Joint Optimization of Relay Strategies and Resource Allocations in Cooperative Cellular Networks

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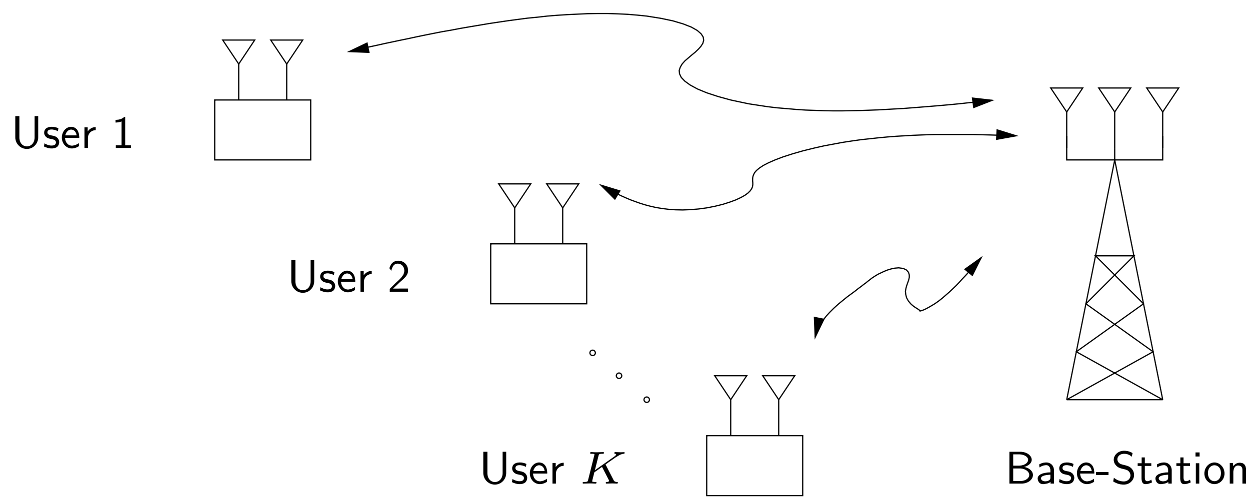
Wireless Cellular Environments

- Cellular network: single basestation, multiple subscribers



  - Fundamental issues: signal propagation and power control.
  - A promising idea: RELAY information

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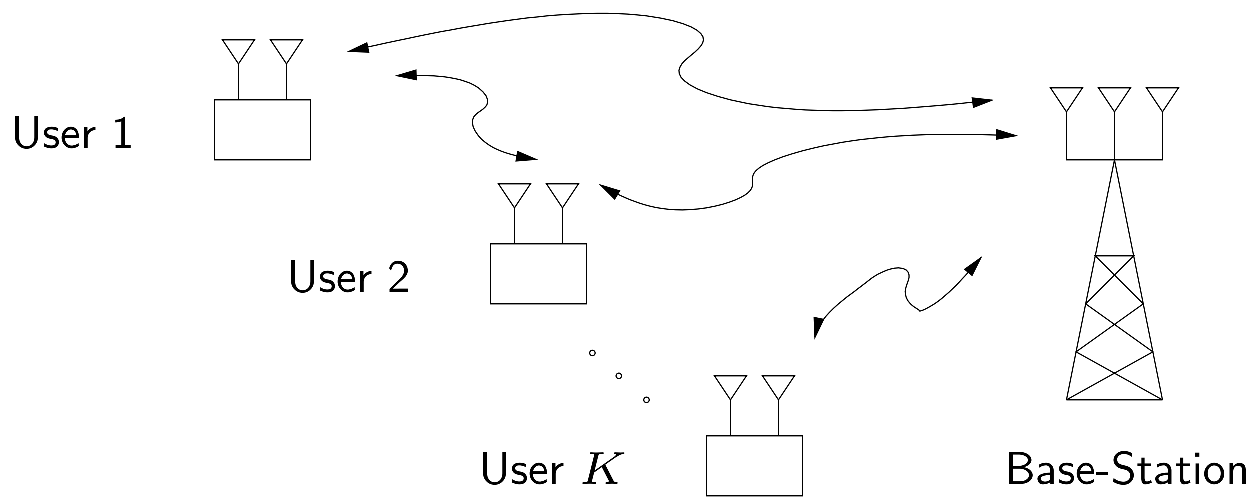


  - Fundamental issues: signal propagation and power control.
  - A promising idea: RELAY information

