Optimal Path Planning for Mobile Backbone Networks

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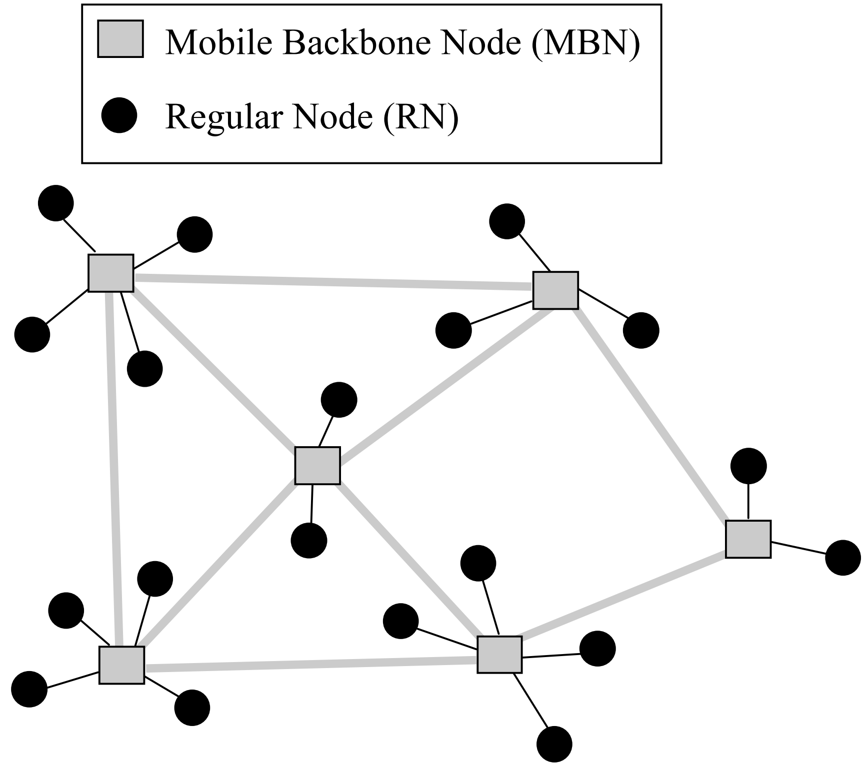
Mobile Backbone Architecture

- **Use dedicated communication nodes to maintain network connectivity**
  - **The MBNs primary purpose is to facilitate communication**
  - **The MBN's form a backbone over which communications takes place**
  - **MBN nodes are moved about to maintain connectivity requirements**
- **Regular nodes (RNs) have unconstrained mobility dictated by their mission**



- **Technical challenges:**
  - **Number and location of MBN's to provide connectivity for RN's**
  - **Algorithms for reconfiguring the network as the RN's move**

Related work: Baker et. al, 1981; Gerla, et. al. 2003; Rubin et. al. 2002

Outline

- Introduction
- MBN placement under disk cover model
  - Network connectivity
- MBN placement under communications model
  - Maximizing network throughput
- MBN trajectory planning
  - Time horizon throughput maximization
  - Energy efficient trajectory

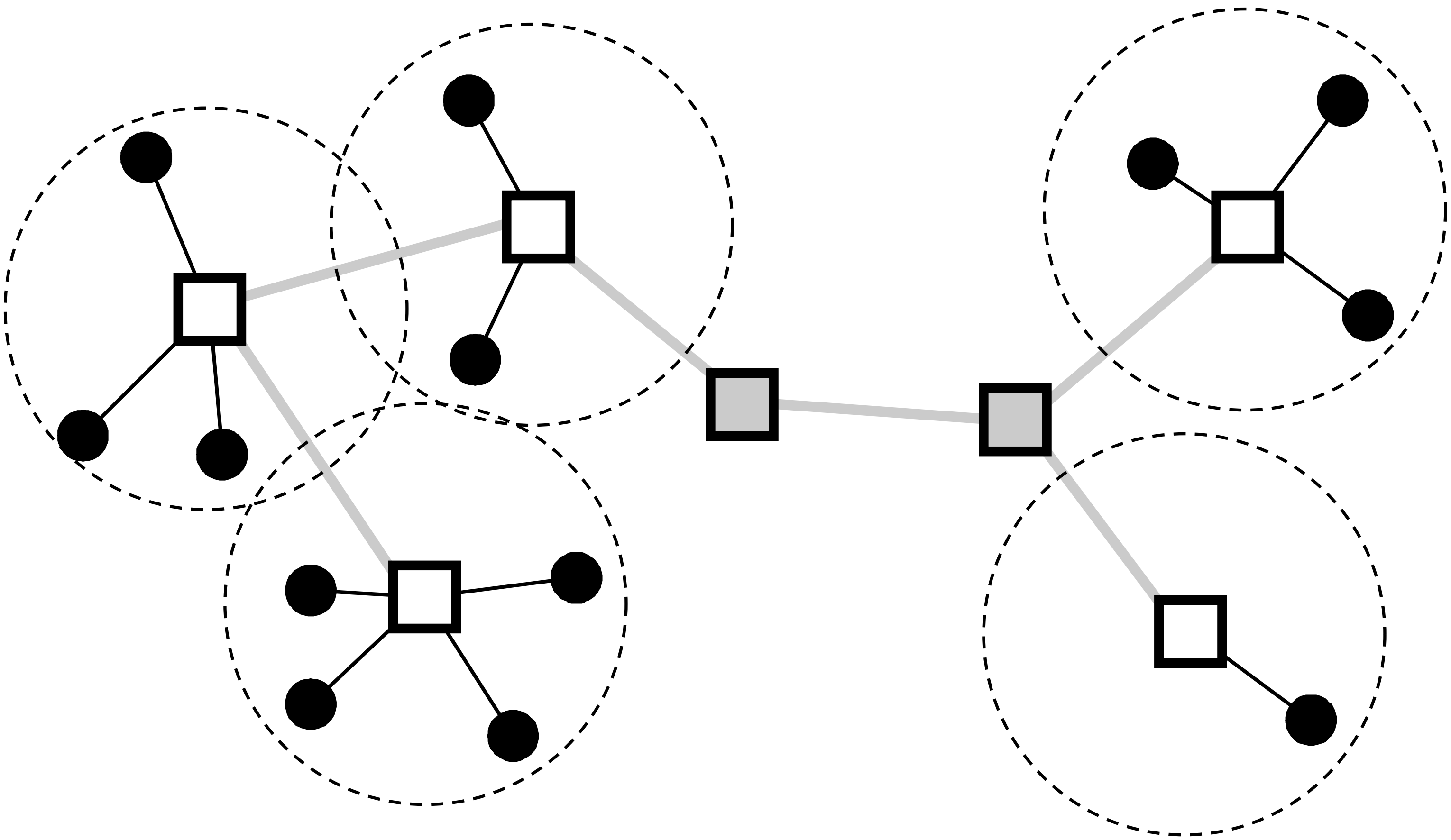
MBN Placement Problem
Disk connectivity model

