On congestion control, multipath routing, and admission control

Fernando Paganini
Universidad ORT Uruguay

• Part I (with Enrique Mallada). Combined congestion control and node-based multipath routing: new results on stability since CISS’06.

• Part II (with Andrés Ferragut, in CISS’08 paper). Achieving network stability and user fairness through admission control of TCP connections.
Congestion control with multipath routing

Source-destination pair ("commodity") $k$, input rate $x^k$ (bps)
Utility $U_k(x^k)$.

Links
- Rate $y_i$
- Capacity $c_i$, or cost $\phi_i(y_i)$.

Source $s(k)$

Destination $d(k)$

Router $i$

$\alpha_{i,j}^d$: fraction of traffic destined to $d$, sent to node $j$. $\alpha_i^d = \{\alpha_{i,j}^d\} \in \Delta_i$ (simplex)

Source-destination pair ("commodity") $k$, input rate $x^k$ (bps)
Utility $U_k(x^k)$.

[Link to Paganini, CISS’06, ECC07, Mallada-P’ Netcoop ’07 for congestion control with multipath routing]
Congestion prices

- Define: link congestion price $p_i$. (e.g., $p_i = \phi'(y_i)$) or $\frac{dp_i}{dt} = \gamma_i[y_i - c_i].$
- $q_i^d$: average price from node $i$ to destination $d$
  
  $$q_d^d = 0, \quad q_i^d = \sum_{j: (i,j) \in L} \alpha_{i,j}^d \left( p_{i,j} + q_j^d \right), \quad i \neq d$$
  
  $q^k := q^d_{s(k)}$, average price seen by source $s(k)$

**Multipath routing control.**

Routers control $\alpha_i^d := \left\{ \alpha_{i,j}^d \right\}_{(i,j) \in L}$ based on seen prices $\pi_i^d := \left\{ p_{i,j} + q_j^d \right\}_{(i,j) \in L}$

- First choice (essentially from Gallager '77): follow negative price gradient. $\dot{\alpha}_i^d = \beta_i E_{\alpha_i^d} \left[ -\pi_i^d \right]$. 

The projection $E_{\alpha_i^d}$ keeps $\alpha_i^d \in \Delta_i$. 

[Diagram of network with nodes and links, showing flow and congestion control. Diagram includes arrows indicating flow, and equations for congestion pricing and multipath routing control.]
Primal congestion control under gradient control of routing fractions

\[ \dot{x}^k = \kappa(x^k) \left[ U'(x^k) - q^k \right] \]

Traffic splitting

\[ \dot{\alpha}_i^d = \beta_i E_{\alpha^d} \left[ -\pi_{i,j}^d \right] \]

Split ratios \( \alpha_i^d \)

Node price recursion

\[ p_l = \phi^l(y_l) \]

Theorem: with these dynamics the system converges globally to the optimum of the SURPLUS problem,

\[ \max S := \sum U^k(x^k) - \sum \phi_i(y_i) \]

[See P' CISS '06], extends Gallager '77.
Also in [Xi & Yeh CISS'06], combined with wireless power control.
Dual congestion control under gradient control of routing fractions

\[ x^k = \arg \max \left[ U(x^k) - q^k x^k \right] \]

\[ \dot{\alpha}^d_i = \beta_i E_{\alpha^d} \left[ -\pi^d_{i,j} \right] \]

**Equilibrium**: solution to optimal WELFARE problem,

\[ \max \sum U_k(x^k) \quad \text{subject to} \quad y \leq c \]

Convergence to equilibrium?

No! Simple examples exhibit harmonic oscillations.
Which model is correct for queuing delay:

Static $p_l = \varphi(y_l)$ or integrator $\dot{p}_l = \frac{1}{c_l} [y_l - c_l]^{+}$?

Example of instability, packet simulation in ns2
Single source, two bottlenecks.

- Dual model of queue is more appropriate.
- Indeed, it predicts correctly oscillation period.
Solving the problem

Adapt $\alpha_i^d$ based on anticipated (rather than current) price

$$\pi^*_{i} = \pi_{i}^d + V_i \dot{\pi}_i^d$$

In control terms, add derivative action. Same equilibrium. Simulations:

Theorems [P'-Mallada, submitted to ToN, CDC'08]
the equilibrium point (optimum $\max \sum U^k(x^k)$) is:
- locally asymptotically stable in an arbitrary network
- globally asymptotically stable in a network of parallel links.

Packet implementation: variants of TCP-FAST and RIP.
On congestion control, multipath routing, and admission control

Fernando Paganini
Universidad ORT Uruguay

• Part I (with Enrique Mallada). Combined congestion control and node-based multipath routing: new results on stability since CISS’06.