Physical Layer vs Network Layer Relay

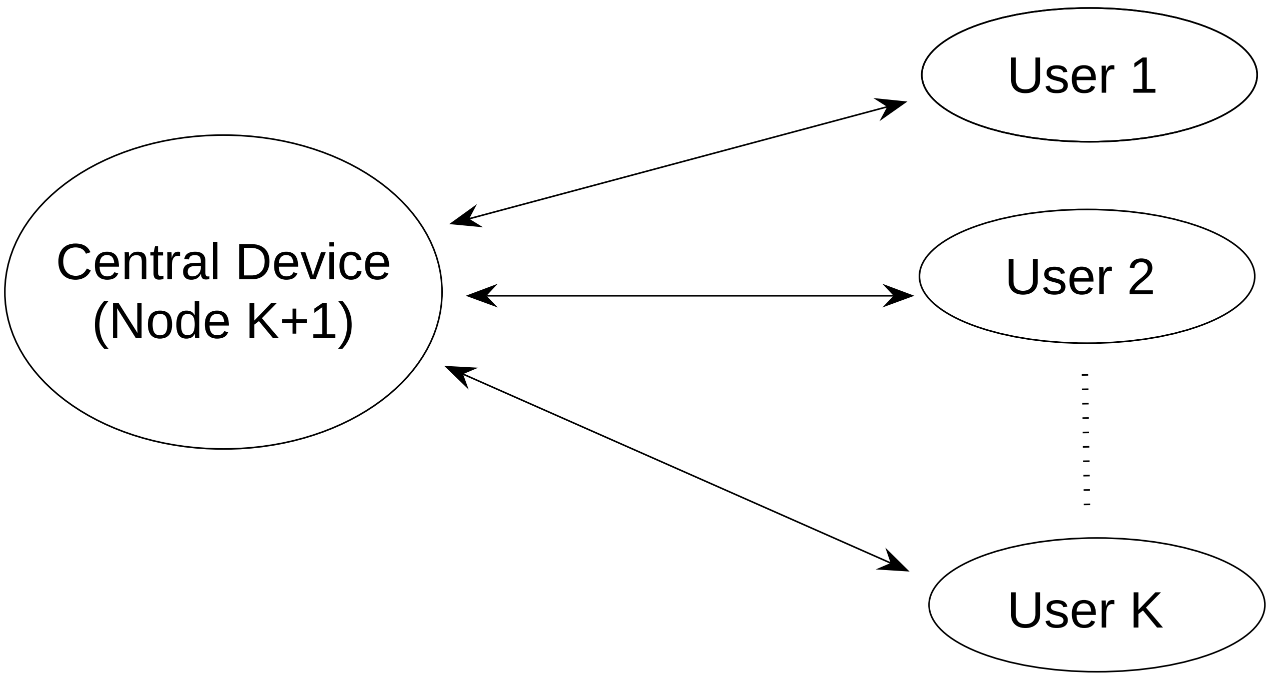
- Relaying may be implemented at different layers.
- Network layer:
  - IP packets may be relayed by one subscriber for another.
- Physical layer:
  - Cooperative diversity: Distributed space-time codes
  - Spatial multiplexing: Distributed spatial processing
- Interaction between physical and network layers is crucial.

This Talk: Crosslayer Optimization

- Who needs help from relays?
  - Intuitively, when channel condition is bad. But how bad?
- Who should act as relay?
  - Intuitively, when the relay has a good channel. But how good?
  - Intuitively, when the relay has excess power. But how much?
- Our perspective:
  - Crosslayer optimization via utility maximization.

Network Description

- Consider a wireless network with a central device and K user nodes



Network Description

- Each user communicates with the central device in both downstream (d) and upstream (u) directions. So, there are $2K$ data streams.

- Assume orthogonal frequency division multiplexing (OFDM) with N tones.

- Assume only one active data stream in each tone, so that there is no inter-stream interference.

- Frequency-selective but slow-fading channel with coherence bandwidth no greater than the bandwidth of a few tones.