- **Assume disk communications model**
  - RN *i* can *communicate* with node *j* ⇔ d(i,j) ≤ r
  - MBN *i* can *communicate* with MBN *j* ⇔ d(i,j) ≤ R

- **Given a set of RNs in the plane, place the *minimum number* of MBNs such that:**
  - (a) Every RN can communicate with at least 1 MBN
  - (b) The network formed by the MBNs is *connected*

- **Results**
  - Problem can be *decomposed* into 2 sub-problems:
    - Geometric Disk Cover (GDC) Problem
    - Steiner Tree Problem w/ Minimum Number of Steiner Pts. (STP-MSP)
  - Developed distributed algorithms for GDC and STP-MSP

Decomposition Illustration



MBN Placement Problem (Disk model)

- **Decomposition Lemma:**
  - Applying α- and β-approximation algorithms for the GDC and STP-MSP respectively, yields an (α+β)-approximation algorithm for the overall problem
  - Implication 1: even optimal solution to both GDC and MTP-MSP yields a 2-approximation for overall problem
  - Implication 2: decomposition yields a 3.5-approximation algorithm using known (centralized) solutions for GDC and STP-MSP
- **Distributed algorithms**
  - Developed distributed approximation algorithms for GDC with mobile RNs by dividing plane into "strips"
  - Developed a distributed algorithm for STP-MSP using discretization
  - Decomposition approach is sub-optimal but amenable to *distributed* implementation
- **Papers**
  - A. Srinivas, G. Zussman and E. Modiano, "Mobile Backbone Networks – Construction and Maintenance," *ACM Mobihoc*, May, 2006.
  - A. Srinivas, G. Zussman and E. Modiano, "Mobile Disk Cover – A Building Block for Mobile Backbone Networks," *IEEE Allerton*, Sept., 2006.

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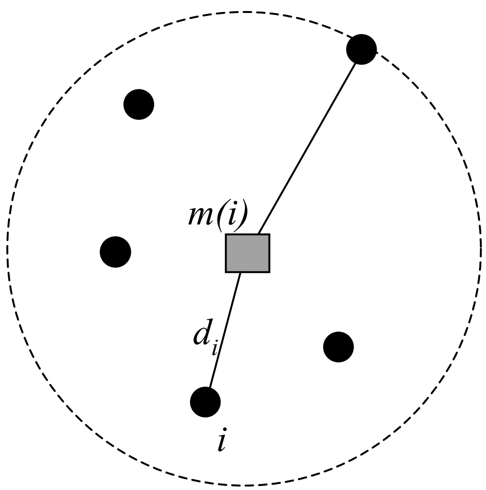
MBN Placement Problem Revisited

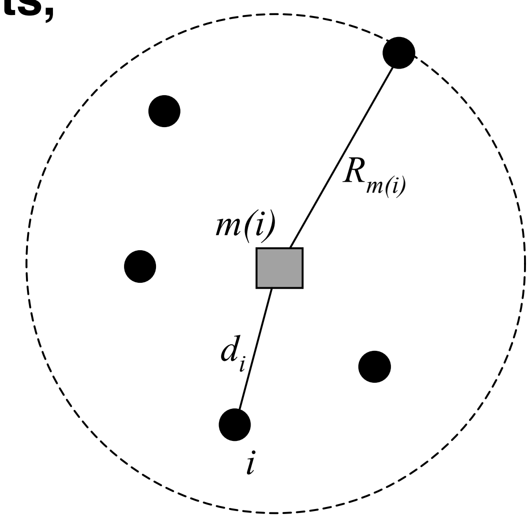
- **Issues with previous formulation**
  - Implicitly assumes arbitrary number of available MBNs
  - Simplistic disk communications model

- **Alternative formulation**
  - Fixed number of MBNs, ***K***
  - Assume a more general communications model:
    - Throughput of RN ***i*** is a decreasing function ***F( ),*** of
      - d_i - distance from its assigned MBN *m(i)* *(propagation effects)*
      - $P_{m(i)}$ - number of RNs assigned to the same MBN *(interference effects)*



Examples of Communication Model

- **Slotted ALOHA (no capture)**
  - **All RNs transmit with equal probability**
  - **Throughput of RN *i* is probability *only i* transmits, multiplied by a distance penalty,**



$$F(\ ) = \left(\frac{1}{P_{m(i)}}\right)\left(1 - \frac{1}{P_{m(i)}}\right)^{P_{m(i)}-1}\left(\frac{1}{d_i^{\alpha}}\right) \approx \frac{1}{e \cdot P_{m(i)} \cdot d_i^{\alpha}}$$

- **CDMA**
  - **Received power is equalized at *m(i)***
  - **Throughput of RN *i* is related to SINR at *m(i),***

$$F(\ ) = \frac{(1/R_{m(i)}^{\alpha})}{(1/R_{m(i)}^{\alpha}) \cdot (P_{m(i)} - 1) + \eta} = \frac{1}{(P_{m(i)} - 1) + \eta \cdot R_{m(i)}^{\alpha}}$$

MBN Placement Problem Revisited

- ***Maximum Fair Placement and Assignment*** **(MFPA) Problem:**
  - **Given N RNs, place *K* MBNs and assign each RN to exactly one MBN such that,**

$$T_{MFPA} = \min_i \left\{ F(d_i, P_{m(i)}) \right\}$$ **is *maximized***

- ***Maximum Throughput Placement and Assignment*** **(MTPA) Problem**
  - **Given N RNs, place *K* MBNs and assign each RN to exactly one MBN such that,**

$$T_{MTPA} = \sum_i F(d_i, P_{m(i)})$$ **is *maximized***

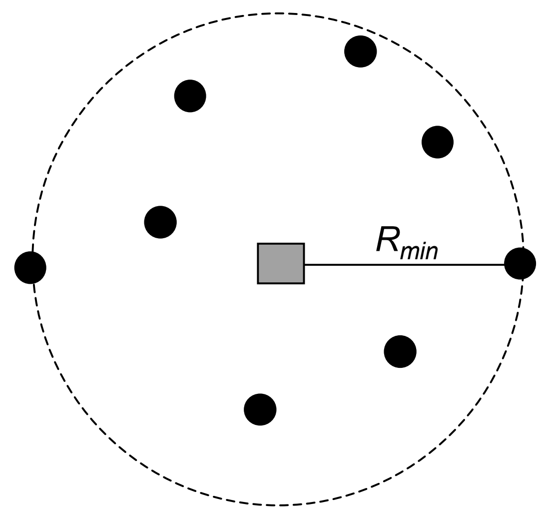
- **Note:**
  - **Above formulations do not consider MBN connectivity**
  - **Above formulations include a non-trivial assignment component**

