Coalitional Games in Receiver Cooperation for Spectrum Sharing

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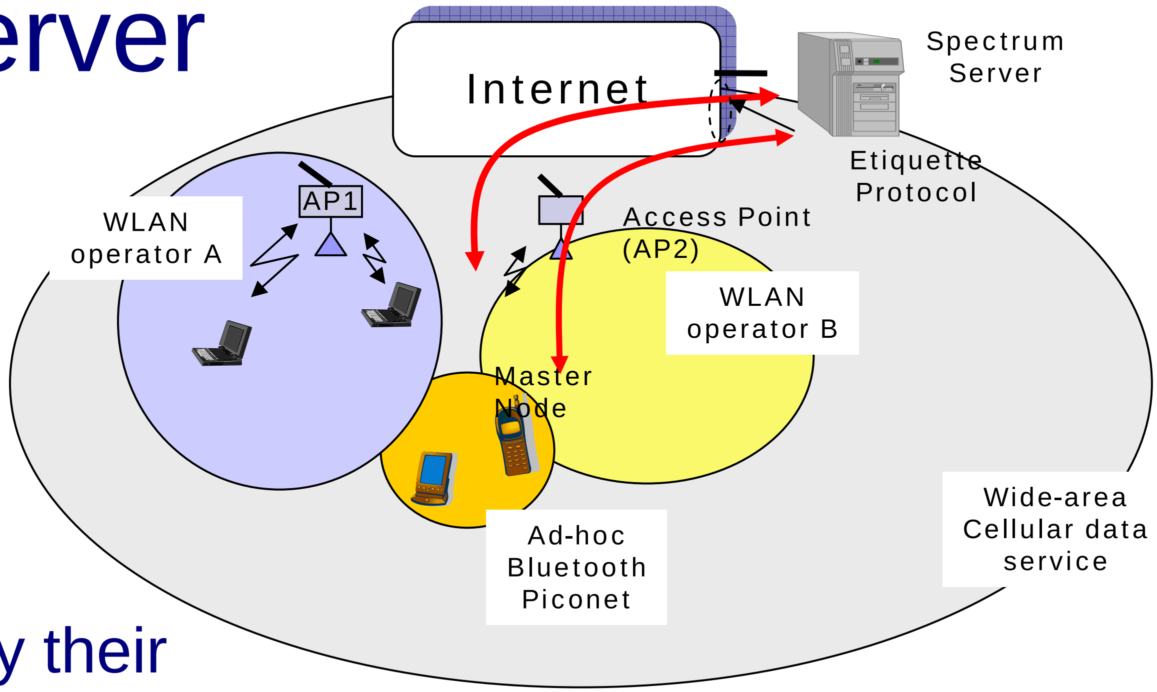
Cooperation in wireless networks

- When wireless links cooperate, does it always result in gains for each cooperating link?
  - Tx cooperation, Rx cooperation
  - Role of 'cognitive radio' in cooperative communications
  - Do cooperating radios have incentives to "break" away?
- When "different" types of devices/networks coexist, Tx cooperation may not always be possible.
  - Rx cooperation may be the only feasible way
  - Central entity required → Spectrum server [Ileri & Mandayam 2005], [Raman, Yates & Mandayam, 2005]

Spectrum Server

- Receivers belonging to disparate links connect to the spectrum server.



- Receivers may relay their received signals to spectrum server for joint decoding in addition to seeking spectrum information.

Coalitions for Spectrum Sharing

We study cooperation between links under the framework of coalitional game theory

- Receiver cooperation in an interference channel (IC).
- Multiuser detection (MUD) in a multiple access channel (MAC).
- Transmitter + Receiver cooperation in an IC.

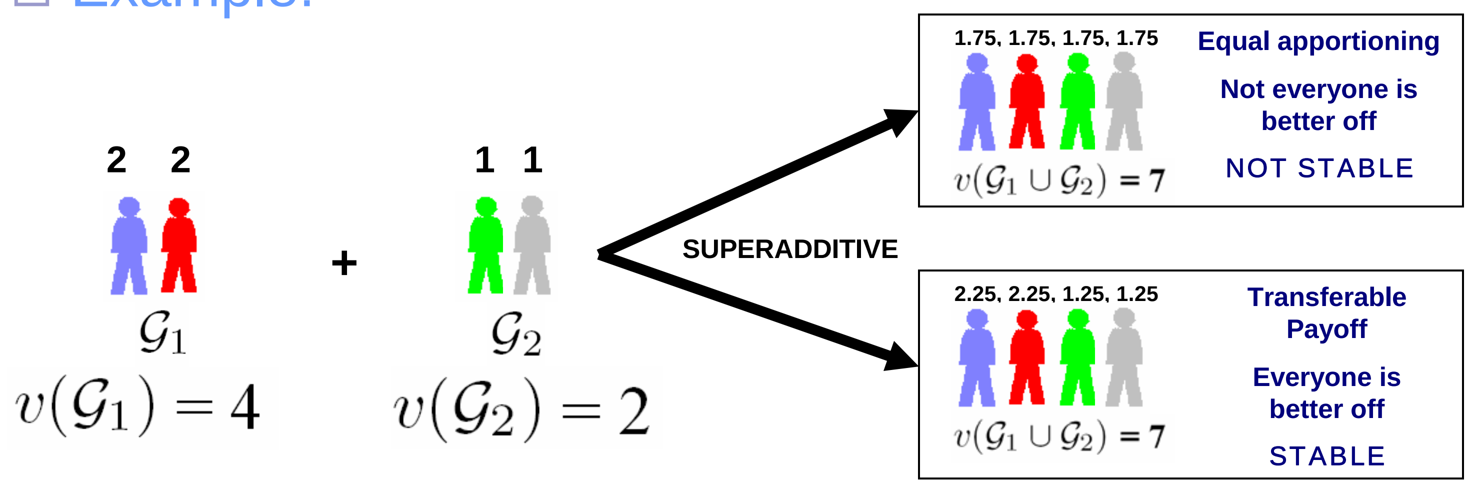
A game-theoretic look at the Gaussian multiaccess channel [La-Anantharam, 2003]

Coalitional Game Theory - Overview

- A coalitional game with transferable payoff $\langle \mathcal{S}, v \rangle$
  - Finite set of links $\mathcal{S}$
  - Value function: $v : \mathcal{G} \rightarrow \mathbb{R}, \forall \mathcal{G} \subseteq \mathcal{S}$
- Payoff: share of the value $v(\mathcal{G})$ to each link
- Characteristic function form game: $v(\mathcal{G})$ is unaffected by “strategy” of members not in coalition $\mathcal{G}$.