• Part II (with Andrés Ferragut, in CISS’08 paper). Achieving network stability and user fairness through admission control of TCP connections.
Stability and user-level fairness

User rate $\rho_k$

Utility $U_k(\rho_k)$

Link rate $y_i$

Capacity $c_i$

Single path routing matrix:

$$R_{lk} = \begin{cases} 
1 & \text{if user } k \text{ uses link } l \\
0 & \text{otherwise} 
\end{cases}$$

$$y = R \rho$$

KELLY'S SYSTEM PROBLEM

$$\max_\rho \sum_k U_k(\rho_k), \quad \text{subject to} \quad R \rho \leq c$$

LINK CAPACITY CONSTRAINTS

USER UTILITY FUNCTION
Contrast with flow-level fairness of TCP

Rate $x_k$ per flow $\rightarrow$
Utility $U_{TCPk}(x_k)$
$n_k$ flows per user.

TCP utility $U_{TCPk}$ (per flow) determined by the protocol, e.g., $U_{TCPk}(x) = \kappa_k x^{-\alpha}$ for $\alpha=2$.

**TCP: NETWORK PROBLEM**

$$\max_x \sum n_k U_{TCPk}(x_k) \quad \text{subject to} \sum_k R_{lk} n_k x_k \leq c_l$$

- Without control of number of connections, fairness per flow is moot (Briscoe'07).
- Incentives to employ many TCP flows (e.g., p2p). Tragedy of the commons?
- If we could control $n_k$, can we induce with TCP the SYSTEM problem allocation? (similar to "user problem" setting weight parameter in Kelly-Maulloo-Tan '98)
On stochastic stability of a network served by TCP
[deVeciana, Lee, Konstantopoulos ’99, Bonald-Massoulié ’01]

User: Poisson ($\lambda_k$) arrivals, $\exp(\mu_k)$ workloads.

For each fixed $\{n_k\}$, service rates $x_k$ determined by TCP congestion control $U'_{TCP_k}(x) = \kappa_k x^{-\alpha}$ for $\alpha > 0$.

Result: $n_k$ Markov chain $\{n_k\}$ stable if and only if $\sum_k R_{lk} \frac{\lambda_k}{\mu_k} < c_l \ \forall l$.

Remark: congestion control ensures neither stability nor fairness.

- Both stability, and resource allocation depend solely on users' "open loop" demands $\frac{\lambda_k}{\mu_k}$.
- Fairness choice per flow (e.g., value of $\alpha$) has minimal impact. A heavy user will compensate a low TCP rate by increasing $n_k$, until $\rho_k$ serves demand, if feasible. If not $n_k$'s grow without bounds.
Closing the loop on $n_k$ for user-level fairness

Assume that for fixed $n_k$, the flow rate $x_k$ is determined by TCP:

$$x_k = f_{TCP_k}(q_k)$$

where $q_k$ is the congestion price seen by the source, and $f_{TCP_k} = (U'_{TCP_k})^{-1}$, TCP demand curve. The user rate is $\rho_k = n_k x_k$.

Objective: control $n_k$ so that the system converges to an equilibrium where $\rho_k = n_k x_k$ solves

$$\max_{\rho} \sum_k U_k(\rho_k), \text{ s.t. } R\rho \leq c,$$

with utilities defined by users.

Control law for continuous $n_k$:

$$\dot{n}_k = \beta \left( U_k^{-1}(q_k) - \rho_k \right).$$

Other recent work on controlling no. of flows:

Chen - Zakhor '06
(for TCP over wireless).
Analysis using dual TCP congestion control,

\[ \dot{n}_k = \beta \left( U^{-1}_k(q_k) - \rho_k \right); \]
\[ \rho_k = n_k x_k; \]
\[ x_k = f_{TCPk}(q_k). \]

Theorem 1 (arbitrary network).
The equilibrium satisfies \( \max_{\rho} \sum_k U_k(\rho_k) \), subject to \( R\rho \leq c \), and is locally asymptotically stable. Proof: passivity argument (as in Wen-Arcak '03).

Theorem 2 (single bottleneck).
Assume time-scale separation: for fixed \( n = \{n_k\} \), let \( \hat{q}_k(n) \), \( \hat{x}_k(n) \) be the equilibrium values from dual congestion control, and \( \hat{\rho}_k(n) = n_k \hat{x}_k(n) \). Then the "slow" dynamics \( \dot{n}_k = \beta \left( U^{-1}_k(\hat{q}_k(n)) - \hat{\rho}_k(n) \right) \) are globally convergent to a point \( n^* \) where the corresponding \( \hat{\rho}_k(n^*) \) are at the optimum welfare point.

\[ \dot{p}_l = y_l [y_l - c_l]_p \]
From fluid control to admission control.

In practice, $n_k$ is discrete (number of TCP connections). Furthermore:

- Real-time control at sources' (application layer) is impractical, incentives?
- Killing an ongoing TCP connection to reduce $n_k$ is undesirable.