Simple example: 1 MBN and N RNs

- **MFPA cost function, i.e.** $T = F\left(R_{m_1}, P_{m_1}\right)$
  - R_{m1} = radius of MBN
  - P_{m1} = number of RN's assigned to the MBN

– **Note: No assignment component in this case**

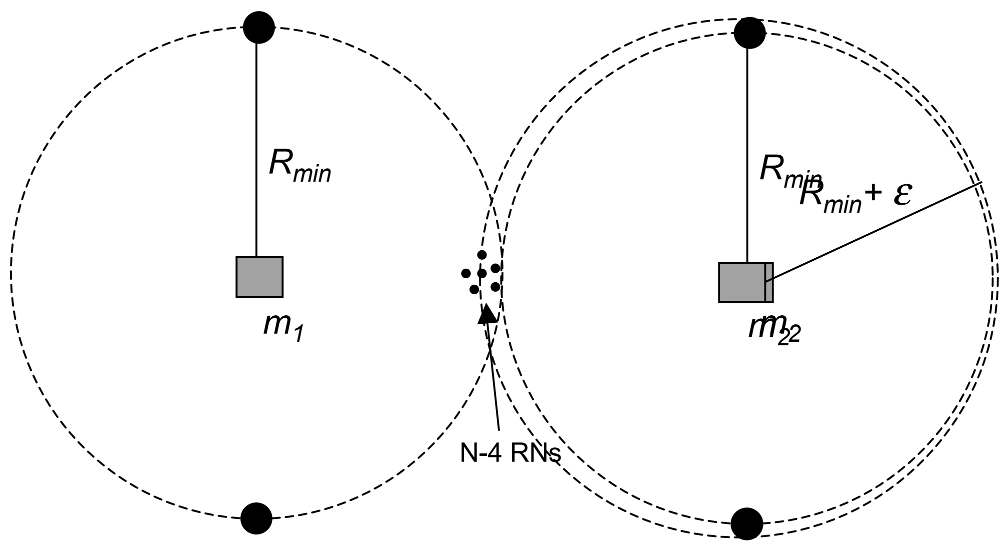
– ***Place MBN to minimize*** R_{m1}
  - **Optimal MBN placement at "1-center" of the RNs (O(nlog n) computation)**



Optimal Throughput is $T_{1\text{-}cen} = F(R_{min}, N)$

Non-trivial example: 2 MBNs and N RNs

- **MFPA cost function, i.e.,** $T = \min_i \{F(R_{m(i)}, P_{m(i)})\}$
- **Assignment problem non-trivial**
- **2-center solution no longer optimal**



2-center solution

$T_{2-cen} = F(R_{min} + \varepsilon, N-2)$

Optimal Solution
(e.g. load-balanced)

$T_{opt} = F(R_{min}, N/2)$

- **Note:**
  - ***Infinite number of possible MBN placements***
  - ***Given a placement, there are 2^N different RN to MBN assignments***

Optimal Solution for K-MFPA

- **Placement of K MBNs to maximize min throughput**
- **Basic idea**
  - **Show that restricting MBN placements to a finite set of locations preserves optimality**
  - **Show that given an MBN placement, can efficiently solve RN assignment sub-problem**
- **Theorem 1:**
  - **Given optimal RN to MBN assignments $P_1,..,P_k$**
  - **Placing MBNs at 1-center locations 1-Center(P_1), …1-Center (P_k) is optimal**
- **Theorem 2:**
  - **The 1-center location of *any subset* of N RNs lies among at most N^3 candidate locations, which can be easily enumerated**
    - **At most $\binom{N^3}{K}$ possible placements of K MBNs**
- **For each placement, need a way optimally assign RNs to MBNs**

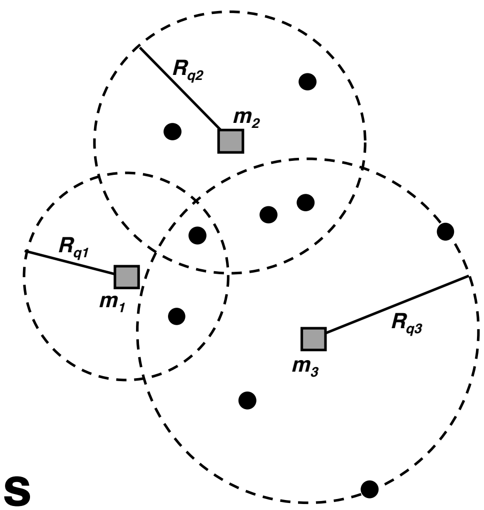
Assignment Sub-problem

- Given K placed MBNs at "1-Center locations" $q_1,\ldots,q_K$, assign every RN to exactly 1 MBN to maximize

$$T = \min_i \left\{ F(R_{m(i)}, P_{m(i)}) \right\}$$

  - Note: there are K^N possible assignments

- Key observation: each candidate location q has an associated radius R_q (given by set of nodes that define this candidate location)
  - Only RNs within R_q of the MBN placed at location q can be assigned to it
  - An RN may still be covered by multiple MBNs
  - Defines a set of "valid assignments", i.e., $z_{ij} = 1$, iff RN i can be assigned to MBN m_j



Mathematical formulation for the assignment problem

- **Mathematical Formulation of MFPA Assignment Sub-problem:**

$$\begin{aligned}&\max_{j\in M} \min F\left(R_j, \sum_{i\in P} x_{ij}\right)\\ &s.t. \quad \sum_{j\in M} x_{ij} = 1, \quad \forall i \in P\\ &\qquad x_{ij} \le z_{ij}, \forall i \in P, j \in M\\ &\qquad x_{ij} \in \{0,1\}\end{aligned}$$