Coalitional Game Theory - Overview

- When $v(\mathcal{G})$ flexibly shared between cooperating links $\rightarrow$ coalitional game with transferable payoff.
- $\langle \mathcal{S}, v \rangle$ is a superadditive game if for any two disjoint coalitions: $v(\mathcal{G}_1 \cup \mathcal{G}_2) \geq v(\mathcal{G}_1) + v(\mathcal{G}_2)$
  - Example:



Coalitional Game Theory - Overview

- $\mathcal{G}$ -feasible payoff vector $\underline{x}_{\mathcal{G}} = (x_m)_{m\in\mathcal{G}}$ $\sum_{m\in\mathcal{G}} x_m = v(\mathcal{G})$
- $\mathcal{S}$ -feasible payoff vector is called a *feasible payoff profile*
- The set of stable coalitions form the *core*
- Core $C(v)$ of $\langle\mathcal{S}, v\rangle$ for all $\mathcal{G}$-feasible vectors $\underline{y}_{\mathcal{G}} = (y_m)_{m\in\mathcal{G}}$

$$C(v) = \{\underline{x}_{\mathcal{S}} : \nexists\, \mathcal{G} \subset \{1, \ldots, M\} \;\; s.t. \;\; y_m > x_m, \forall m \in \mathcal{G}\}$$

- Under superadditivity and transferable payoffs

$$C(v) = \left\{\underline{x}_{\mathcal{S}} : \sum_{m\in\mathcal{G}} x_m \geq v(\mathcal{G}), \;\; \forall \mathcal{G} \subset \mathcal{S}\right\}$$

Receiver cooperation in an interference channel

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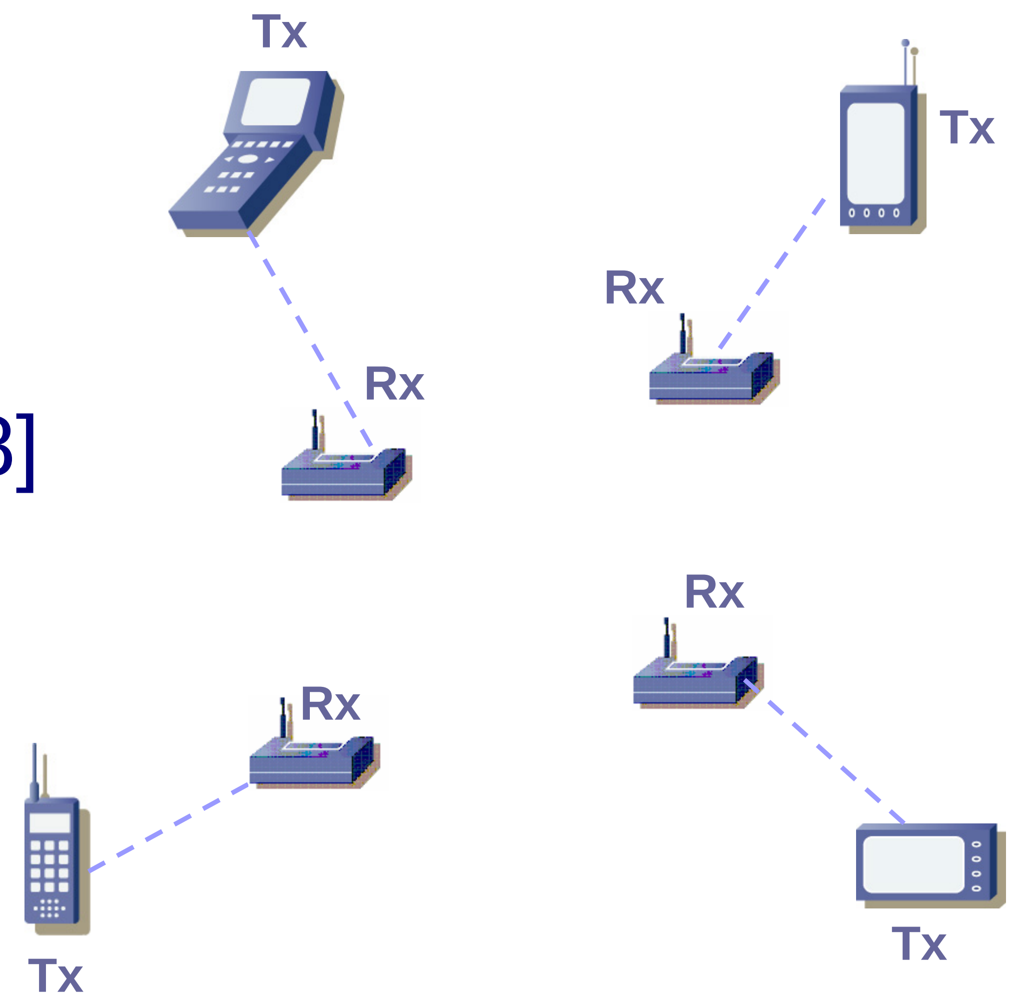
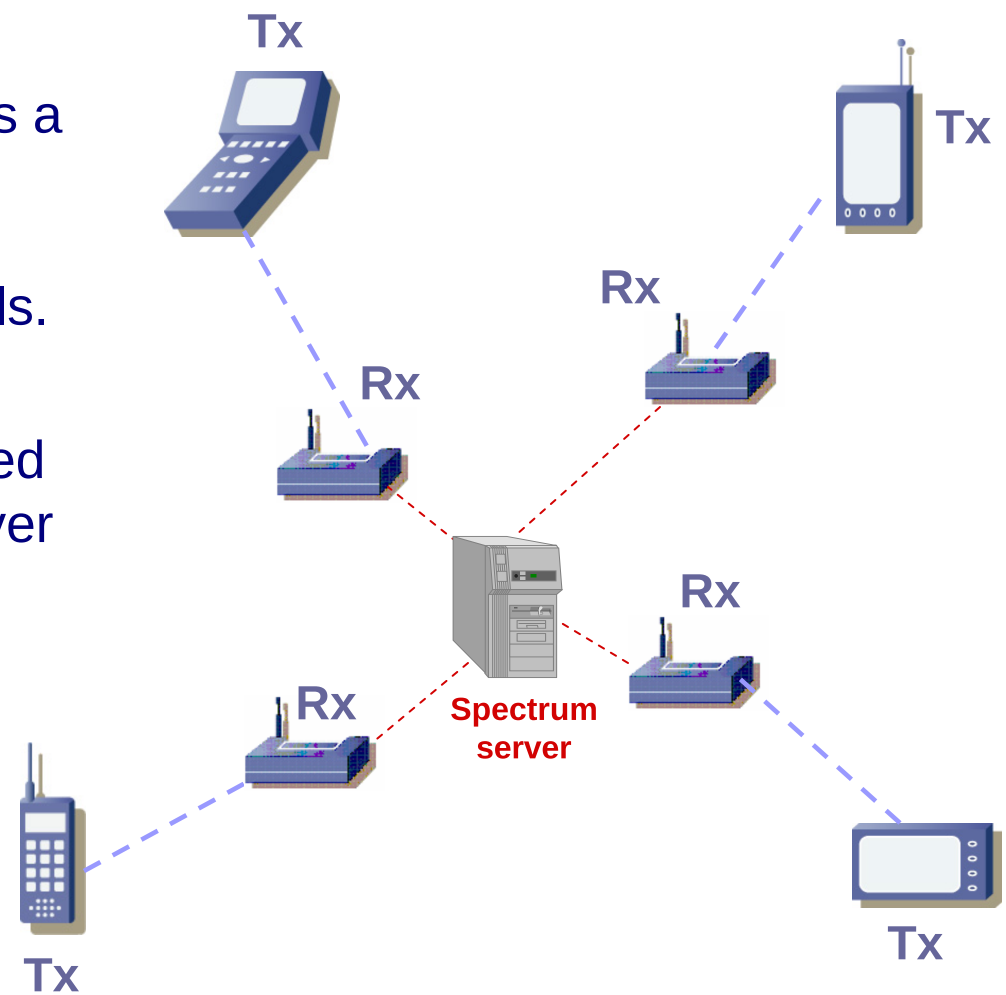
Rx cooperation in an IC