More practical alternative:

- Control increase of $n_k$ (admit new connections), rely on natural termination.
- Admission control carried out by edge router.
- User utility $U_k(\rho_k)$ describes the SLA: admit new connection $\Leftrightarrow U_k^{1-1}(q_k) > \rho_k$

Stochastic model. Poisson($\lambda_k$) arrivals, exp($\mu_k$) workloads.
Active sessions served with rate $x_k$ obtained from the network.

Continuous time Markov chain with state $n = \{n_k\}$.

Transition rates: $q_{n,n+e_k} = \lambda_k \mathbf{1}\{U_k^{1-1}(\hat{q}_k(n)) > \hat{\rho}_k(n)\}$; $q_{n,n-e_k} = \mu_k \hat{\rho}_k(n)$
Single bottleneck, two users, utility $U_k(\rho_k) = K \log(\rho_k)$.

State space and transition rates:

$U^{-1}_2(\hat{q}_2(n)) = \hat{\rho}_2(n)$

Irreducible set around $n=0$ is bounded $\Rightarrow$ Stability, Independently of $\lambda_k, \mu_k$
Fairness? Simulations: JAVA-based tool with random arrivals and workload, simulated dual congestion control.

TCP utility $U'_{TCP_k}(x) = \kappa_k x^{-2}$. User utility $U'_k(x) = \kappa' x^{-5}$. Emulates max-min fairness.
Fluid modeling of admission control.

Simulation results show admission control achieves optimal allocation, provided the offered loads \( \frac{\lambda_k}{\mu_k} \) are larger than the equilibrium fairshare \( \rho^*_k \).

We seek analytical proof, and also understanding of the non-greedy case.
Fluid modeling of admission control.

Try a large network asymptotic, scaling capacity and user demand curve.

\[ c^{(L)} = c_0 L, \quad U^{(L)}_k(\rho) = U^{(1)}_k \left( \frac{\rho}{L} \rho_0 \right). \]

Rescaled simulation plot \( \frac{n^{(L)}_k}{L} \)

Optimal point
Fluid limit.

\[ \dot{n}_k = \frac{\lambda_k 1}{\left[U_k^{-1}(\hat{\rho}_k(n)) > \hat{\rho}_k(n)\right]} - \mu_k \hat{\rho}_k(n) \]

For \( \frac{\lambda_k}{\mu_k} > \rho^*_k \) (optimal fairshare) fluid simulations converge to optimal point.

We can prove it in simple cases.

![Optimal point on graph]
Fluid limit for the case of non-greedy users:

\[ \dot{n}_k = \lambda_k \frac{1}{\{U_k^{-1}(\hat{\rho}_k(n)) > \hat{\rho}_k(n)\}} - \mu_k \hat{\rho}_k(n); \text{ assume a certain user has } \frac{\lambda_k}{\mu_k} < \rho^*_k \]

Example: \( c = 1 \), user 1 with \( \frac{\lambda_1}{\mu_1} = 0.3 \), greedy user 2.

\[ n = (n_1, n_2) \]

Conjecture (verified in simulations so far): converges to solution of \( \max \rho \sum_k \bar{U}_k(\rho_k) \), s.t. \( R \rho \leq c \), where \( \bar{U}_k(\bullet) \) corresponds to a demand curve saturated at rate \( \rho_k = \frac{\lambda_k}{\mu_k} \). Non-greedy user is protected.
Back to multipath, work in progress.

Suppose: edge router can choose in which path to route a new flow.
Given: prices $q_k^r$ of the various candidate routes, a natural policy is:

- Admit new connection $\Leftrightarrow \min_r q_k^r < U_k^r(\rho_k)$, where $\rho_k = \sum_r \rho_k^r$
- If admitted, select cheapest path.

Example (symmetric users):

![Diagram of network with three paths and three users with average load 300. Path 1 has cost C1=150, Path 2 has cost C2=120, and Path 3 has cost C3=90. Two users are shown: User 1 and User 2, each with an average load of 300.]
Multipath admission control
Simulations

Conjecture: converges to optimal multipath welfare allocation,

$$\max_{\rho} \sum_{k} U_k(\rho_k), \quad \text{subject to} \quad \rho_k = \sum_{r} \rho_k^r, \quad \sum_{k} \sum_{r \in l} \rho_k^r \leq c_l.$$
Conclusions and Future Work

- We studied two cross-layer resource allocation problems:
  I. Congestion control and multipath routing.
  II. Congestion control and admission control.
- Objective: welfare optimization. We designed decentralized control laws based on prices, achieve these equilibria.
- Dynamic analysis: local stability proofs in arbitrary networks, global results for simpler cases.
- Beware on simplistic models for delay!
- Simulation studies confirm and generalize the above theory.
- Future work:
  - Part I: Global proof in arbitrary networks, loss-based implementation.
  - Part II: Stability proofs for fluid limit model, ns2 implementation.
  - Combination: multipath admission control.