Utility Maximization

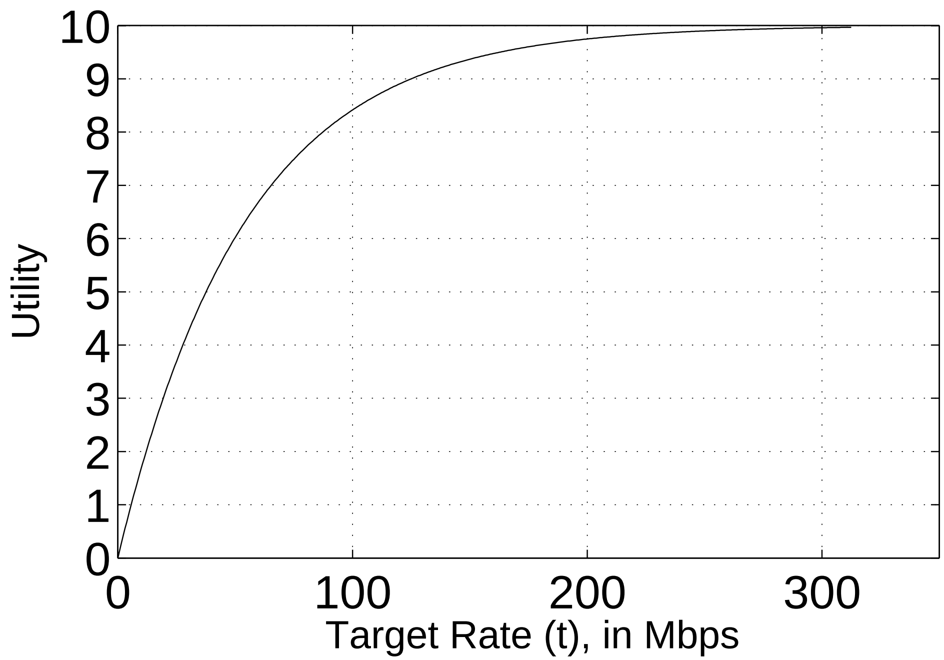


Figure 1: $U(t) = 10(1 - e^{-1.8421\text{x}10^{-8}t})$

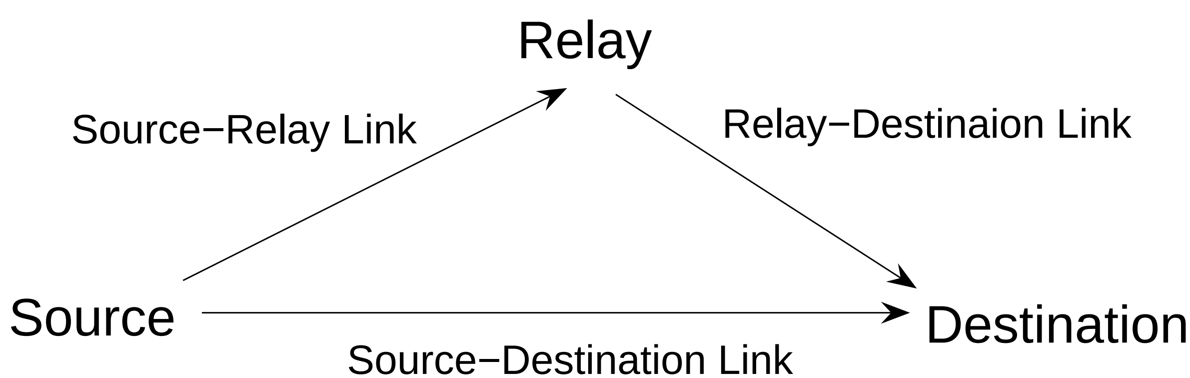
Goal: Maximize System Utility

- Let $\boldsymbol{r} = \left[\boldsymbol{r}_{(\boldsymbol{1},\boldsymbol{d})}, \boldsymbol{r}_{(\boldsymbol{2},\boldsymbol{d})}, \ldots, \boldsymbol{r}_{(\boldsymbol{K},\boldsymbol{u})}\right]^T$ be achievable rates.
- Define $\boldsymbol{P}$ as a $(K+1) \times N$ matrix of transmit power.
- Define $\boldsymbol{R}$ as a $2K \times N$ rate matrix for each frequency tone and data stream.
- The system optimization problem:

$$\begin{array}{ll} \text{maximize} & \displaystyle\sum_{m \in \mathcal{M}} U_m(\boldsymbol{r_m}) \\ \text{subject to} & \boldsymbol{P1} \preceq \boldsymbol{p^{max}} \\ & \boldsymbol{R1} \succeq \boldsymbol{r} \end{array}$$

Modes of Operation

- "No relay" mode: only direct transmission
- "Relay" mode: allow the use of a relay



- Problem: decide whether to relay and who to act as relay in every tone.