$\rightarrow$

$$\begin{aligned}&\min_{j\in M} \max H\left(R_j, \sum_{i\in P} x_{ij}\right)\\ &s.t. \quad \sum_{j\in M} x_{ij} = 1, \quad \forall i \in P\\ &\qquad x_{ij} \le z_{ij}, \forall i \in P, j \in M\\ &\qquad x_{ij} \in \{0,1\}\end{aligned}$$

$\rightarrow$

$$\begin{aligned}&\min \quad W\\ &s.t. \quad \sum_{i\in P} x_{ij} \le G(W; R_j)\\ &\qquad \sum_{j\in M} x_{ij} = 1, \quad \forall i \in P\\ &\qquad x_{ij} \le z_{ij}, \forall i \in P, j \in M\\ &\qquad x_{ij} \in \{0,1\}\end{aligned}$$

- **x_{ij} = 1 if RN i assigned to MBN m_j**

- **P ≡ set of RNs, M ≡ set of placed MBNs, H() = 1/F(), G = H-1()**

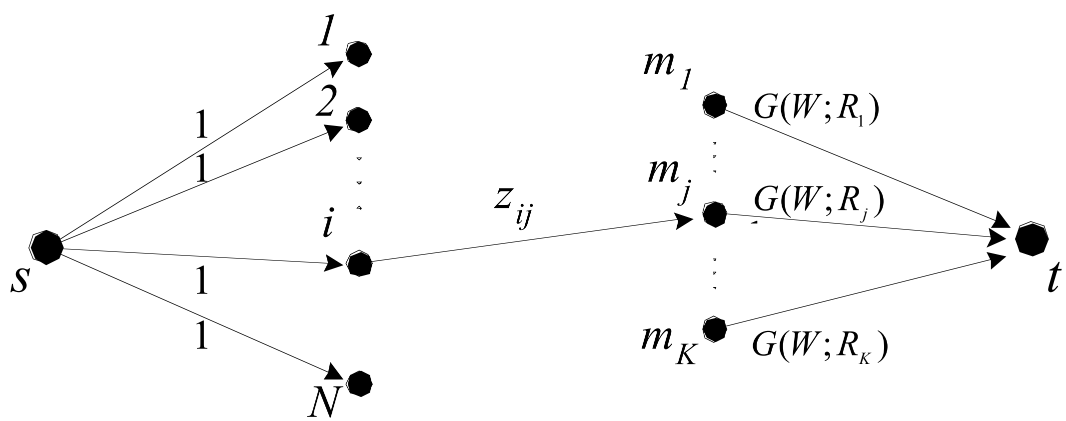
- **W corresponds to the inverse of the throughput**
  - **G gives the maximum number of RNs that can be assigned to the MBN while satisfying the throughput implied by W**

- **Finding the optimal W* - Max-flow formulation**

Assignment Sub-problem Details

- **Lemma:**
  - **Optimal W* must satisfy $G(W;R_j) \in Z_+$, for some $j \in M$**
    - **At most KN possibilities for W**

- **For a given value of W, we can formulate the feasibility problem as the following integer max-flow problem:**



- **Theorem:**
  - **A s-t max-flow of size N exists ⇔ the assignment sub-problem is feasible for W**
    - **Flow variables x_{ij}'s specify RN to MBN assignments**

Lower Complexity Heuristics

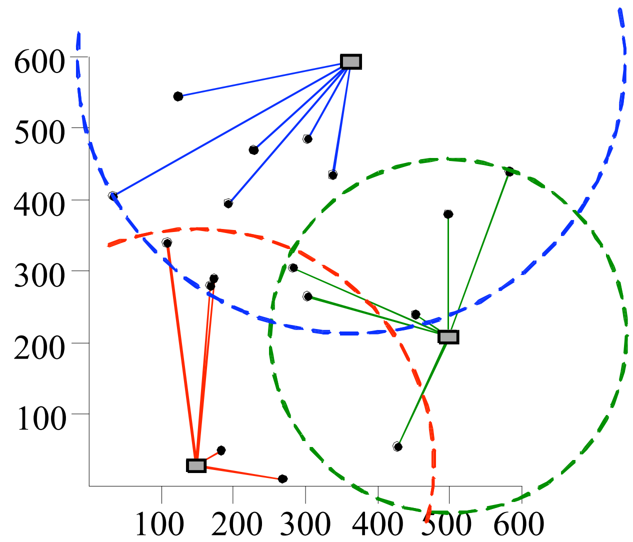
- **Developed lower complexity sub-optimal heuristics for both MTPA and MFPA**
  - **Extended-Diameter Algorithm**
    - **Prune number of placements to** $\binom{N^2}{K}$
    - **Consider pairs of nodes as defining the optimal 1-center locations**
    - **Achieves 3-approximation ratio in worst case**
  - **Farthest Point Heuristic**
    - **Applies a approximation algorithm of K-center algorithm for placement, and assigns RNs to their closest MBNs**

      Start with single 1-center at an arbitrary node

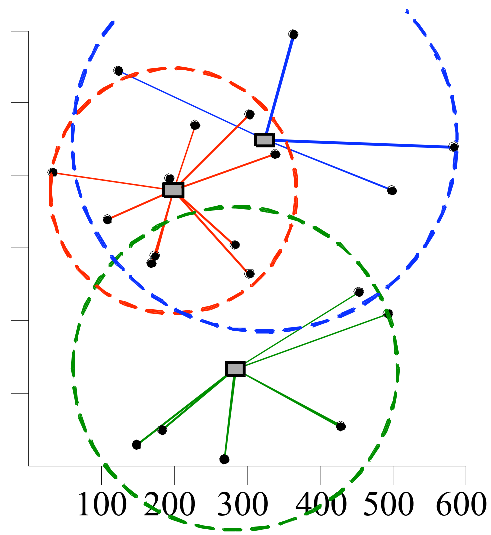
      Repeatedly add MBNs at "furthest" point from existing 1-centers
    - **Running time of O(KN logN)**

Simulation Results

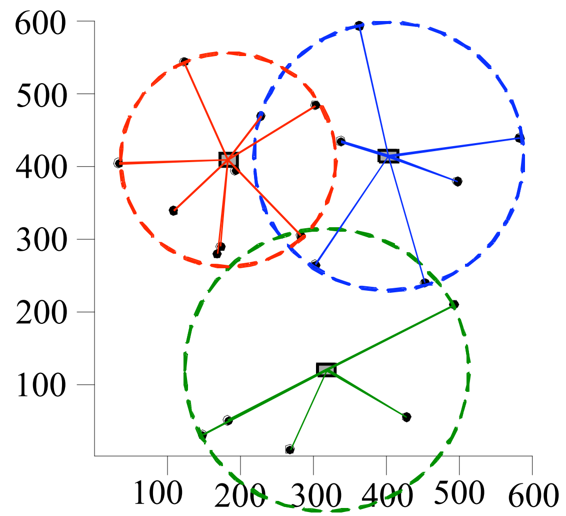
- **Single Example, 3 MBNs, 20 RNs, MFPA objective function**