- Interference Channel (IC): Network of M transmit-receive links [Carleial, ’78]



Rx cooperation in an IC

- A spectrum server serves as a central entity that enables disparate devices to jointly decode their received signals.
- Receivers relay their received signals to the spectrum server which then jointly decodes them.



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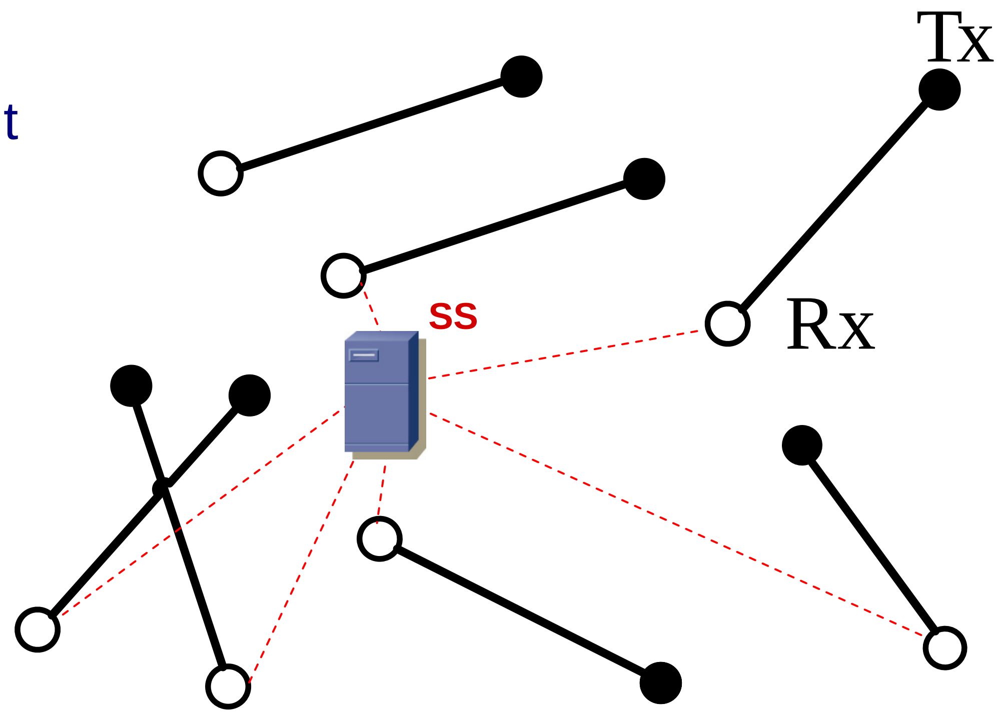
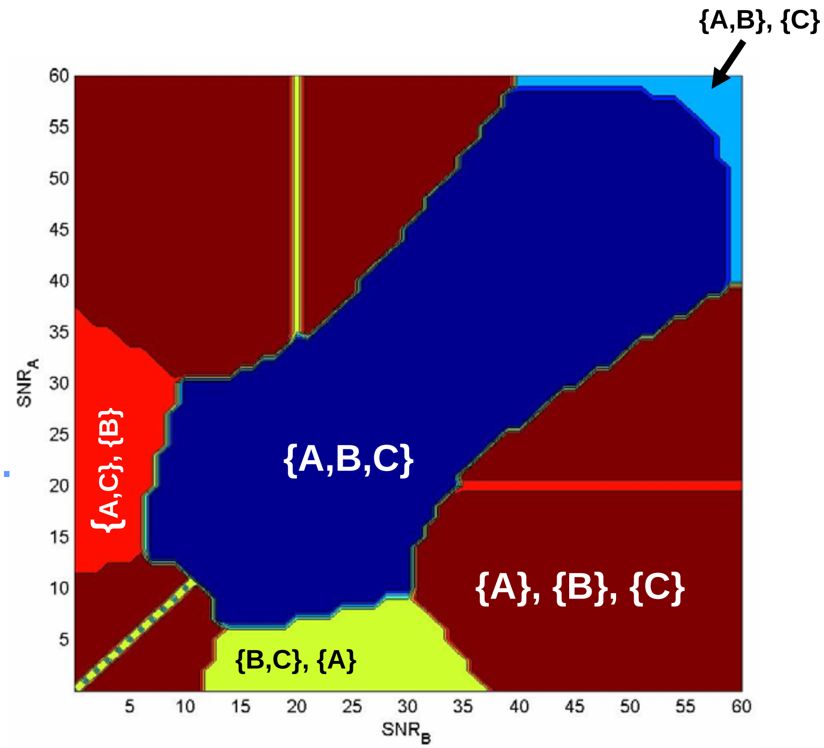
Rx cooperation in an IC