Cooperative Network

- Any user node that is not the source or destination can be a relay.
- Relaying can happen in every frequency tone.
- Choice of relay in each tone can be different.
- Assume per-node power constraints.
- Give "selfish" nodes an incentive to act as relay:
  - Relaying decreases available power for its own data streams;
  - But cooperative relaying increases the overall system utility.

Optimization Framework

- The system utility maximization problem:

$$\begin{aligned} \text{maximize} \quad & \sum_{m \in \mathcal{M}} U_m(\boldsymbol{r_m}) \\ \text{subject to} \quad & \boldsymbol{P1} \preceq \boldsymbol{p^{max}} \\ & \boldsymbol{R1} \succeq \boldsymbol{r} \end{aligned}$$

- Main technique: Introducing a pricing structure.
  - Data rate is a commodity $\Longrightarrow$ price discovery in a market equilibrium

Dual Decomposition

- Form the so-called Lagrangian of the optimization problem:

$$L = \sum_{m \in \mathcal{M}} U_m(\boldsymbol{r_m}) + \boldsymbol{\lambda^T}\Big(\boldsymbol{R1} - \boldsymbol{r}\Big),$$

which can be decomposed into two maximization subproblems:

$$\max_{\boldsymbol{r}} \quad \sum_{m \in \mathcal{M}} \Big(U_m(\boldsymbol{r_m}) - \boldsymbol{\lambda_m r_m}\Big),$$

$$\max_{\boldsymbol{P},\boldsymbol{R}} \quad \sum_{m \in \mathcal{M}} \boldsymbol{\lambda_m} \sum_{n \in \mathcal{N}} \boldsymbol{R_{mn}} \qquad \text{s.t.} \quad \boldsymbol{P1} \preceq \boldsymbol{p^{max}}$$

Application Layer Subproblem

- Data rates come at a price $\boldsymbol{\lambda_m}$.
- The price for each data stream $\boldsymbol{m}$ is different.

$$\max_{\boldsymbol{r}} \sum_{m \in \mathcal{M}} \left(U_m(\boldsymbol{r_m}) - \boldsymbol{\lambda_m} \boldsymbol{r_m} \right)$$

- Each data stream optimizes its rate based on the price.
  - In practice, the optimal $\boldsymbol{r_m}$ can be found by setting the derivative of $(U_m(\boldsymbol{r_m}) - \boldsymbol{\lambda_m} \boldsymbol{r_m})$ to zero.

Physical Layer Subproblem

- The price $\boldsymbol{\lambda_m}$ indicates desirability for $\boldsymbol{r_m} \iff$ priority.

$$\begin{array}{ll} \max\limits_{\boldsymbol{P},\boldsymbol{R}} & \sum\limits_{m\in\mathcal{M}} \boldsymbol{\lambda_m} \sum\limits_{n\in\mathcal{N}} \boldsymbol{R_{mn}} \\ \text{s.t.} & \boldsymbol{P1} \preceq \boldsymbol{p^{max}} \end{array}$$

- The data rate maximization problem can be further decomposed

$$\sum_{m\in\mathcal{M}} \boldsymbol{\lambda_m} \sum_{n\in\mathcal{N}} \boldsymbol{R_{mn}} + \boldsymbol{\mu^T}(\boldsymbol{p^{max}} - \boldsymbol{P1})$$

Summary of Results So Far

- Use pricing to decompose the utility maximization problem into two subproblems:
  - Application layer problem
  - Physical layer problem

- Next: How to solve the physical-layer problem with relay?
  - Decode-and-forward
  - Amplify-and–forward

Dual Optimization for OFDM Systems

- Primal Problem: $\max \sum_{n=1}^{N} f_n(x_n) \quad \text{s.t.} \sum_{n=1}^{N} h_n(x_n) \le \mathbf{P}$.

- Lagrangian: $g(\lambda) = \max_{x_n} \sum_{n=1}^{N} f_n(x_n) + \lambda^T \cdot \left(\mathbf{P} - \sum_{n=1}^{N} h_n(x_n) \right)$.

- Dual Problem: $\min g(\lambda) \quad \text{s.t.} \lambda \ge 0$.

