Distributed Interference Pricing for OFDM Wireless Networks with Non-seperable Utilities

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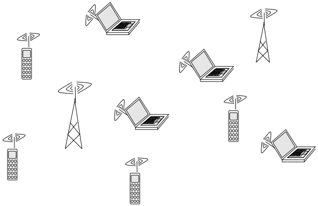
Spectrum Sharing



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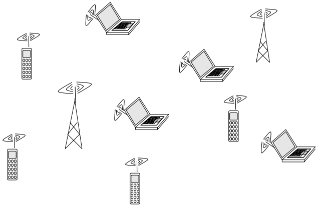

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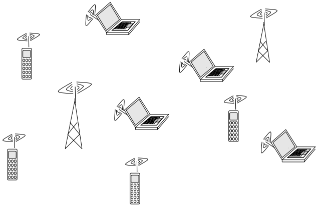
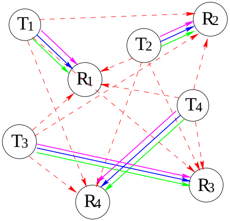


- Interference management essential when multiple transmitters share wireless spectrum.
- In OFDM systems: power allocation (over frequency) aids in this.
- Centralized power allocation well-studied (e.g. cellular downlink)
- Interest here is on distributed approaches (e.g. ad hoc networks, mesh networks, etc.).
  - Want to efficiently allocate power with limited information exchange.

OFDM Network model



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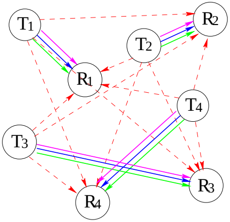

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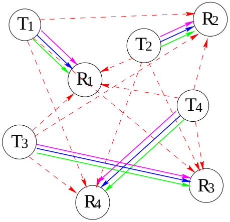

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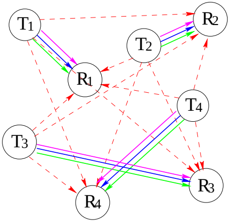

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- Each transmitter uses same bandwidth of B Hz.
- Band divided into K equal-sized subchannels.
  - OFDM tones or groups of tones.
- Each subchannel modeled as Gaussian interference channel with flat-fading.
  - H_{ij}^{k} = gain of channel k between user i and j.

Power Allocation

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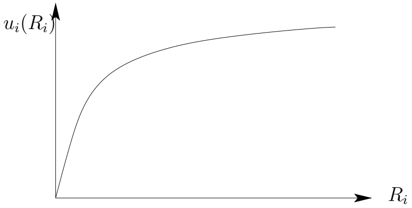
- Each user's rate depends on allocation of all users $\mathbf{P}$.
- Total rate for user i:

$$R_i(\mathbf{P}) = \frac{B}{K} \sum_{k=1}^{K} \log(1 + \gamma_i^k(\mathbf{P})),$$

where

$$\gamma_i^k(\mathbf{P}) = \frac{p_i^k H_{ii}^k}{n_0^k + \sum_{j \ne i} p_j^k H_{ji}^k}.$$

User Preferences



- All users are rate-adaptive with *elastic* demands.
- User's QoS preferences given by a utility function

$$u_i(R_i(\mathbf{P})).$$

  - $u_i(\cdot)$ is increasing, twice differentiable, strictly concave function of R_i.
  - in general not concave in $\mathbf{P}$.

Performance Objective

$$\max_{\mathbf{P}} \sum_{i=1}^{M} u_i(R_i(\mathbf{P}))$$

$$s.t. \sum_{k=1}^{K} p_i^k \leq P_i^{max} \ \forall i,$$

$$p_i^k \geq 0 \ \forall i, k.$$

- Note not separable across users.

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- Here study case where utility is not carrier separable.

