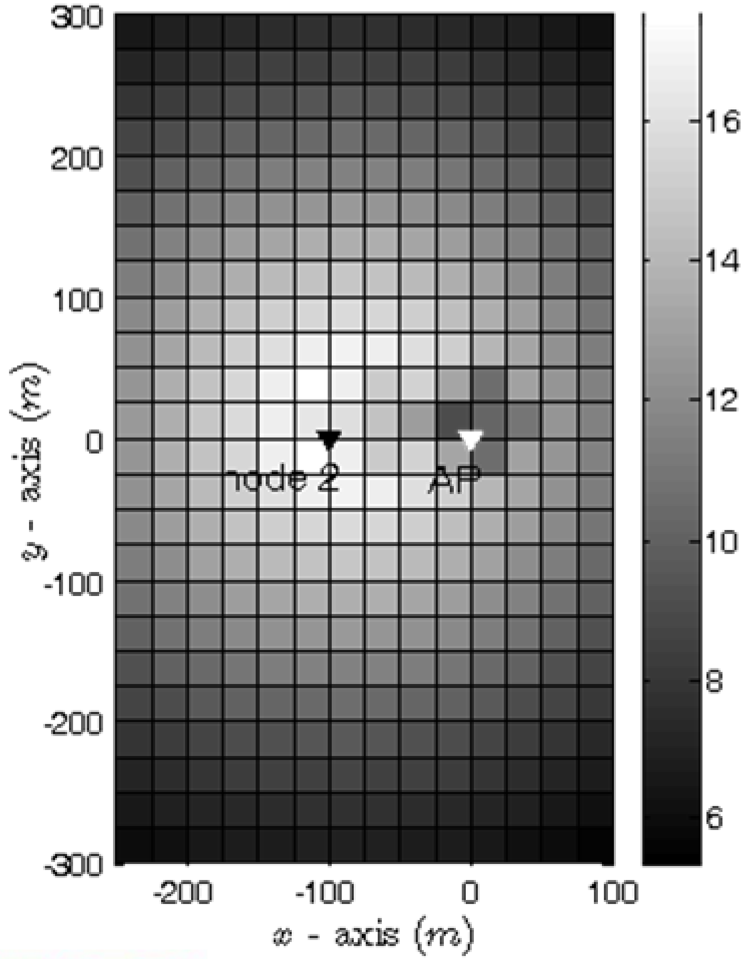
Coverage Area - Outage Prob < 0.01



How Much is Average Delegated Bandwidth?



- Node 1 delegates bandwidth for node 2 to cooperate
- Bandwidth measured in MHz
- The closer to node 2, the more bandwidth delegated

N -Node Relay Game with BE

- Under flow splitting and multi-hop, BE can still be defined
  - Exponentially increasing relay configurations
  - Requires simplification and heuristic algorithms
- NBS requires convex optimization tools
  - Geometric approach no longer feasible
- Only flows of few hops are of practical interest, restricting the number of relays

Conclusion

Bandwidth Exchange

- Simple resource-bartering incentive mechanism
- Triggers extensive cooperation
  - greatly enhances connectivity
  - also enhances rates
- Motivates the need to explore other resource exchange mechanisms