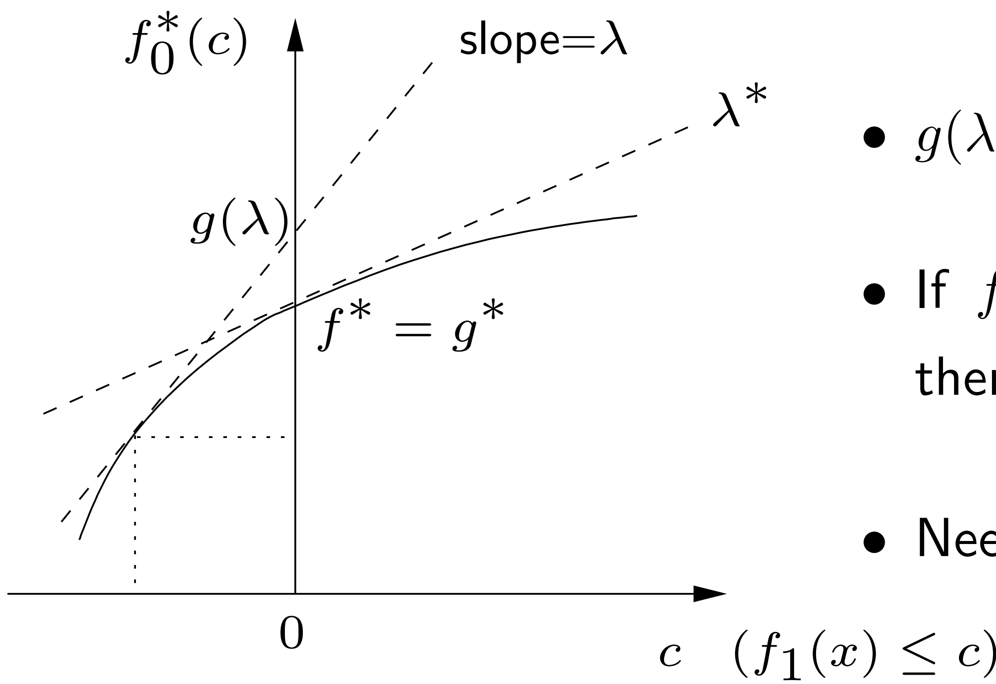
If $f_n(x_n)$ is concave and $h_n(x_n)$ is convex, then duality gap is zero.
For OFDM systems,

$$\boxed{\text{Duality gap is zero even when } f_n(x_n) \text{ and } h_n(x_n) \text{ are non-convex.}}$$

This leads to efficient dual optimization methods.

Why Zero Duality Gap?

$$\boxed{\text{Define } f_0^*(c) = \max_x f_0(x), \ \ s.t. \ f_1(x) \le c.}$$



- $g(\lambda) = \max_x f_0(x) - \lambda f_1(x)$
- If $f_0^*(x)$ is concave in $f_1(x)$, then $\boxed{\min g(\lambda) = \max f_0(x)}$
- Need concavity of $f_0^*(c)$.

Capacity for Direct Channel (DC)

- Channel Equation

$$y_D = \sqrt{p_S} h_{SD} x_S + n_D$$

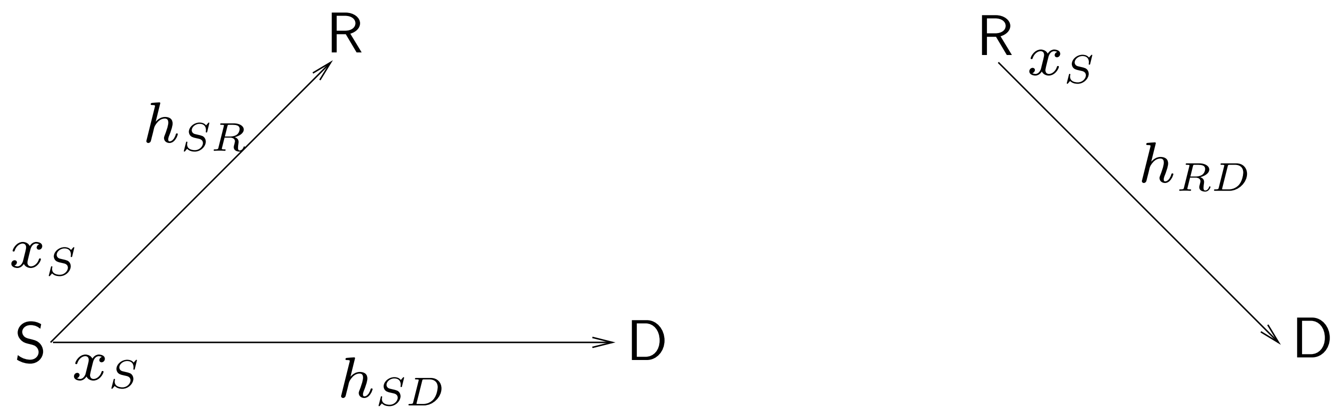
- The capacity in bits/sec is

$$r_{DC} \leq I(x_S; y_D) = W \log_2 \left(1 + \frac{p_S |h_{SD}|^2}{\Gamma N_o W} \right)$$

