Resource and Power Costs in Dynamic Spectrum Allocation

Joydeep Acharya
Roy D. Yates
Wireless Information Network Laboratory (WINLAB)
Rutgers University New Jersey, USA

CISS, Princeton University
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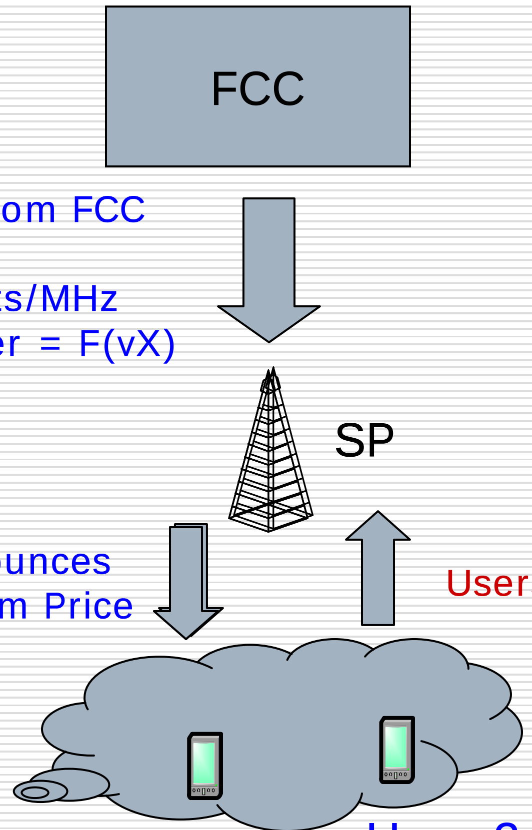
The Spectrum Debate

- What everyone agrees on:
  - Spectrum use is inefficient
  - FCC licensing has yielded false scarcity

- Proposed Solutions
  - Open Access (Commons)
  - Spectrum Property Rights
    - **Dynamic Spectrum Allocation (DSA)**

Proposed DSA Model



Cost Model for the SP

- Spectrum Cost
  - $C(X) = CX$, X: sum of spectrum from all users
  - Constant C depends on
    - Geographical region, urban/rural

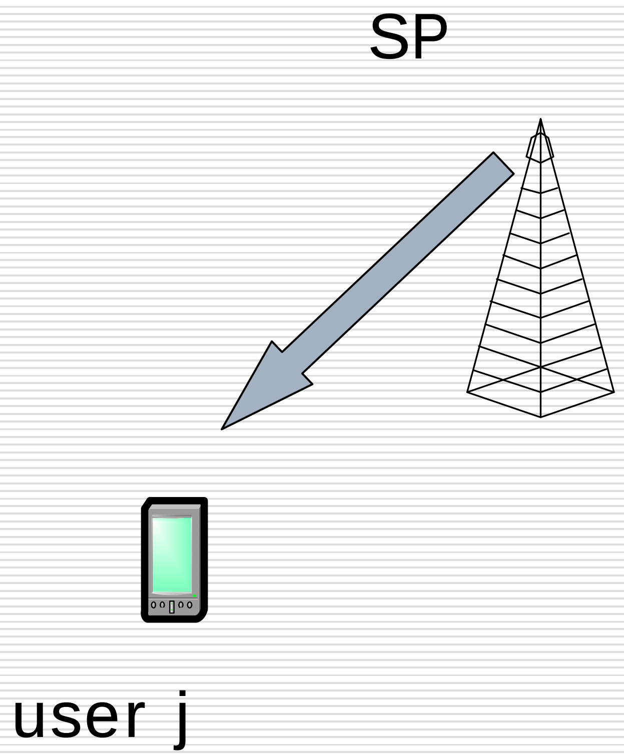
- Power Cost
  - Transmit power = νX
  - $F(\nu,X) = T\nu X$
  - Constant T may depend on
    - Presence of other providers in band ‘X’