Farthest Point Heuristic



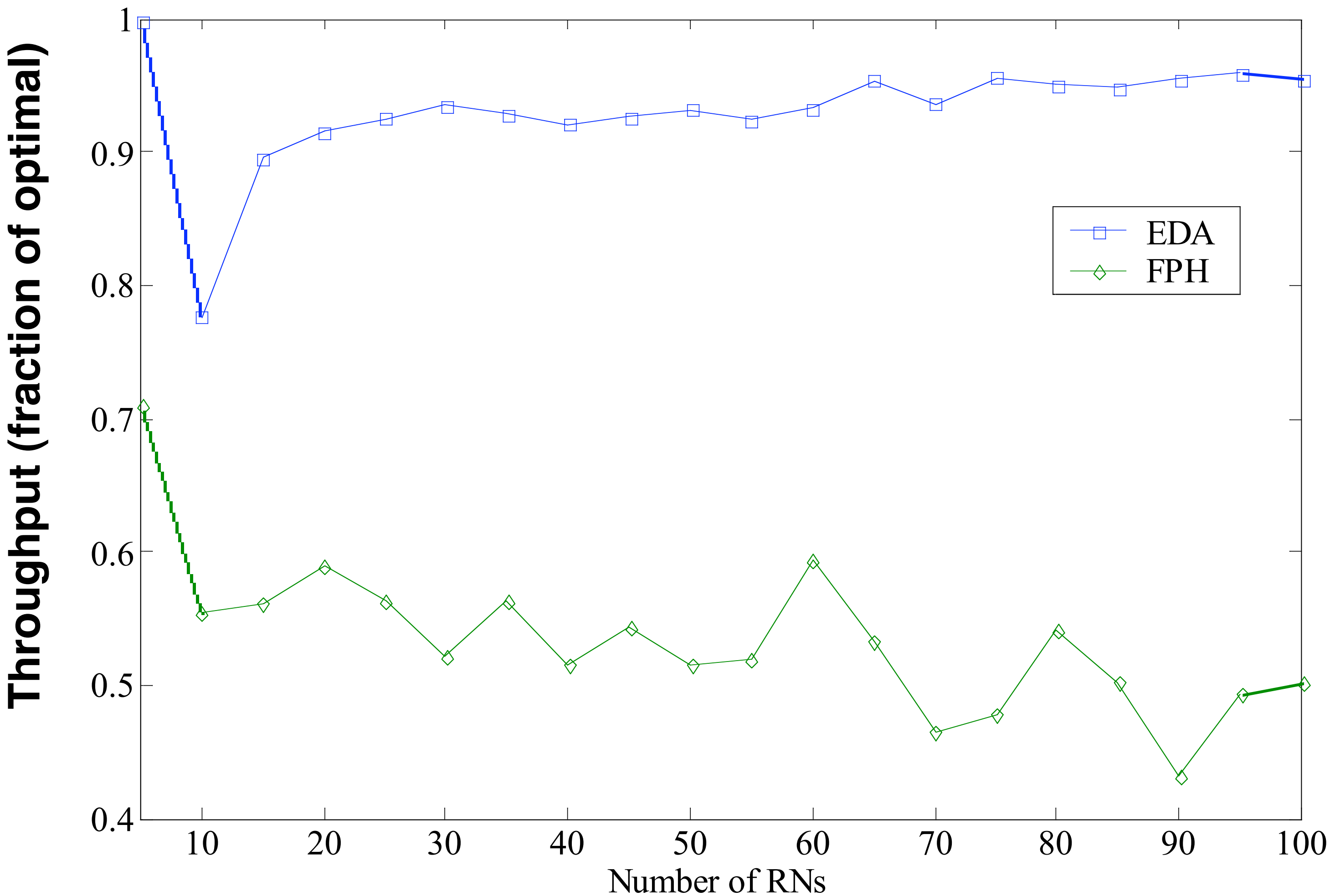
Extended Diameter Algorithm



Optimal Algorithm

Simulation Results

- RNs randomly distributed, 2 MBNs, MFPA objective function



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  - Time horizon throughput maximization

MBN Path Planning (MPP) Problem

- **Motivation**
  - In many scenarios RN trajectories are known a-priori
    - Previous formulations place MBNs *reactively* based on independent snapshots of the network
    - This requires MBNs to move arbitrarily fast in response to RN's mobility
    - Want algorithms that incorporate RN trajectory knowledge and solve for entire MBN path a-priori
    - Account for limited MBN mobility
- **Our focus: single MBN and multiple RNs**

MBN Path Planning (MPP) Problem Formulation

- **Goal is to determine optimal *path* of a single MBN - $M[t]$**
  - **Given initial MBN position, $M(0) = M_0$**
  - **Assume a finite time horizon $t \in [0,T]$**
    - Discrete-time, $K=T/\Delta t$ intervals of size Δt
  - **Consider time-average system throughput**
  - **$F[d_{max}(t)]$ = decreasing function of distance from farthest RN**

- **Assume *N* RNs, trajectory information known a-priori**
  - **Given RN positions $p_i[t], t = 0,\ldots,K$ ($K=T/\Delta t$)**
- **Limited MBN mobility**
  - ***Hard Constraint – bound MBN velocity by V*: $d[M(t\text{-}1), M(t)] \le V\Delta t$**