Relaying Schemes

- Two strategies: decode-and-forward (DF) and amplify-and-forward (AF)
- Need two time slots to implement relay.
- Source S only transmits in the first time slot
  - Relay R may simply amplify and forward its signal (AF).
  - Relay R may decode first, then re-encode the information (DF).
- The effective bit rate and power must be divided by a factor of 2.

Achievable Rate with DF



- The channel equations are

$$\begin{aligned}
y_{D1} &= \sqrt{p_S} h_{SD} x_S + n_{D1}, \\
y_{R1} &= \sqrt{p_S} h_{SR} x_S + n_{R1}, \\
y_{D2} &= \sqrt{p_R} h_{RD} x_S + n_{D2}.
\end{aligned}$$

Computing the DF Capacity

- Relay node R has to decode x_S successfully in the first time slot, so

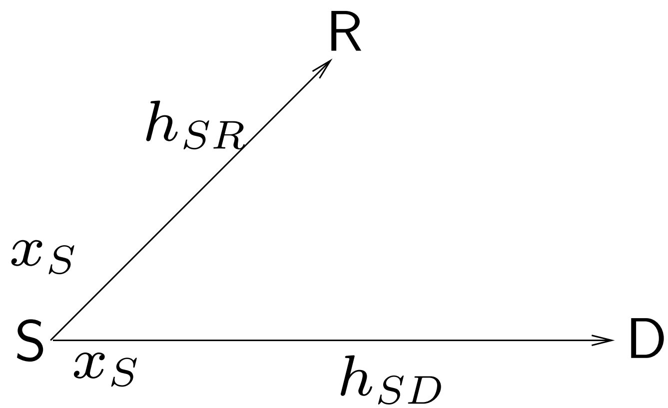
$$r_{DF} \quad \leq \quad I(x_S; y_{R1}) = W \log_2 \left(1 + \frac{p_S |h_{SR}|^2}{\Gamma N_o W} \right)$$

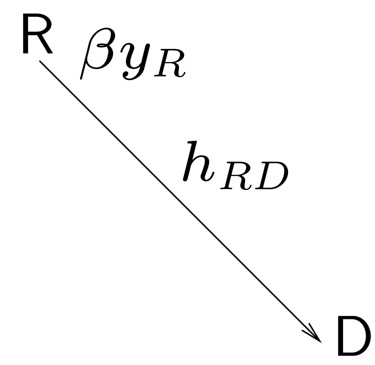
- Destination D also has to successful decode x_S, we need

$$r_{DF} \leq I(x_S; y_{D1}, y_{D2}) = W \log_2 \left(1 + \frac{p_S |h_{SD}|^2 + p_R |h_{RD}|^2}{\Gamma N_o W} \right)$$

essentially maximum-ratio combining at the destination.

Achievable Rate with AF





- The channel equations are

$$
\begin{aligned}
y_{D1} &= \sqrt{p_S} h_{SD} x_{S1} + n_{D1}, \\
y_{R1} &= \sqrt{p_S} h_{SR} x_{S1} + n_{R1}, \\
y_{D2} &= \beta y_{R1} h_{RD} + n_{D2}.
\end{aligned}
$$

Computing Capacity of AF

- β is the power amplification factor at R, and

$$\beta = \sqrt{\frac{p_R}{p_S|h_{SR}|^2 + N_o W}}$$

- To decode successfully at D,

$$\begin{aligned} r_{AF} &\leq I(x_S; y_{D1}, y_{D2}) \\ &= W \log_2 \left(1 + \frac{1}{\Gamma} \left[\frac{p_S|h_{SD}|^2}{N_o W} + \frac{\frac{p_R p_S |h_{RD}|^2 |h_{SR}|^2}{p_S|h_{SR}|^2 + N_o W}}{N_o W \left(1 + \frac{p_R |h_{RD}|^2}{p_S|h_{SR}|^2 + N_o W} \right)} \right] \right) \end{aligned}$$