User j: spectrum & rates

- h_j = downlink gain
- Spectral efficiency

$$K_j = \log(1+\nu h_j/N_0)$$

- x_j = allocated spectrum
- Rate $\quad R_j = K_j x_j,$

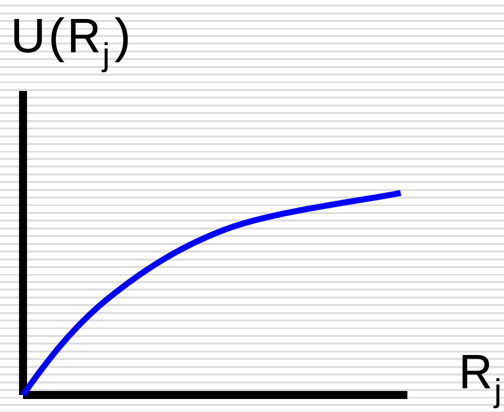


User j: utility functions

- Logarithmic

$$U(R_j) = \log(1+R_j)$$

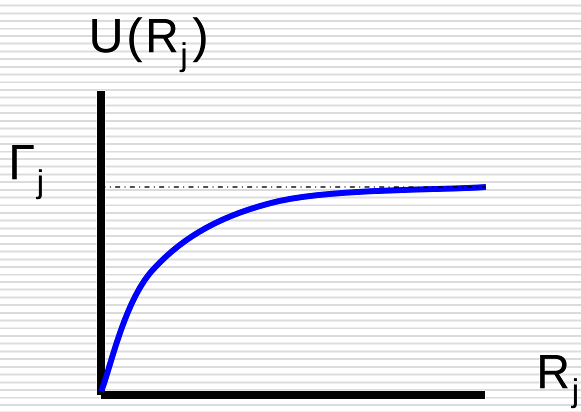
  - Elastic data application (large file download)



- Exponential

$$U(R_j) = \Gamma_j[1-\exp(-R_j/\Gamma_j)]$$

  - Application with target rate Γ_j



Objective of the SP and Users

- SP maximizes its net revenue
  - Expense: Spectrum purchase and transmit power
  - Income: Charges the users
- Users maximize their utility minus cost
  - Expense: Charge paid to the SP
  - Gain: Increase in utility due to spectrum

1 SP = Monopolistic Pricing

- SP charges a two part tariff from users
  - μ = variable spectrum usage fee **per Hz**
  - κ = fixed connection price
- Users decide how much spectrum to buy
  - X = Aggregate spectrum of all users
- SP buys spectrum X from FCC,
  - SP Cost: $C_e X = (C+T\nu)X$
- SP optimizes prices (μ,κ) to maximize profits

Technology, Pricing and Spectrum Costs

- Higher power spectral density ν
  - Raise $\nu \Rightarrow$ Raise spectral eff, $K_j = \log(1+h_j\nu/N_0)$

    $\Rightarrow$ improved transceiver technology

- Optimal technology parameters K_j depends
  - Spectrum cost C and power cost T
    - Higher C (relative to T) may lead to better technology

- Price paid by users
  - (μ,κ) depend upon C, T and ν
  - High prices $\Rightarrow$ user can refuse service

Related Work

- [Varian] Congestion pricing in wireline networks
- [Oi] Monopolistic two part tariffs
  - Fixed Connection Fee
  - Variable Usage Cost
- [Ozdaglar et al] Marginal user principles for resource allocation

System Model: user optimization

- x_j, $U_j(x_j,\nu)$: Spectrum and utility of user j
- Payment of user j is

$$\rho(x_j) = \begin{cases} \mu x_j + \kappa, & x_j > 0 \\ 0, & x_j = 0. \end{cases}$$

Users declines service due to high price

- Optimization of user j

How much spectrum to obtain

$$\max_{x_j} U_j(x_j,\nu) - \rho(x_j)$$

$$\Rightarrow \quad \mu = \partial U_j(x_j,\nu)/\partial x_j$$ [Demand Function]

System Model: SP optimization

- SP sets (μ, κ)
- *L* users request spectrum $X = \sum_{j=1}^{L} x_j$
- Optimization by SP:

What prices to set

$$\max_{\mu,\kappa} \mu X + \kappa L - (C + T\nu)X$$

The Marginal User Principle

- An example of a Stackelberg Game
  - Users follow prices set by the SP
  - SP can anticipate the user response

- SP increases (μ,κ) until one (marginal) user has zero net utility

  - If price is increased any further

    $\Rightarrow$ marginal user will refuse service

    - The marginal user is indifferent to the service

How to choose (μ,κ)

- Depends on the details
- Spectrum usage μ depends on spectrum cost $C+T\nu$
- Once μ has been decided
  - κ chosen to extract surplus from marginal user
  - Surplus of user j = $U_j(x_j,\nu) - \mu x_j$