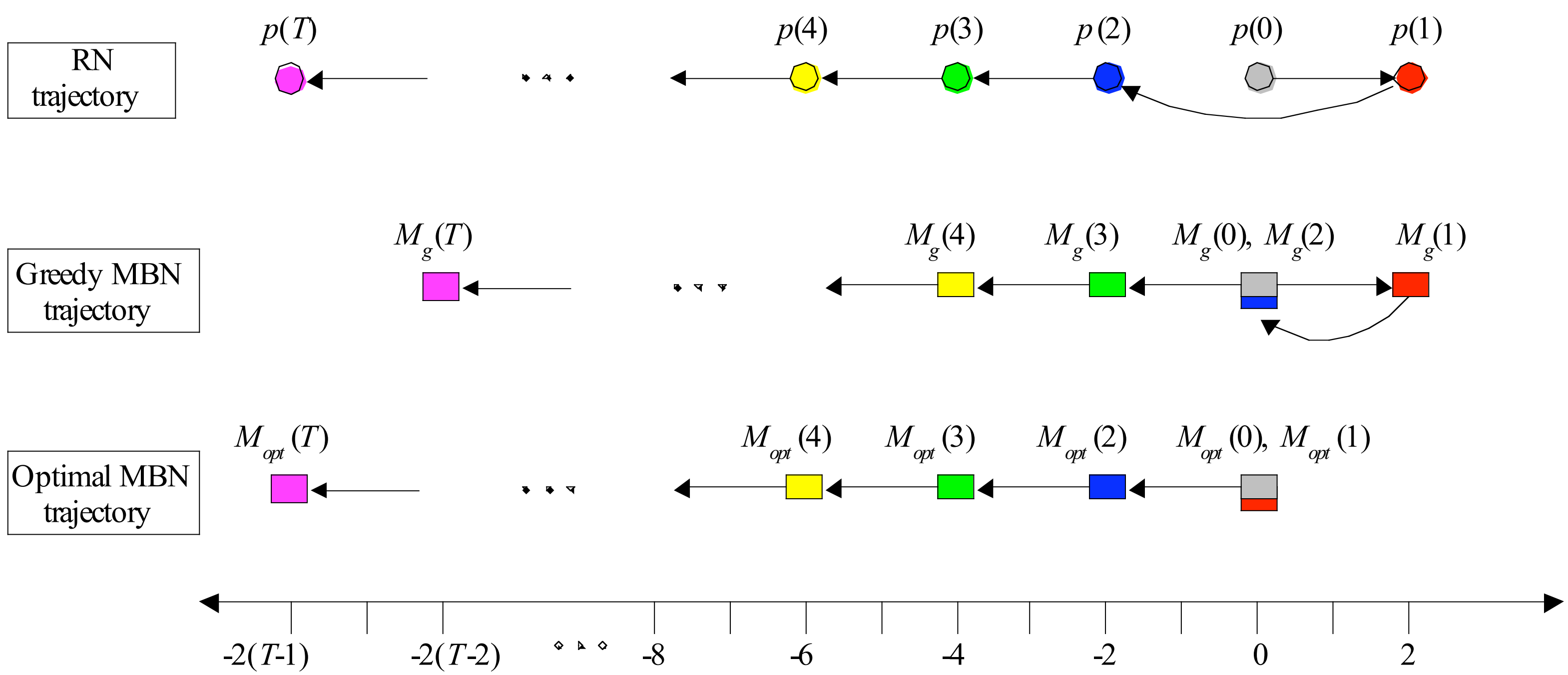
$$\max_{M^*} \quad \frac{1}{K}\sum_{t=1}^{K} F[d_{\max}(t)]$$

$$s.t. \quad d[M(t-1), M(t)] \le V\Delta t$$

$$M(0) = M_0$$

Sub-optimality of greedy algorithm

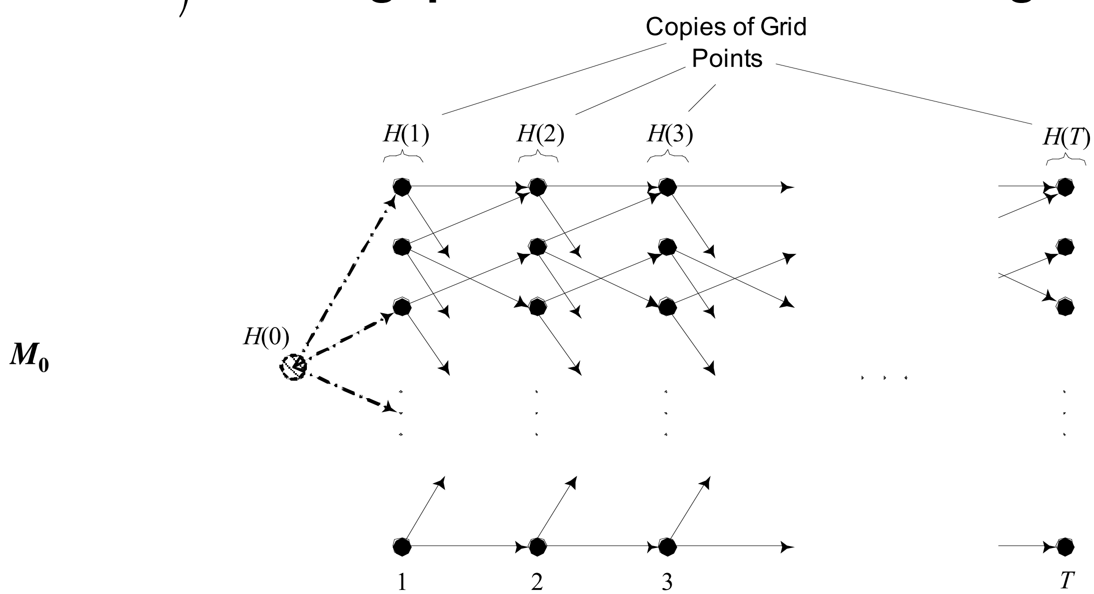
- **Illustrative Example (Single RN, 1-D):**
  - **Greedy vs. Optimal solutions ($V = 2$, $\Delta t = 1$)**



- **To guarantee good performance, want algorithms that solve for the entire MBN path at once**

Dynamic Programming Algorithm (discrete locations)

- **Grid the plane with horizontal/vertical spacing $\varepsilon \leq V\Delta t$**
  - **Define graph G=(V',E)**
  - **Let *V'* represent *K* copies of grid points $H(1),\ldots,H(K)$**
  - **$v \in H(t)$ represents a potential location for the MBN at time *t***
  - **Define edge weight for (*u,v*), $d[u,v] \leq V\Delta t,\ u \in H(t),\ v \in H(t+1)$ as $F\left(\max_i d[v, p_i(t)]\right)$ - throughput function evaluated at grid point**



- **Optimal MBN path *constrained to grid points* corresponds to the longest (max-weight) path in the above graph**

Performance grid-based DP algorithm

- **How does the grid-based DP algorithm compare to the unconstrained optimal solution?**

- **Lemma: on an unbounded plane, the objective of the DP algorithm is lower bounded by,**