Rate and Power Tradeoff

- For each fixed (p_S, p_R) and for each relay mode, we have found the achievable rate. So:

$$r = \max(r_{DC}, r_{AF}, r_{DF})$$

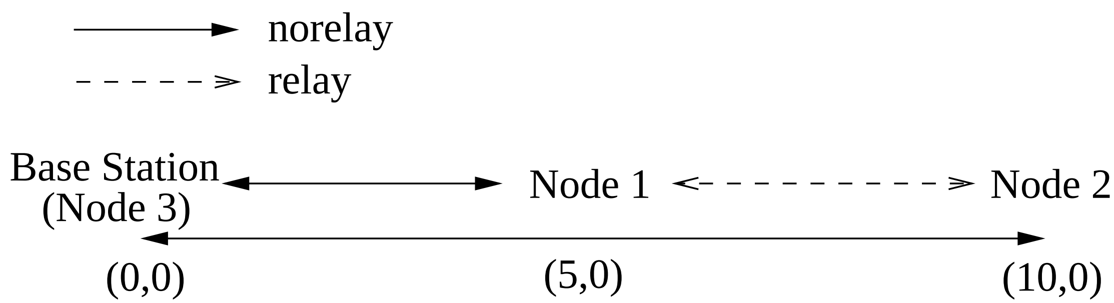
- Conversely, for a fixed desirable r, we can find the minimal power needed.

- Goal of optimization: find the optimal tradeoff between power and rate.

Optimization Algorithm

- For each data stream $\boldsymbol{m}$ and in each tone n, we search for the best relay and the best relay strategy:
  - No Relay:
  $$\max \boldsymbol{\lambda_m r_{m,n}} - \boldsymbol{\mu_S p_S}$$
  - Relay:
  $$\max \boldsymbol{\lambda_m r_{m,n}} - \boldsymbol{\mu_S p_S} - \boldsymbol{\mu_R p_R}$$
- Then, we search for the optimal data stream in each tone.
- Finally, we search for the optimal power price vectors $\boldsymbol{\mu_R}$ and $\boldsymbol{\mu_S}$.

Simulations: 2-User Example



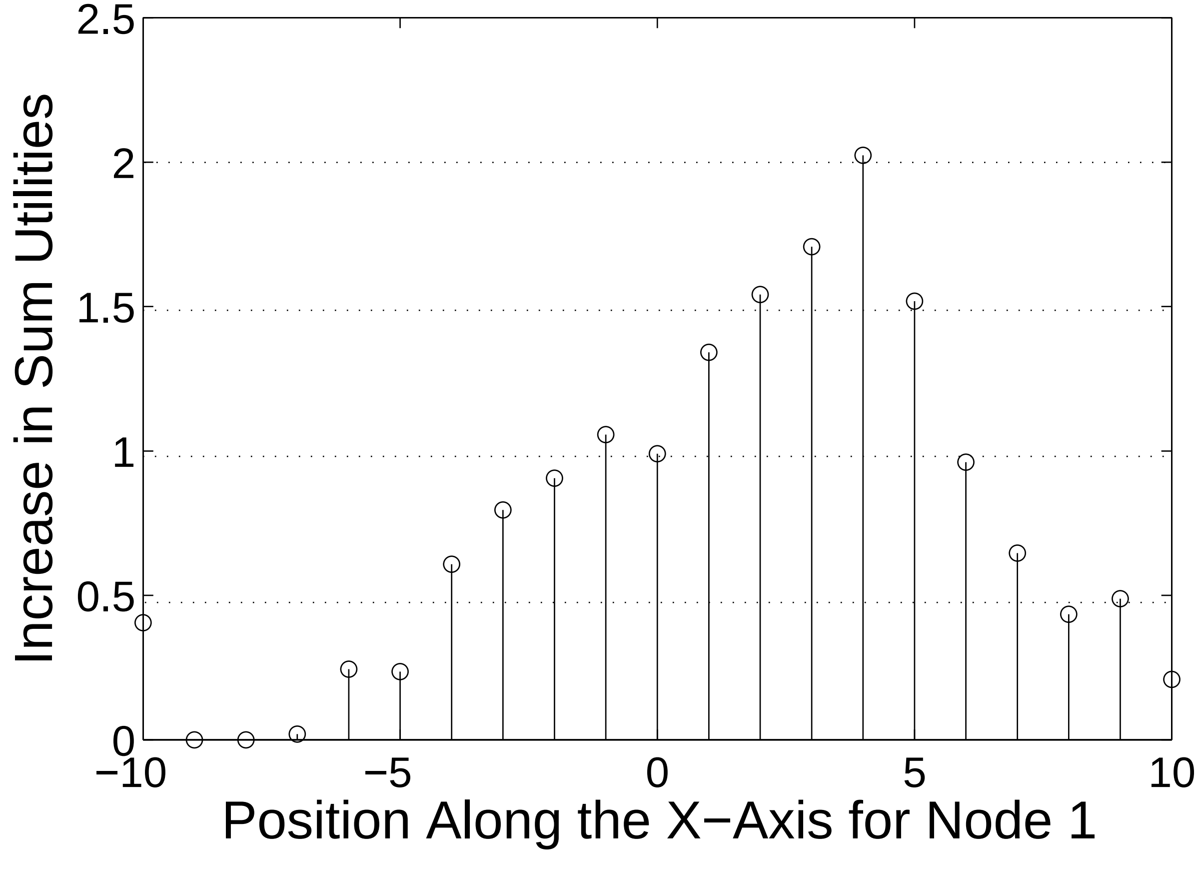
- The total bandwidth is 80MHz with $N = 256$ OFDM tones.
- Assume large-scale (distance dependent) fading with path loss exponent equal to4, as well as small-scale i.i.d. Rayleigh fading.
- Maximum utility for downstream and upstream are 10 and 1 respectively.
- Relaying increases system utility by about 10%.