The Marginal User

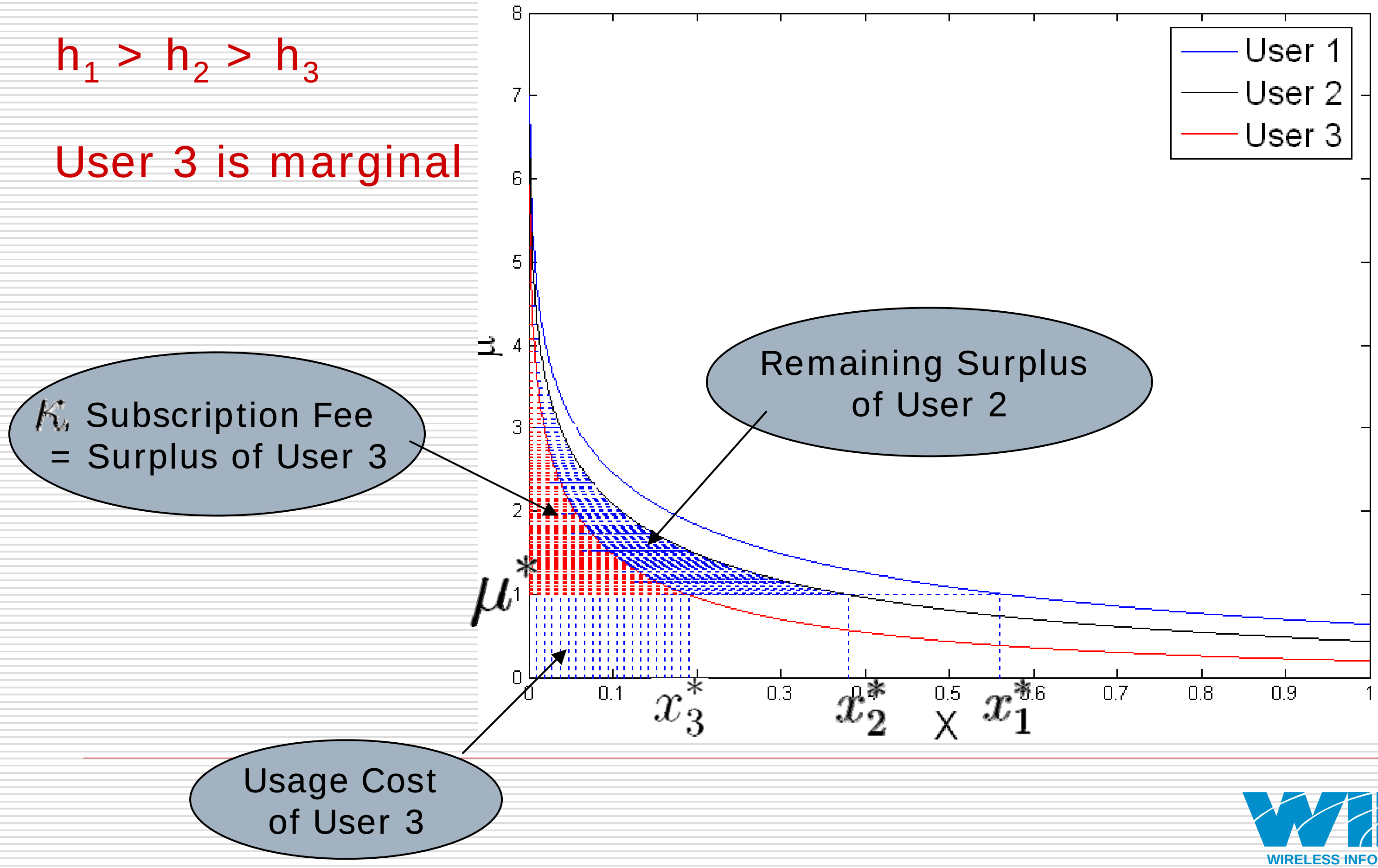
- Recall $R_j = K_j x_j = \log(1+\nu h_j/N_0)$
- Logarithmic utilities
  - Weakest link gain user is marginal
- Exponential utilities
  - $\Gamma_j = \Gamma$ for all users, weakest link user is marginal
- Marginal user achieves least rate for given spectrum

