A Lower-Bound on the Number of Rankings Required in Recommender Systems Using Collaborativ Filtering

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Overview

- Motivation
- Problem Formulation
- Results

Netflix Prize

- Netflix:
  - Online Video Rental
  - Users Rank Movies: *,**,...,*****
  - System Provides Recommendations (Ranking Predictions)
- Netflix Prize
  - Improve Netflix Prediction System by 10%
  - Prize: 1 Million Dollar
  - Data Set of User Rankings
    - Training Set
    - Data Set
- Training Set
  - 480,000 Users
  - 30,000 Movies
  - 100,000,000 Rankings

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- 1 Million Dollar
- Interesting Questions
  - Is it possible to improve by 10%?
  - Is it a difficult problem?
  - How many rankings are needed to make "good" predictions
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- Important Problem: Collaborative Filtering
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- Interesting Problems/Models
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- N users
- I_N items to be ranked
- Each user ranks m_N items (chosen at random)
- Ranking: $[0, 1]$
- Correlation
  - C classes, $c = 1, ..., C$
  - Ranking vector: $r_c = (r_c(1), ..., r_c(I_N))$
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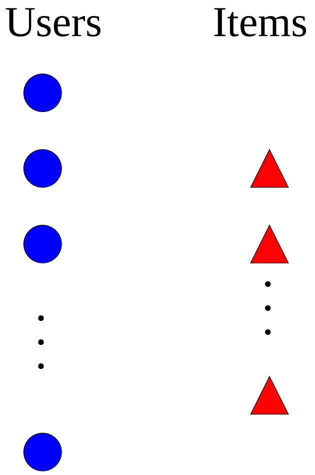

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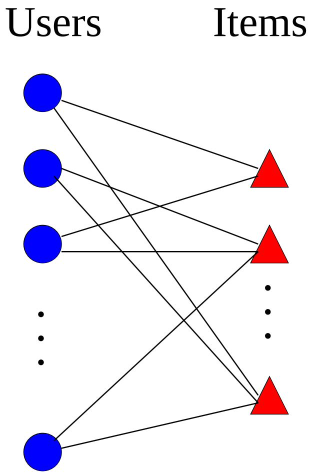

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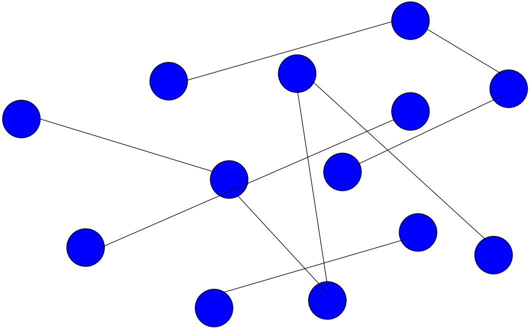

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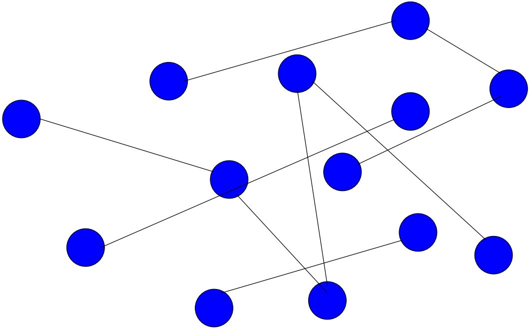

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Lower-Bound on m_N

- Complete Separation: $r_{c'}(i) \neq r_{c''}(i), i = 1, ..., I_N$
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- Notation
  - Fixed class c
  - N
- Probability that an edge exists between two users

$$p(I_N, m_N) \approx \frac{m_N^2}{I_N} = \frac{1}{I_N} \cdot m_N \cdot m_N$$

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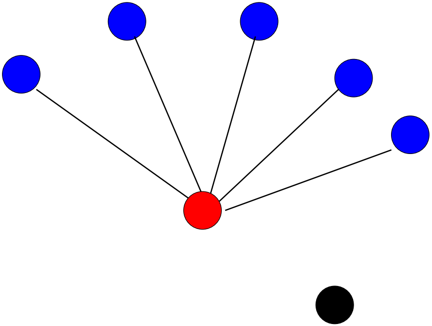

Results

- P_N: Probability of Full Connectivity
- Note
$$\frac{Nm_N^2}{l_N} = N\frac{m_N^2}{l_N} \approx Np(l_N, m_N)$$
- If
$$\frac{Nm_N^2}{l_N} = \omega(logN)$$
then $\lim_{N\to\infty} P_N = 1$,
- if
$$\frac{Nm_N^2}{l_N} = \log N + a + o(1)$$
then $\lim_{N\to\infty} P_N \leq e^{-e^{-a}}$.

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- Many-User Case

$$\lim_{N\to\infty} \frac{N}{I_N \log N} = \infty$$

- Balanced Case

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Back to Netflix

- Lower-Bound

$$\frac{Nm_N^2}{I_N} \approx \log N$$

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  - $N = 480,000$
  - $I = 30,000$
  - $m \approx 200$
- For Netflix

$$\frac{Nm^2}{I} \approx 1.3N$$

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