ADP Algorithm [HBH06]

- Each user i announces one "interference price" on each subchannel k:

$$\pi_i^k = -\frac{\partial u_i(R_i(\mathbf{P}))}{\partial I_i^k}.$$

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- User i updates power $\mathbf{p}_i = \{p_i^1, \ldots, p_i^K\}$ to maximize surplus:

$$s_i(\mathbf{p}_i, \mathbf{P}_{-i}) = u_i(R_i(\mathbf{P})) - \sum_{k=1}^{K} p_i^k \left(\sum_{j \neq i} \pi_j^k H_{ij}^k \right).$$

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- Only need to know "adjacent" channel gains (H_{ij}) and interference prices.

Single-channel analysis

- For class of utility functions, in [HBH06] the ADP algorithm is shown to globally converge to the optimal power allocation.
  - Convergence typically much faster than gradient-based methods.
- Proof based on viewing ADP as best response updates in a game.
  - Under given conditions, game is "supermodular".

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- Proof based on viewing ADP as best response updates in a game.
  - Under given conditions, game is "supermodular".
- For OFDM model with non-separable utilities, this analysis breaks down.
- But in numerical example ADP still appears to converge.
- This work attempts to understand this.

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  2. Each user changes power allocation to be a convex combination of best response and current allocation.

$$\mathbf{p_i}(n+1) = (1-\alpha_i)\mathbf{p_i}(n) + \alpha_i \arg\max_{\mathbf{p_i}\in\Pi_i} s_i(\mathbf{p_i};\mathbf{P_{-i}}(n))$$

for given "step-size" $\alpha \in (0,1)$.

Convergence

Proposition There exists an $\alpha_i > 0$ for each user i so that the total utility under the modified ADP algorithm converges monotonically to a fixed point satisfying the KKT conditions of the total utility maximization problem.

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- analytical bounds on α_i given but appear to be very loose.

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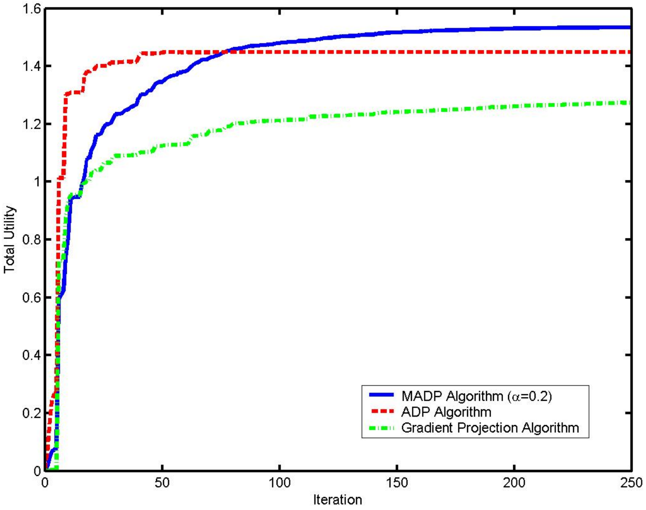
- Then can use Descent Lemma to lower bound change in total utility in terms of gradient of Utility and Lipschitz constant.
- Gradient of total utility can be expressed in terms of interference prices and linearized utility.
- This plus some algebra can show that for small enough step-size total utility must increase.

Numerical Examples

- 10 users/10 subchannels
- Transmitters/receivers randomly placed in a square area.
- Subchannel gains i.i.d. Rayleigh with distance-based mean.
- $u_i(R_i) = 1 - \exp(0.1R_i)$.
- Three algorithms: ADP, modified ADP, and distributed gradient algorithm.

Example Convergence



Convergence comparison

Threshold	MADP	ADP	Gradient
1%	72.92	39.71	> 98.99
0.5%	102.71	46.32	> 147.42
0.1%	180.02	61.6	> 230.36

- Averaged over 100 realizations.
- Depends on step-size(s) used, but similar trends for other values.

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Conclusions

- Presented algorithms for distributed power control in OFDM networks..
- Can achieve good performance with only local channel information and limited information exchange.
  - required information = "interference prices".
- Convergence proof for modified ADP algorithm.
  - "un-modified version" performs better in examples, but no convergence proof.
- Also ADP algorithms seem to converge to "better" optima?