Usage Cost & Connection Fee

Demand Functions for 3 users

$h_1 > h_2 > h_3$

User 3 is marginal



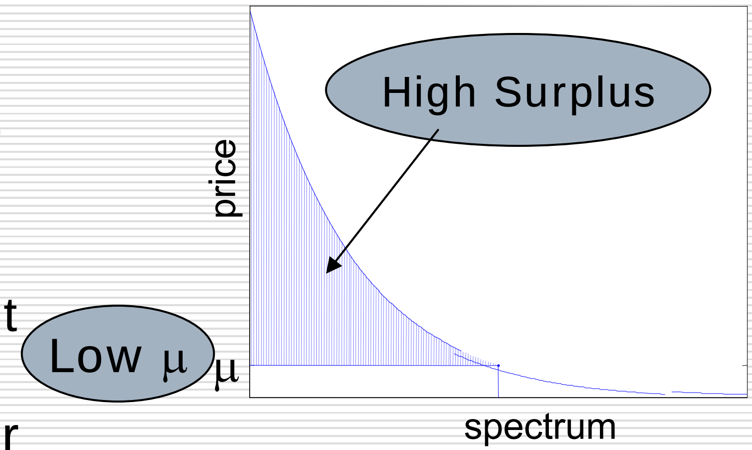
Users' spectrum cost

- Effective Spectrum cost to SP: $C_e = C + T\nu$
- Users' usage cost = μ, connection fee = κ
- **Logarithmic utilities**
  - Spectrum overpriced: $\mu > C_e$ always
  - Utility grows unbounded $\Rightarrow$ always demand
  - Can't arbitrarily overprice
    - Derived upper bound, function of link gains, C_e and L

Users' spectrum cost

- **Exponential utilities**
    - Lots of spectrum, low $\mu \Rightarrow$ high user surplus
  - Low $C_e \Rightarrow \mu < C_e$ (spectrum underpriced)
    - SP lowering $\mu \Rightarrow$ users buy lots of spectrum
    - Utility saturates at target rate for high spectrum
      - Reduced demand for add'l spectrum
  - SP: Hikes connection fee κ to extract user surplus

Demand Function



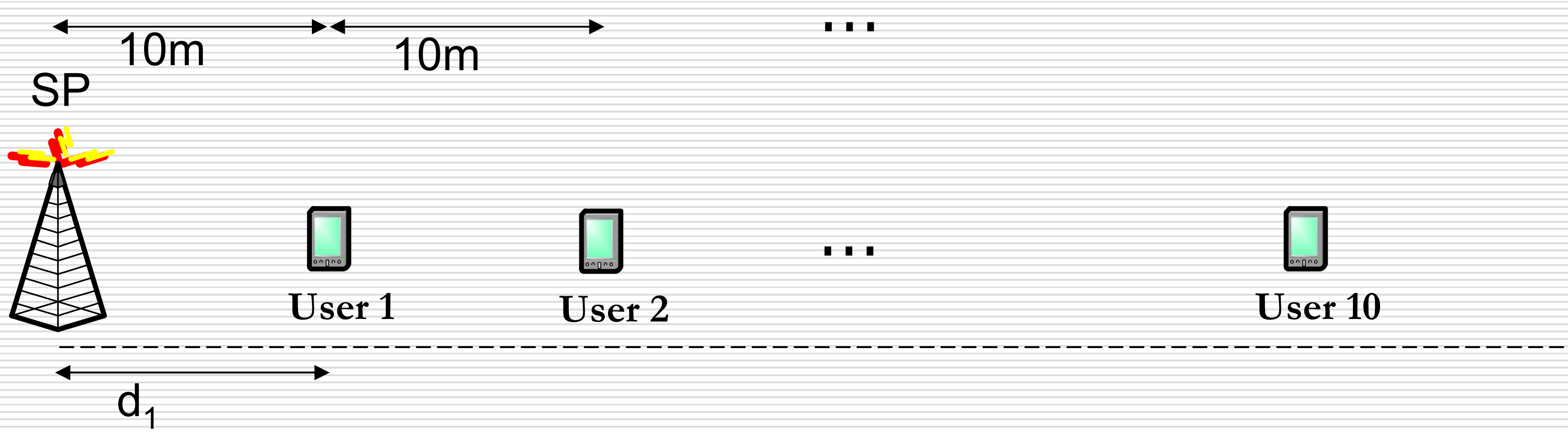
Elasticity of Demand (Alternate Interpretation)