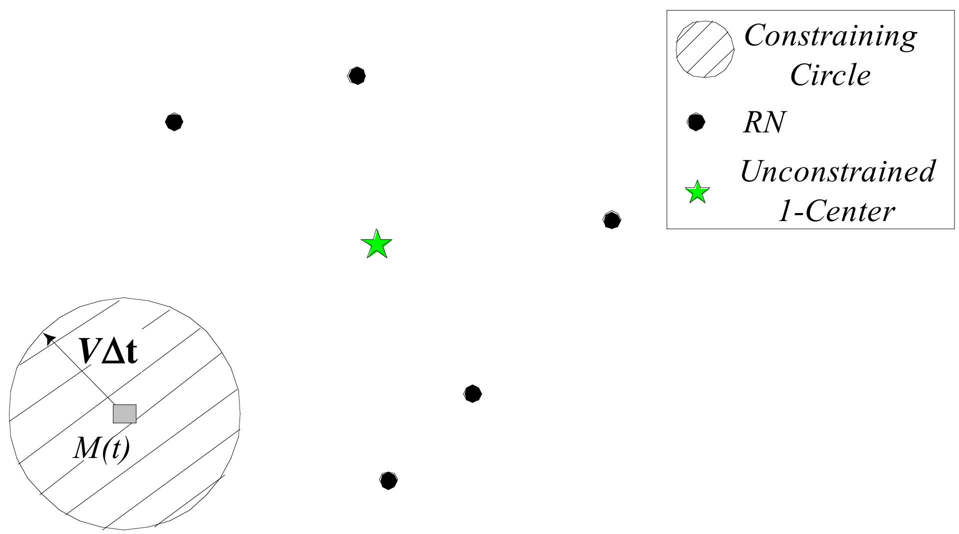
$$\frac{1}{K}\sum_{t=1}^{K} F\left[d_{\max}^{opt}(t) + 2\sqrt{2}t\varepsilon\right]$$

- **Grid-based solution deviates from optimal**
  - **Difference increases with time**
  - **Constraining MBN to travel on "grid" results in the MBN falling behind the optimal location**
  - **Above bound is "tight"**

Greedy Approach

- **Motivation**
  - **DP approach has high computational complexity**
  - **For many practical scenarios, greedy can perform well**

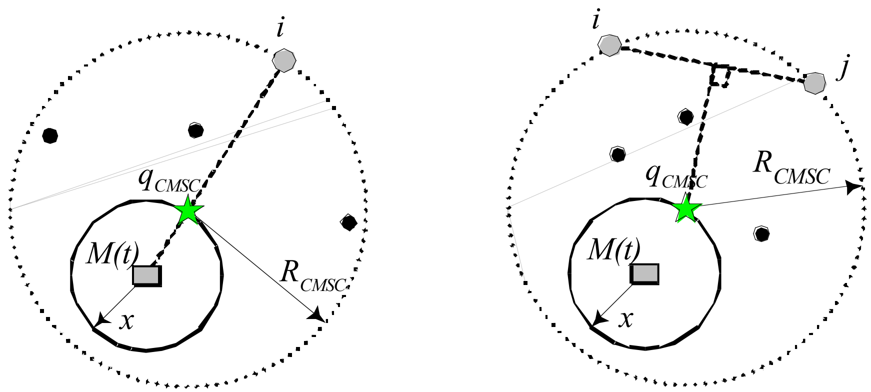
- **Greedy High-Level Idea:**
  - **For each time-step $t = 0,\ldots,K-1$, compute the location for $M(t+1)$ that maximizes $F[d_{max}(t+1)]$ subject to** $d[M(t),M(t+1)] \leq V\Delta t$
  - **Problem reduces to finding the *circular constrained 1-center***



Circularly constrained 1-center

- Lemma: if the unconstrained 1-center lies outside the constraining circle C. Then, the constrained 1-center must lie on the boundary of C (δC)

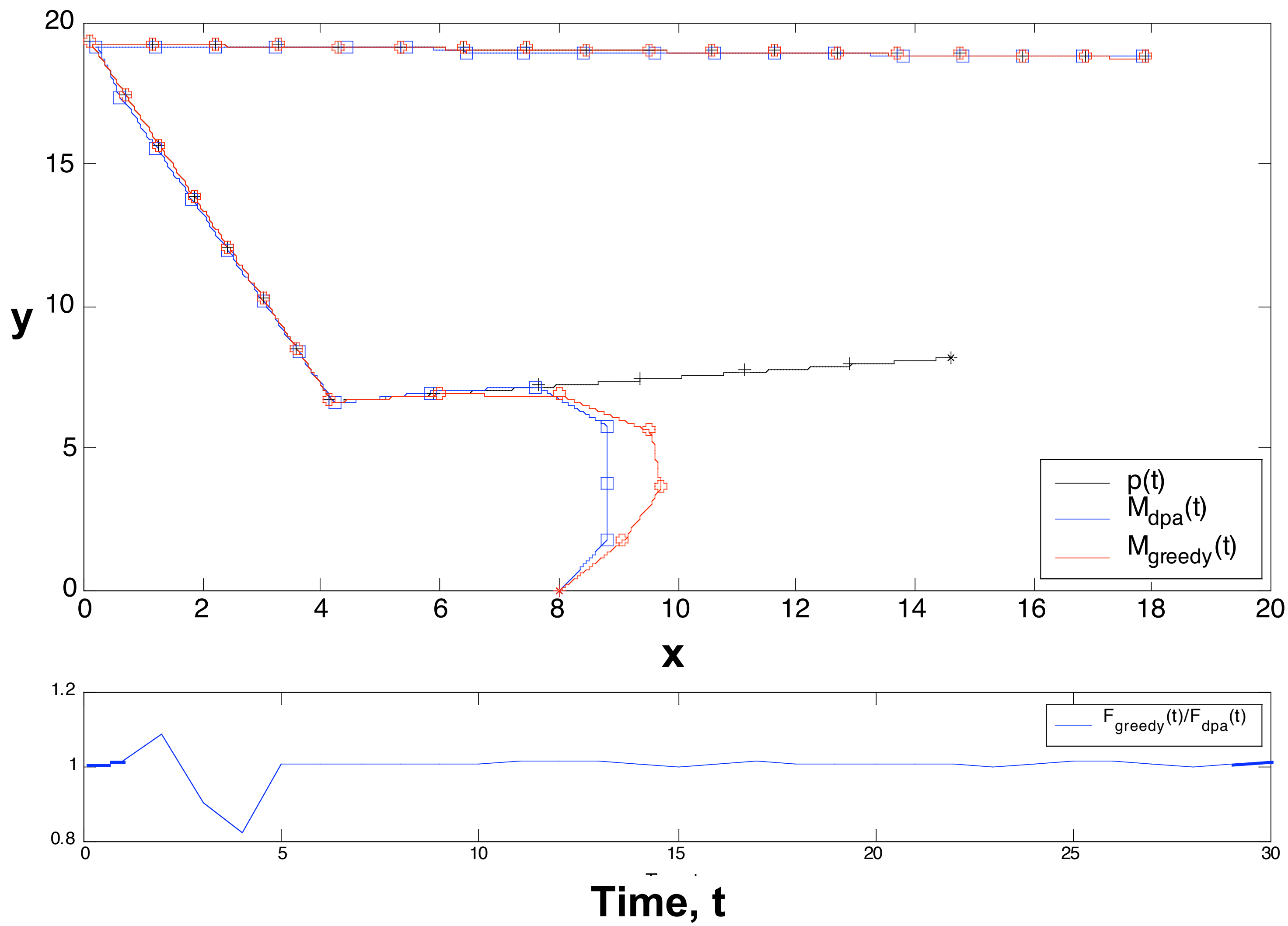
- Lemma: The constrained 1-center, q, can be defined by either,
  - A single RN i. If this is the case, then q is located on the intersection between the line segment <i,M(t)> and δC
  - By a pair of RNs i, j. If this is the case, then q is located on the intersection between the perpendicular bisector of <i, j >and δC