- Receiver cooperation converts the interference channel into a single-input multiple-output multiple access channel **(SIMO-MAC)**
- How to apportion value amongst links in a coalition?



Why should receivers join coalitions ?

- Is the 'grand coalition' (GC) always stable?
- Example:
  - 3 users A,B and C in a MAC.
  - Recd. SNR of C is fixed.
  - With equal apportioning of $v(\mathcal{G})$, grand coalition is not always stable.
  - Grand coalition not the obvious solution.
  - → Depends on apportioning



Stable coalition structures with equal splitting of rate within coalitions when 3 users A, B and C are jointly decoded and the Rx SNR of user C is fixed at 20 dB

System Model of IC

- IC with additive white Gaussian noise and flat fading
- Set of links $\mathcal{S} = \{1, 2, \ldots, M\}$
- i^{th} link input/output: X_i , Y_i

$$Y_i = \sum_{k=1}^{M} h_{k,i} X_k + Z_i, \qquad Z_i \sim \mathcal{CN}(0, 1)$$

- $h_{i,k}$: fading gain between i^{th} xmitter and k^{th} receiver
- Power constraint at each transmitter:

$$E|X_i|^2 \leq P_i \quad i \in \mathcal{S}$$

- Transmitter of each link uses Gaussian codebooks

Rx Cooperation Coalitional Game

- Receivers in a coalition jointly decode received signals.
- Cooperation turns IC into a SIMO-MAC.
- Signals from links not in coalition treated as interference by the coalition.
- Model coalitional game with transferable payoffs.
- Use coalitional game theory to understand likely coalitions formed and their *stability.*
- Stability: Links in a coalition do not have rate incentives to leave the coalition (core).

Receiver Cooperation in an IC

- A coalition of links in $\mathcal{G} \subseteq \mathcal{S}$ form a Gaussian SIMO-MAC of $|\mathcal{G}|$ transmitters and a $|\mathcal{G}|$ –antenna receiver
- $\underline{R}_{\mathcal{G}} = (R_i)_{i \in \mathcal{G}}$ is a vector of rates for links in $\mathcal{G}$
- $C_{\mathcal{G}}$: Capacity region of a $|\mathcal{G}|$ - link Gaussian SIMO-MAC

$$C_{\mathcal{G}} = \left\{ \underline{R}_{\mathcal{G}} : \sum_{k \in A} R_k \leq I(X_A; Y_{\mathcal{G}} | X_{\mathcal{G} \setminus A}); \forall A \in \mathcal{G} \right\}$$

- Value $v(\mathcal{G})$ of a coalition: maximum sum-rate achieved by links in $\mathcal{G}$: $v(\mathcal{G}) = \max_{\underline{R}_{\mathcal{G}} \in C_{\mathcal{G}}} \sum_{i \in \mathcal{G}} R_i = I(X_{\mathcal{G}}; Y_{\mathcal{G}})$
- Dominant face of the capacity region $C_{\mathcal{G}}$ is given by $\sum_{i \in \mathcal{G}} R_i = v(\mathcal{G})$ and is denoted by $D(C_{\mathcal{G}})$.

Rx Cooperation Coalitional Game

- Is **superadditive!**
- Since it is also a **transferable payoff game,** then:

Theorem 1: The grand coalition (coalition of all links) maximizes spectrum utilization for the receiver cooperation IC coalitional game.

Theorem 2: The core of the receiver cooperation IC coalitional game is non-empty. In fact, every point on the dominant face $D(C_S)$ of the capacity region C_S of the grand coalition belongs to the core.

- Core is **non unique** → Fairness of payoff profiles in the core?

Fair Rate Allocations

- With transferable payoff, what is a fair allocation of rates to the links?
  - Can we attribute fairness criteria to points on the dominant face?
- Two bargaining solutions proposed
  - Nash bargaining solution – gains over interference channel performance
  - Proportional Fairness solution

Nash Bargaining Solution (NBS)

- NBS: Maximizes the product of rate gains achieved by each link over its interference channel performance
- Utility function: rate gains via cooperation over IC rates

$$\underline{R}_{\mathcal{S}}^{NBS} = \arg \max_{\{\underline{R}_{\mathcal{S}}: R_i > R_i^{IC}\}} \Pi_{i=1}^{M} (R_i - R_i^{IC})$$ where $$R_i^{IC} = I(X_i; Y_i)$$

- Properties of NBS:
  - Pareto optimal (maximizes sum-rate)
  - Symmetric (independent of link labels)
- Pareto optimality of NBS $\Rightarrow \underline{R}_{\mathcal{S}}^{NBS} \in D(C_{\mathcal{S}})$
- Suffices to search for NBS on the dominant sum-rate face $D(C_{\mathcal{S}})$ of $C_{\mathcal{S}}$.