- Ratio of % change in demand to % change in price

$$\epsilon = -\frac{\mu}{X}\frac{\partial X}{\partial \mu} > 0$$

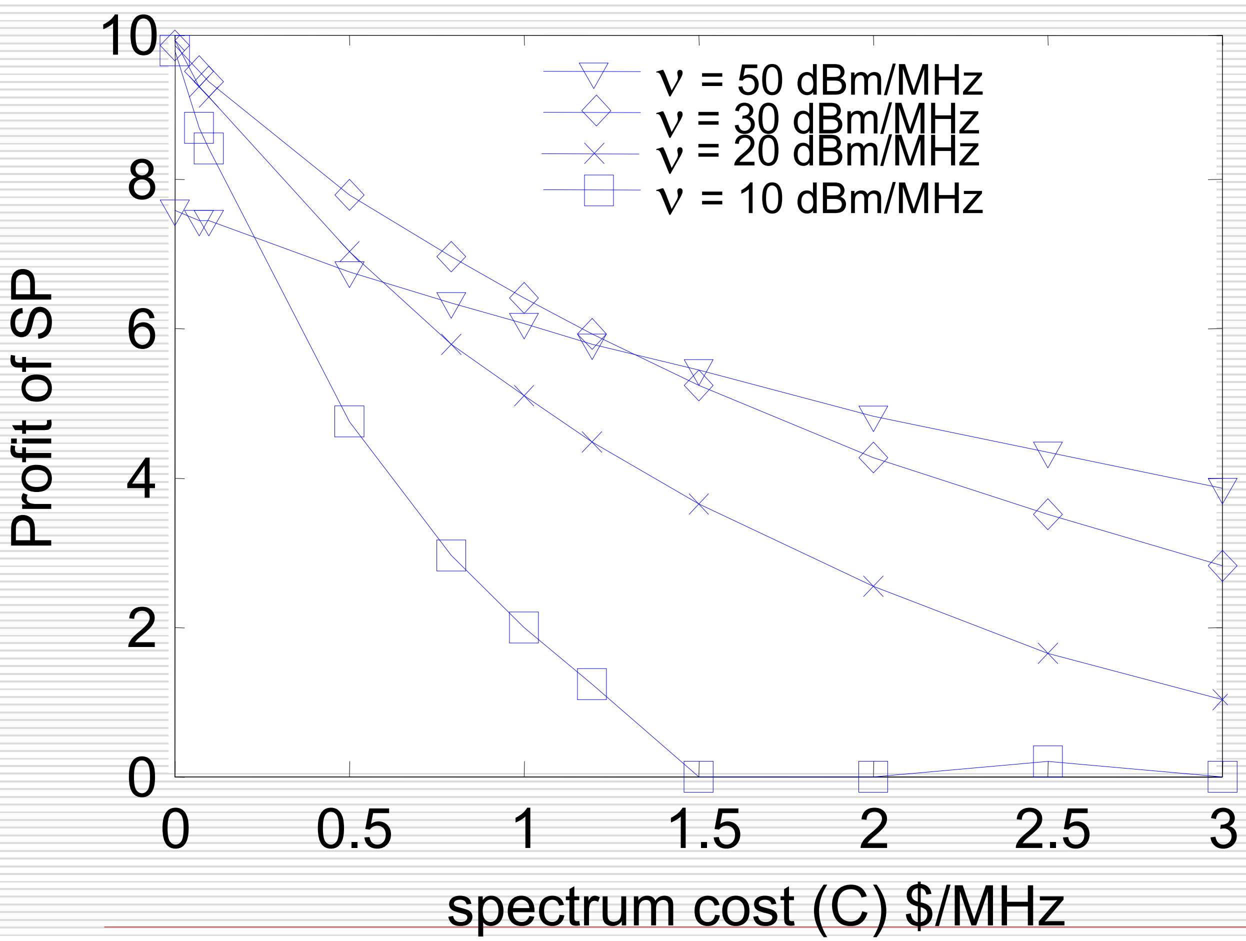
- Logarithmic Utilities: Elastic demand ($\varepsilon > 1$) always
  - When price is increased, % fall in demand is higher
  - SP can't arbitrarily overprice spectrum
- Exponential Utilities: Inelastic demand ($\varepsilon < 1$) for low μ
  - Low μ: Enough spectrum for users to be rate saturated
  - Price changes in this regime, % change in demand is less

Numerical Results



- 1 SP, 10 users, linear network
- Each user placed at uniform distance of 10m from each other
- d_j : Distance between SP user j, $d_j = 10j$
- N_0 : (Noise Spectral density) = -174 dBm/Hz
- Channel model is path loss based (COST-231)
- Link loss/Hz, $h_j = -31.5 - 35*\log(d_j) - \log(N_0)$ dB
- Other topologies considered: result trends are same