Only need to examine $O(N^2)$ possible locations

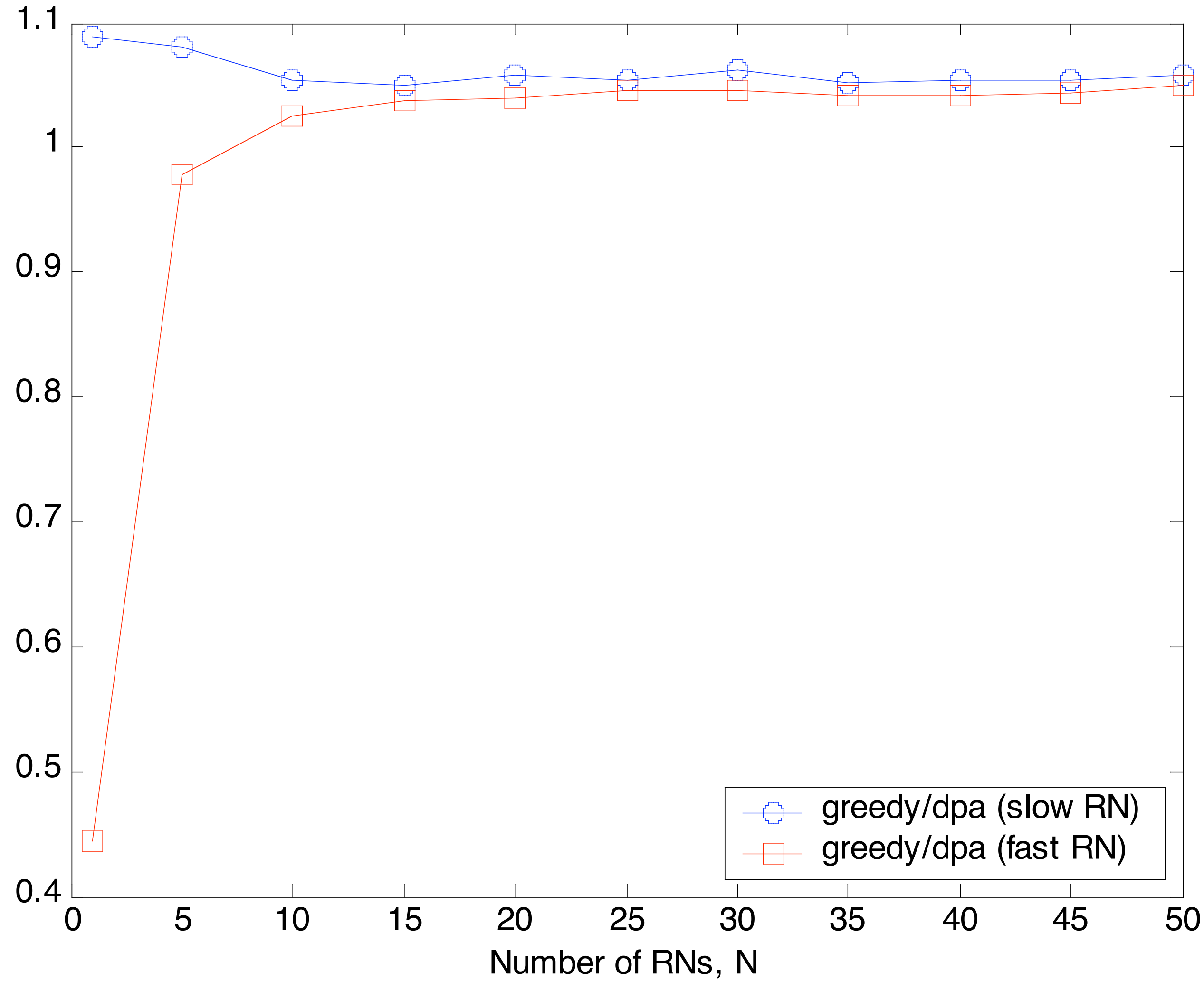
Simulation Results

- **Simplified CDMA throughput function, $F[d_{max}(t)] = 1/(d_{max}(t)^2 + 1)$**
- **Single RN, 2-D random waypoint example, $\Delta t = 1$, 20X20 plane,**



Simulation Results

- Simplified CDMA throughput function, $F[d_{max}(t)] = 1/(d_{max}(t)^2 + 1)$
- 2-D random waypoint example, $\Delta t = 1$, 100s time period, 20X20 plane, velocity $V \in [0,2]$



Summary

- **MBN architecture utilizes dedicated communication nodes to provide support**
  - In contrast to traditional "peer" view of ad hoc networks
  - MBN's trajectory/position can be controlled

- **Developed algorithms for MBN Placement**

  1) Disk communications model
  2) SINR-based communications model

- **Developed algorithms for planning the MBN trajectory**

- **Future directions: controlled mobility wireless networks**

  - Can we take advantage of "controlled mobility" to:
    - Increase network throughput
    - Reduce energy consumption
    - Reduce delays