Proportional Fairness Solution

- An allocation of rates is proportional fair *iff*

$$\sum_{i=1}^{M} \frac{R_i - R_i^{PF}}{R_i^{PF}} \leq 0 \Leftrightarrow \arg\max \sum_{i=1}^{M} \log R_i$$

- For the IC coalitional game, $\underline{R}_{\mathcal{S}}^{PF}$ simplifies as

$$\underline{R}_{\mathcal{S}}^{PF} = \arg \max_{\underline{R}_{\mathcal{S}} \in C_{\mathcal{S}}} \Pi_{i=1}^{M} R_i$$

- PF solution is a special case of the NBS with utility modeled simply as the rate achieved.
- Suffices to find PF solutions on the dominant face $D(C_{\mathcal{S}})$ of $C_{\mathcal{S}}$.

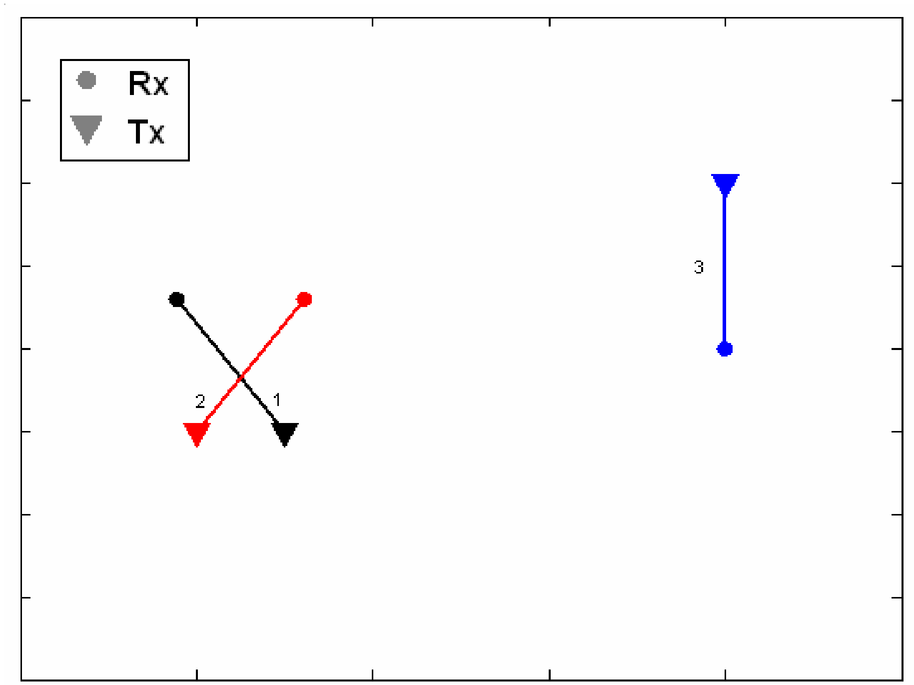
Rx cooperation game in IC-Illustration of Results

- Three-link IC with channel gains

$$h_{k,i} = \frac{A_{k,i}}{d_{k,i}^{\alpha/2}}, \quad \forall i, k \in \mathcal{S} = \{1, 2, 3\}, i \neq k.$$

- α: path-loss exponent set to 3
- Consider three network topologies
- For each topology, the transferable payoff allocations of NBS and PF presented (GC sum-rate optimal)
- Also consider an equal rate (ER) strategy
  - Non-transferable payoff strategy where value $v(\mathcal{G})$ split equally among the members of $\mathcal{G}$.
  - Grand coalition is not necessarily stable (i.e., in the core)

Topology 1

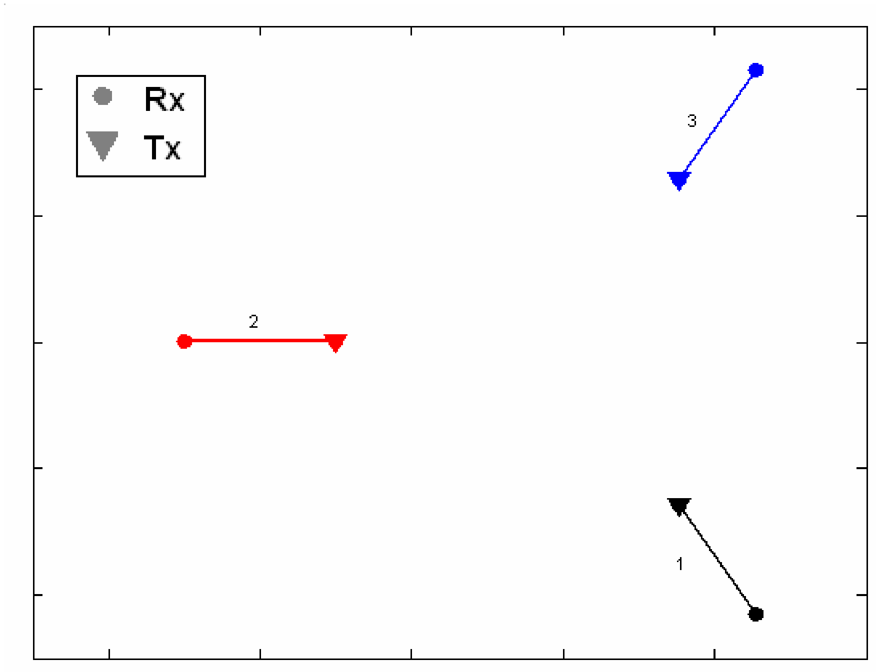


Coalition	R_1	R_2	R_3	Sum-rate
Transferable Payoff Allocation Strategies				
$\{1,2,3\}_{NBS}$	1.4391	1.4346	1.0671	3.9408
$\{1,2,3\}_{PF}$	1.4372	1.4365	1.0671	3.9408
Non-transferable Payoff Allocation Strategy (Equal Rate)				
{1,2,3}	1.3136	1.3136	1.3136	3.9408
{1,2},{3}	1.4174	1.4174	0.9355	3.7703
{2,3}, {1}	0.4170	0.2055	0.2055	0.8280
{3,1}, {2}	0.2115	0.4129	0.2115	0.8359
{1},{2},{3}	0.4170	0.4129	0.9355	1.7654
Stable ER Coalition: {1,2},{3}				