SP Profit vs Spectrum Cost C



Exponential Utilities
Target Rate = 1 Mbps
Power Cost, T=10
Total cost C_e = C+Tν

- High C
  C_e = C+Tν ~ C
  User Utility ↑ as ν ↑
  SP incentive: High ν

- Low C
  C_e = C+Tν ~ Tν
  SP incentive: Low ν

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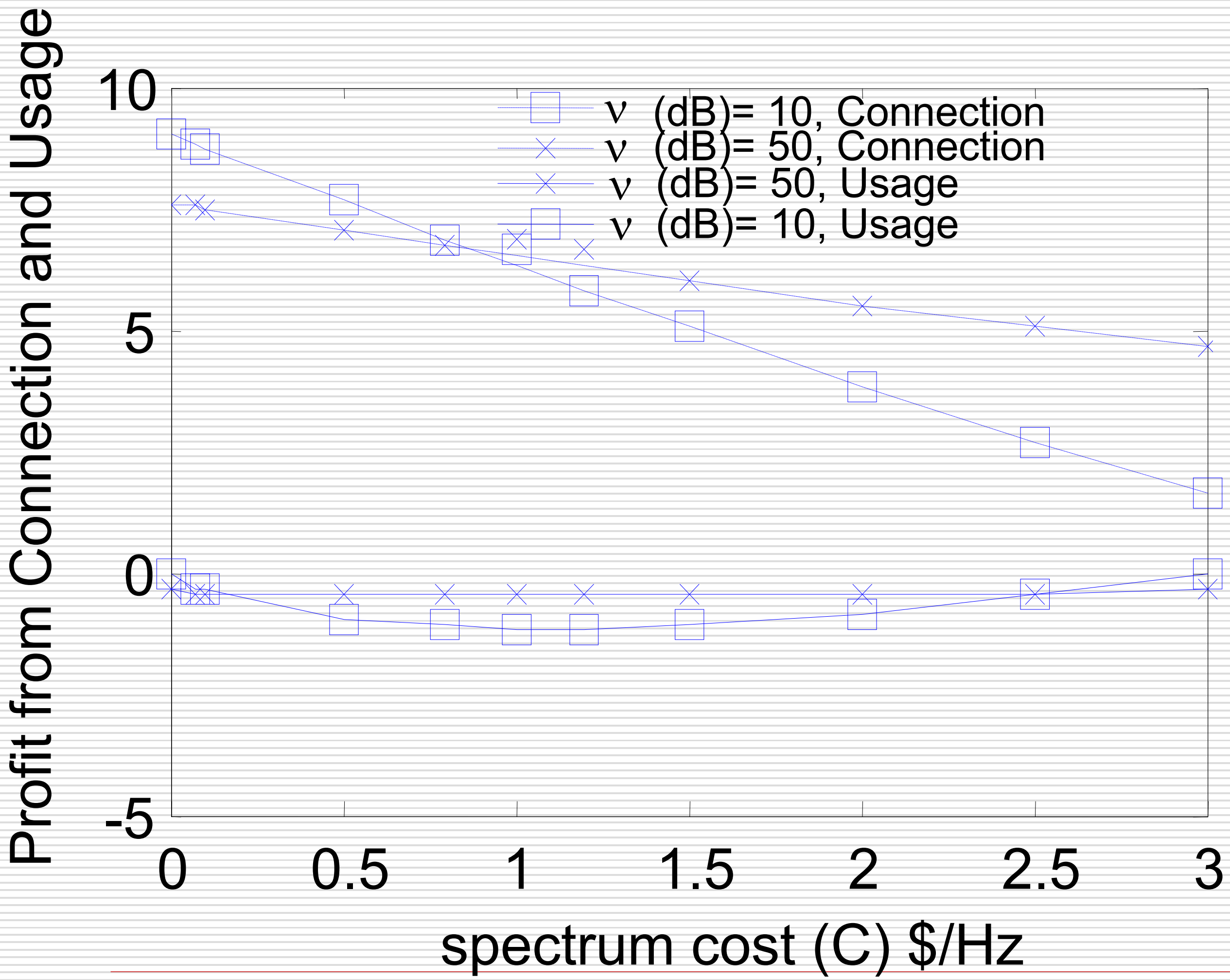
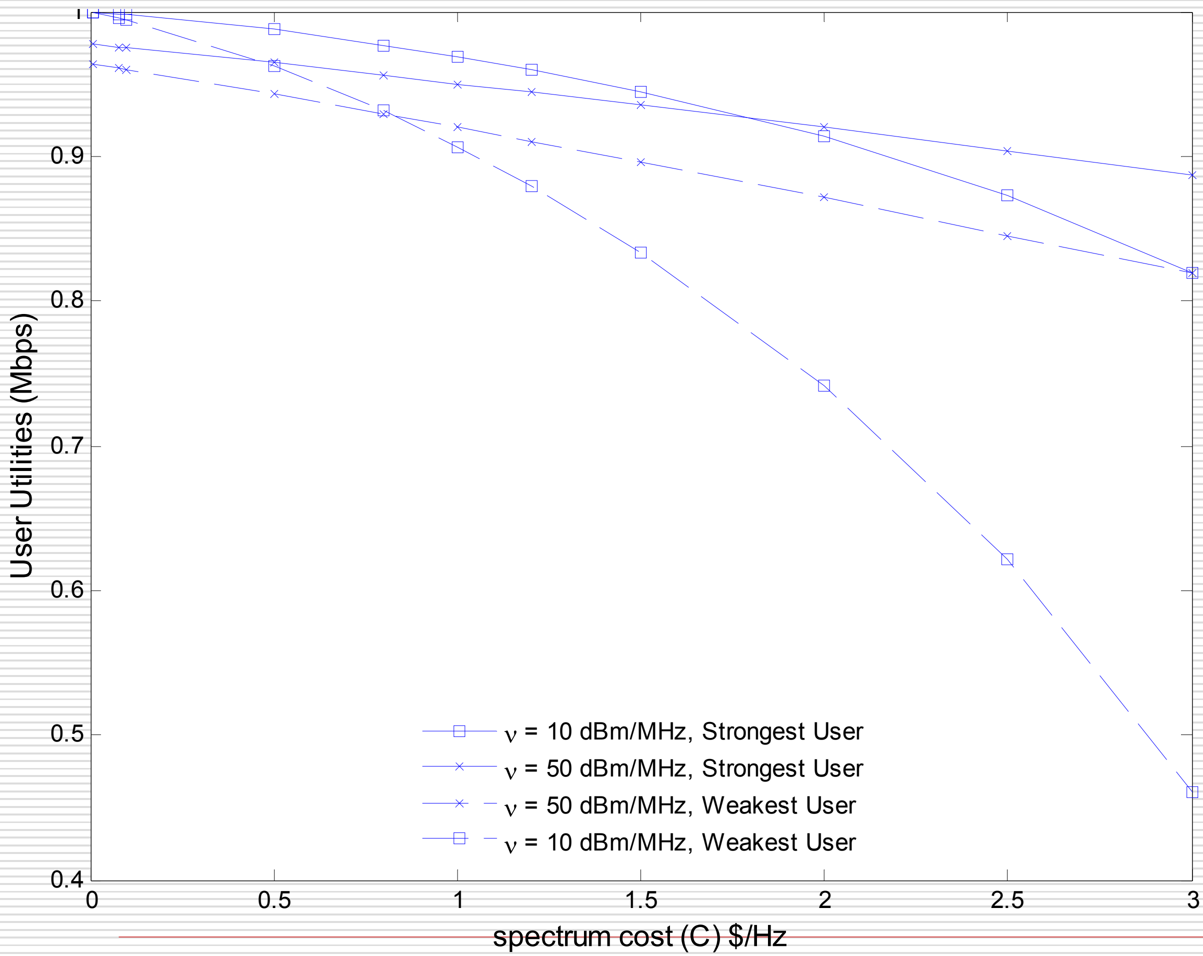
Breakdown of SP Profit from Usage and Connection Fee



SP Profit = $(\mu - C_e)X + \kappa L$
(usage) (connection)
(P_U) (P_C)

- P_C dominates
- $P_U < 0 \Rightarrow$ underpricing
- When ν is high
  $C_e = C + T\nu \sim T\nu$
  Less variation with C

User Net Utility

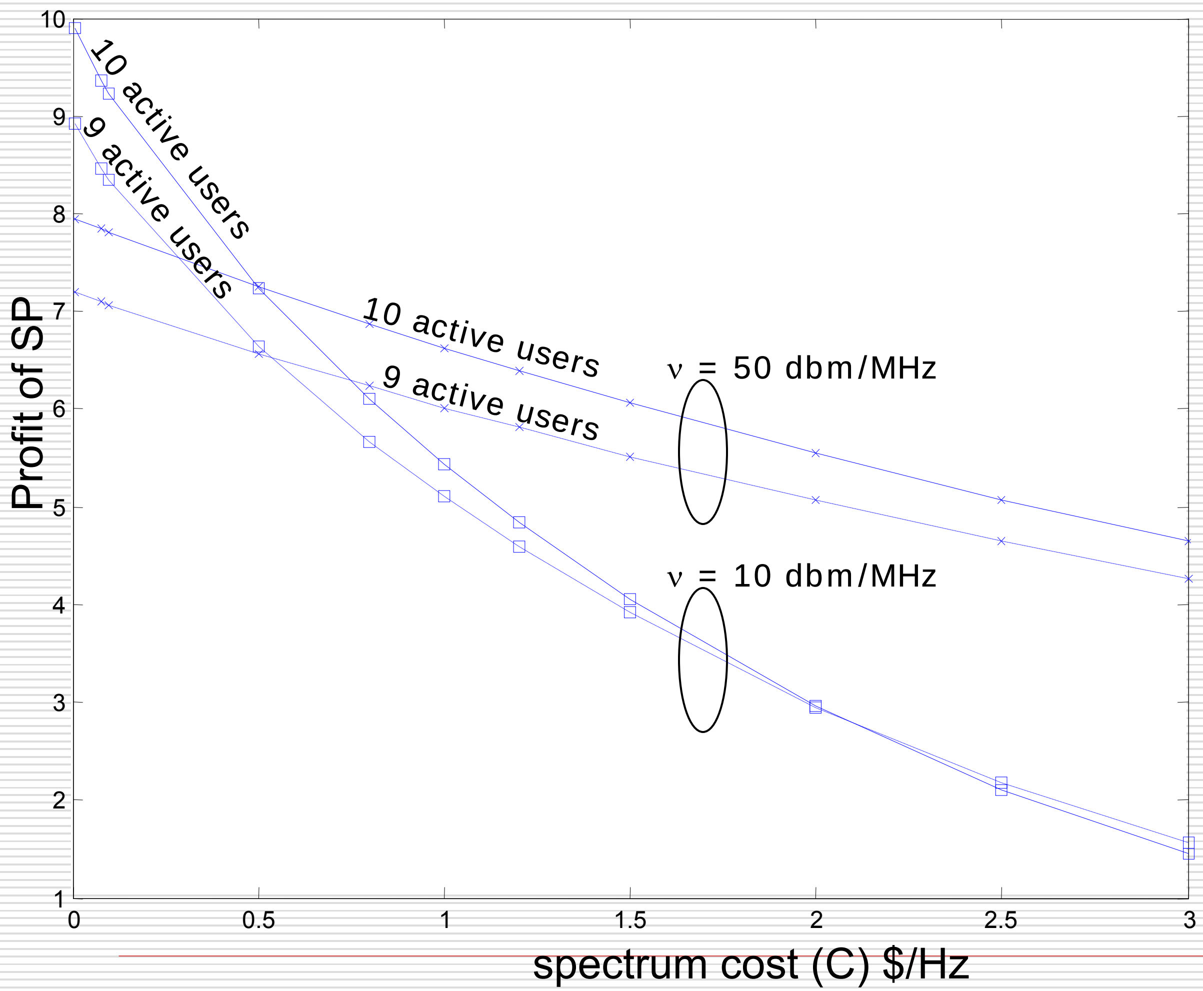


Exponential Utilities
Target Rate = 1 Mbps
Power Cost, T=10
Total cost $C_e = C + T\nu$

- Low C and high ν
  $C_e = C + T\nu \sim T\nu$
  SP costs rise with ν
  For high ν
  - SP buys less spectrum
  - User net utility reduced

- High C and high ν
  $C_e = C + T\nu \sim C$
  SP costs indifferent to ν
  User utility ↑ as ν ↑

Discarding users for profit



Initially L=10 users
User 10 is marginal user

SP raises prices ...
User 10 refuses service
User 9 is now marginal

- As C ↑, discarding users becomes profitable (esp. if ν is low)

Observations

- Summary
  - Usage price depends on SP's spectrum cost
  - Connection price depends on the marginal user surplus
  - Lots of Details
    - Spectrum is over/under-priced based on user application, elasticity of demand
    - SP profit breakdown (usage+connection) depends on application
  - Service Provider Profits ~ Marketing
    - Surveys to learn user utilities
    - Extracting the marginal user surplus
- Future Issues
  - Multiple SPs and SP competition
    - Effect on user surplus, SP profits, technology deployment?

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