Results for Topology 1

- NBS and PF lead to different allocations but both are sum-rate maximizing stable GCs.
- Equal rate (ER) allocation:
  - Grand coalition is NOT stable
  - Links 1 and 2 achieve better rates via the {1,2} coalition though 3 prefers the GC
  - ER tuple does not lie on the sum-rate face of $C_{\mathcal{S}} \Rightarrow$ Proportional fair rate solution is not the equal rate tuple

Topology 2 (Perfect Symmetry)

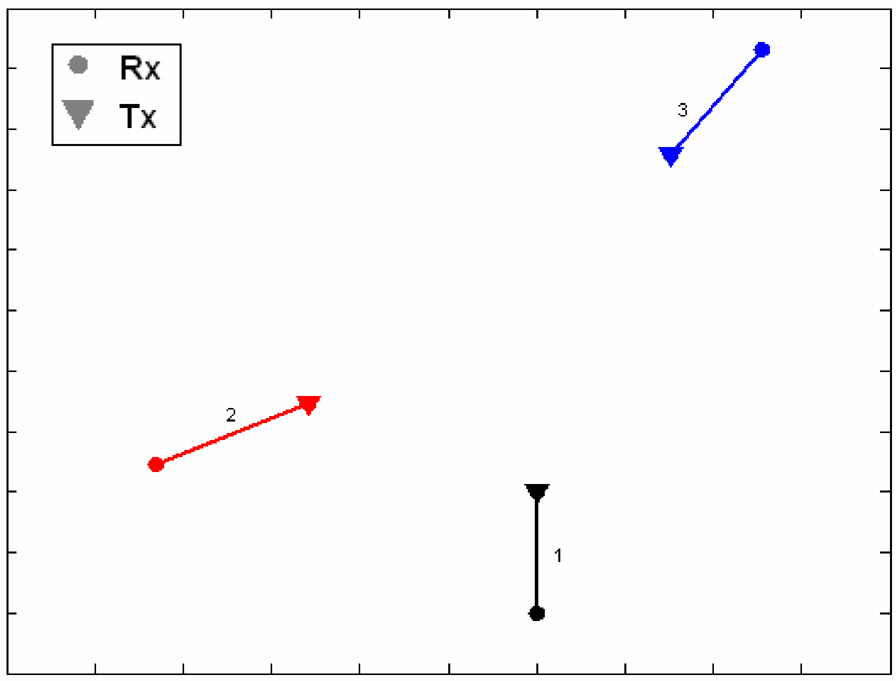


Coalition	R_1	R_2	R_3	Sum-rate
Transferable Payoff Allocation Strategies				
$\{1,2,3\}_{NBS}$	0.9988	0.9988	0.9988	2.9964
$\{1,2,3\}_{PF}$	0.9988	0.9988	0.9988	2.9964
Non-transferable Payoff Allocation Strategy (Equal Rate)				
{1,2,3}	0.9988	0.9988	0.9988	2.9964
{1,2},{3}	0.9671	0.9671	0.9673	2.9015
{2,3}, {1}	0.9673	0.9671	0.9671	2.9015
{3,1}, {2}	0.9671	0.9673	0.9671	2.9015
{1},{2},{3}	0.9673	0.9673	0.9673	2.9019
Stable ER Coalition: {1,2,3}				

Results for Topology 2

- Symmetric geometry and inter-node distances implies
  - NBS, PF and ER lead to identical allocations
  - Grand coalition is sum rate maximizing and stable in all three cases

Topology 3



Coalition	R_1	R_2	R_3	Sum-rate
Transferable Payoff Allocation Strategies				
$\{1,2,3\}_{NBS}$	0.9868	0.9868	1.0246	2.9982
$\{1,2,3\}_{PF}$	0.9994	0.9994	0.9994	2.9982
Non-transferable Payoff Allocation Strategy (Equal Rate)				
{1,2,3}	0.9994	0.9994	0.9994	2.9982
{1,2},{3}	0.9774	0.9774	0.9758	2.9306
{2,3}, {1}	0.9230	0.9209	0.9209	2.7648
{3,1}, {2}	0.9210	0.9231	0.9210	2.7651
{1},{2},{3}	0.9230	0.9230	0.9759	2.8219
Stable ER Coalition: {1,2,3}				

Results for Topology 3

- Slightly skewed geometry of links 1 and 2 towards each other changes allocations
  - NBS and PF lead to different allocations
  - PF and ER lead to identical allocations
  - Grand coalition is sum rate maximizing and stable in all three cases

Multiuser detection (MUD) in a multiple access channel (MAC)

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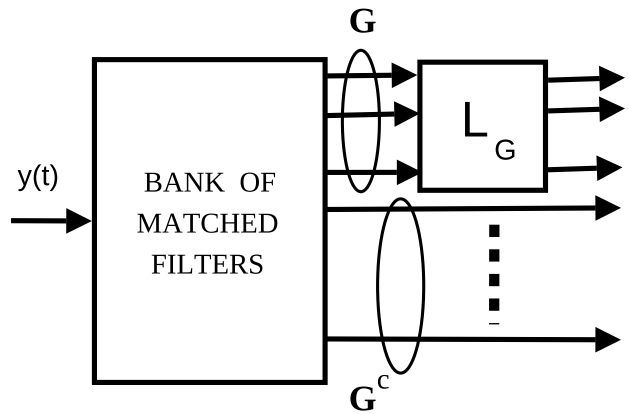
Coalitional Games in Linear Multiuser Detection in a MAC

- Recd. signal model

$$y(t) = \sum_{i=1}^{M} \sqrt{P} h_i b_i s_i(t) + \sigma n(t), \quad t \in [0, T]$$

- MF bank output:

$$\mathbf{y} = \mathbf{R}\mathbf{A}\mathbf{b} + \mathbf{n}$$



- Recd. signal vector of coalition $\mathcal{G}$:

$$\mathbf{y}_{\mathcal{G}} = \mathbf{R}_{\mathcal{G}}\mathbf{A}_{\mathcal{G}}\mathbf{b}_{\mathcal{G}} + \widetilde{\mathbf{R}}_{\mathcal{G}^c}\mathbf{A}_{\mathcal{G}^c}\mathbf{b}_{\mathcal{G}^c} + \mathbf{n}_{\mathcal{G}}$$

- Vector of detected signals for coalition $\mathcal{G}$:

$$\mathbf{y}_{\mathcal{G}}^{out} = \mathbf{L}_{\mathcal{G}}\mathbf{y}_{\mathcal{G}}$$

For Decorr Rx: $\mathbf{L}_{\mathcal{G}} = \mathbf{R}_{\mathcal{G}}^{-1}$

For MMSE Rx: $\mathbf{L}_{\mathcal{G}} = (\mathbf{R}_{\mathcal{G}} + \sigma^2\mathbf{A}_{\mathcal{G}}^{-2})^{-1}$

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Coalitional Games in Linear Multiuser Detection

- We consider linear multiuser detectors for coalitions
- Model as a non-transferable payoff game.
- SINR achieved by a user in a coalition is its payoff
- Users within a coalition benefit from the interference suppression offered by their MUD.

Why would receivers form coalitions ?

- In a game with non transferable payoff, the grand coalition cannot be guaranteed.
  - Users may have incentives to form other coalitions and leave the grand coalition.
  - E.g. Noise enhancement for a user may outweigh the interference suppression offered by the MUD.

Decorrelating detector MAC game

- *SINR* of user *i* in coalition $\mathcal{G}$: [Li & Ephremides]

$$SINR_i^{decorr}(\mathcal{G}) = \frac{P_i}{\frac{\sigma^2}{1-\rho}\frac{1+\rho(|\mathcal{G}|-2)}{1+\rho(|\mathcal{G}|-1)} + \left[\frac{\rho}{1+\rho(|\mathcal{G}|-1)}\right]^2 \sum_{j\in\mathcal{G}^c} P_j}$$

where ρ is the cross correlation between any two users.

- *Theorem:* In the decorrelating detector MAC game, the grand coalition is stable and sum-rate maximizing in the high SNR regime.
  - The theorem follows from:

$$\lim_{\sigma^2\to 0} SINR_i^{decorr}(\mathcal{G}) < \lim_{\sigma^2\to 0} SINR_i^{decorr}(\mathcal{S})$$

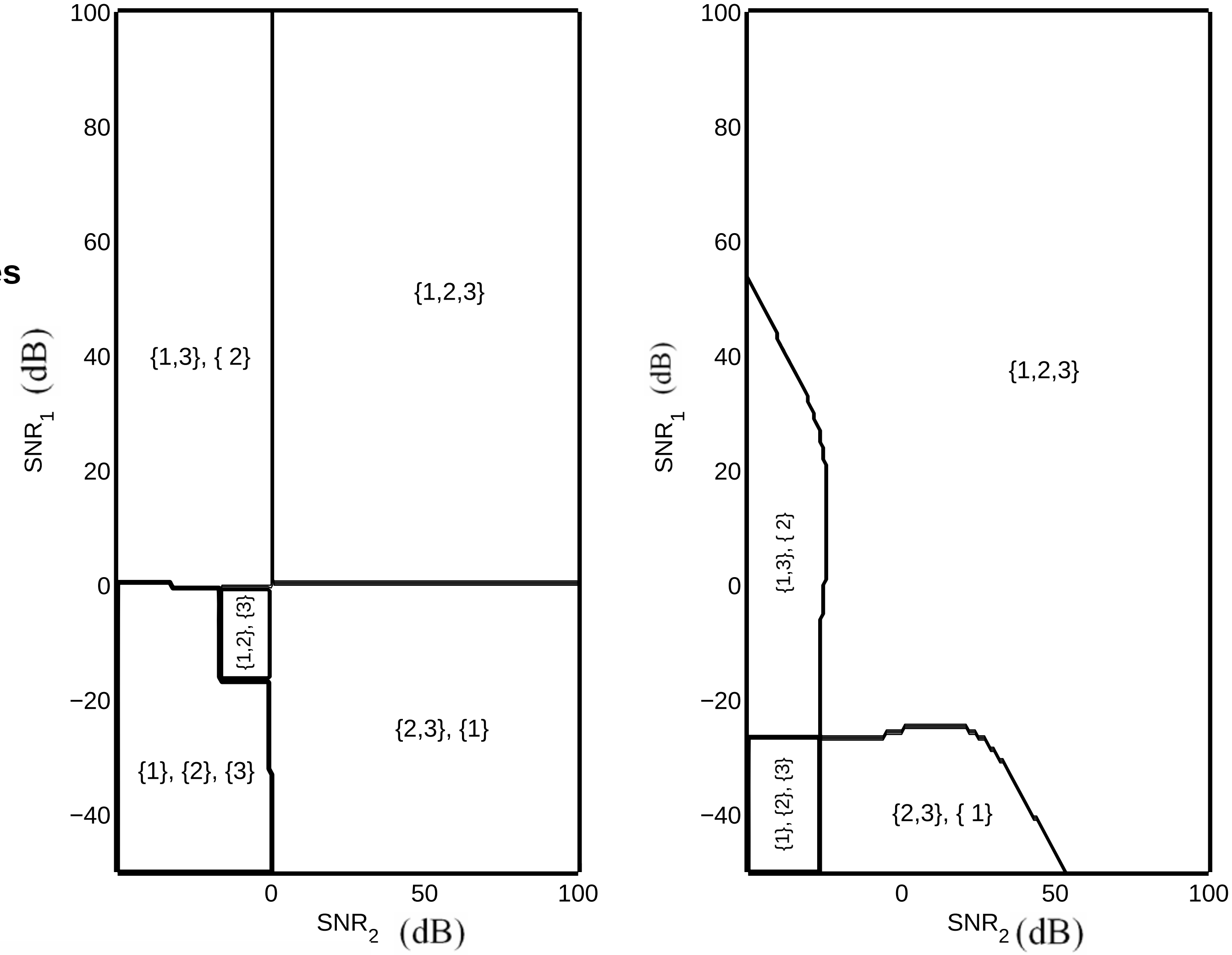
Decorrelating detector MAC game

- In general, however, there is no guarantee that the grand coalition of users should form or that the stable coalition structure should be the one that maximizes sum-rate.
- To illustrate this we use the mapping:

$$R_i(\mathcal{G}) = \log\left(1 + SINR_i(\mathcal{G})\right)$$

Decorrelating detector MAC game

Stable Coalition structures →



Sum-rate maximizing coalition structures ←

Received SNRs of users 1 and 2 are varied while the received SNR of user 3 is kept fixed at 27 dB

MMSE Rx. MAC game

- The grand coalition is always stable and sum-rate maximizing
  - Minimizing the MSE is equivalent to maximizing the SINR
  - Mapping of SINR to rate is monotonically non-decreasing.

Transmitter cooperation in an interference channel

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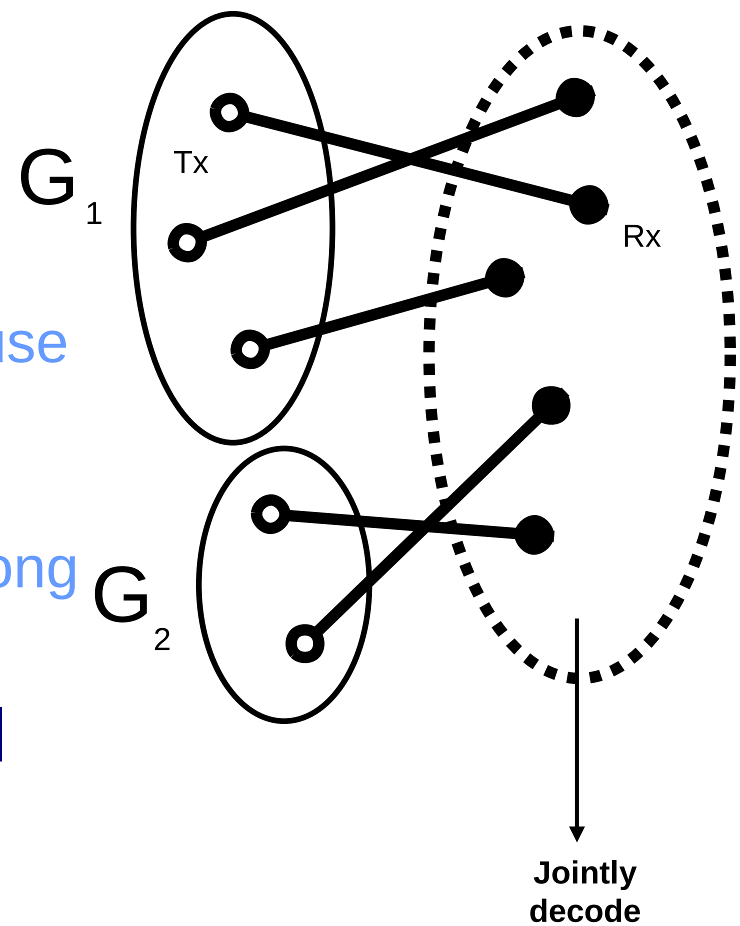
Coalitions of Transmitters in an IC

- Transmitters in a coalition are allowed perfect cooperation
  - Through ideal noise-free inter-user links.
  - Cooperating transmitters jointly encode their transmit signals.
- All receivers always jointly decode their signals.

Tx cooperation games in the IC

- Txs can form cooperative coalitions.
- We are interested in:
  - Which coalitions make optimal use of available spectrum (sum-rate maximization) ?
  - Which coalitions are stable (belong to the core) ?
- Virtual MIMO with individual power constraints.



Tx cooperation games in the IC

- If the value of a coalition is defined as:

$$v(\mathcal{G}) = \max_{X_{\mathcal{G}}} I(X_{\mathcal{G}}; Y_{\mathcal{S}}) \ \text{ s.t. } \ E[X_i^2] \leq P_i \ \ \forall i \in \mathcal{S}$$
$$= \max_{X_{\mathcal{G}}} H(Y_{\mathcal{S}}) - H(\sum_{i \in \mathcal{G}^c} \mathbf{h}_i X_i + \mathbf{n})$$

- $v(\mathcal{G})$ depends on the actions of players outside $\mathcal{G}$ *!*
- Tx cooperation game not of characteristic function form!

How to make it of characteristic form?

- Alter value $v(\mathcal{G})$ to reflect the max. sum-rate achievable by $\mathcal{G}$ under worst case interference from $\mathcal{G}^c$.
- Assume the users in $\mathcal{G}^c$ attempt to 'jam' users in $\mathcal{G}$ to minimize the value of the breakaway coalition $\mathcal{G}$.
  - This is similar in spirit to La & Anantharam's work on the MAC.

Tx cooperation games in the IC – preliminary results

- It can be shown that the game is superadditive.
- The only constraint for apportioning rate in a grand coalition is: $\sum_{i \in \mathcal{S}} R_i \leq v(\mathcal{S})$
- Since it has transferable payoff, the core either contains payoff profiles for the grand coalition or it is empty.