Simulations: 2-User Example

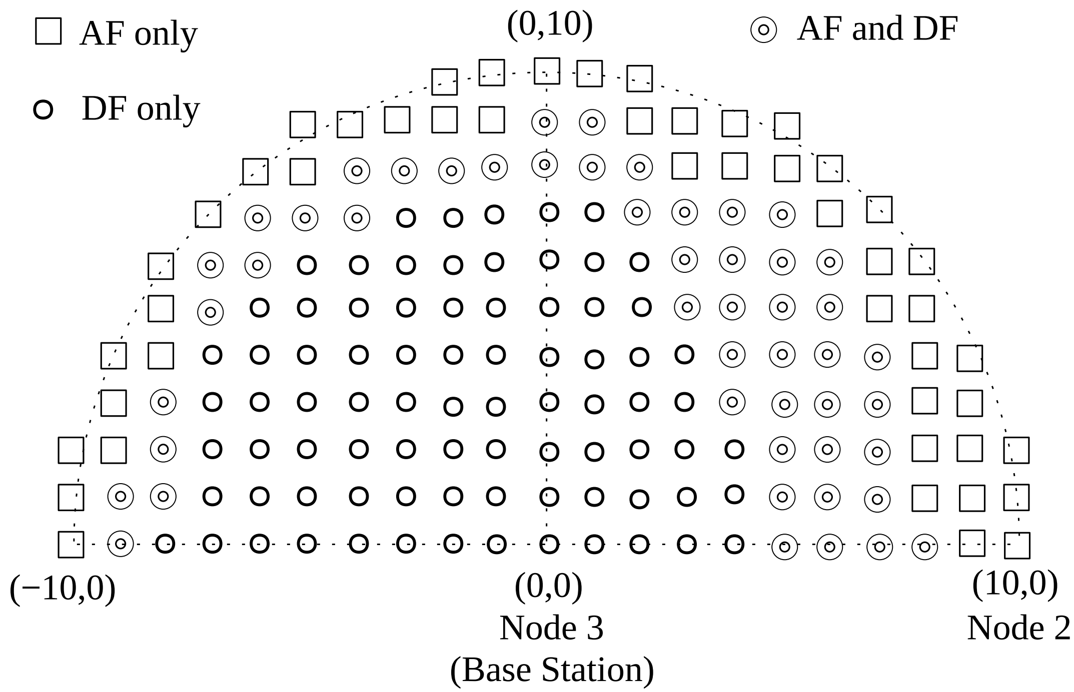
Stream	No Relay	Relay	Percentage Change
$(1, d)$	130.0Mbps	115.9Mbps	-10.8%
$(2, d)$	50.8Mbps	88.8Mbps	74.8%
$(1, u)$	27.0Mbps	19.4Mbps	-28.2%
$(2, u)$	16.2Mbps	15.8Mbps	-2.5%

Node	Percentage of power spent as relay
1	47.6%
2	0%

Varying Node 1 Along the X-Axis

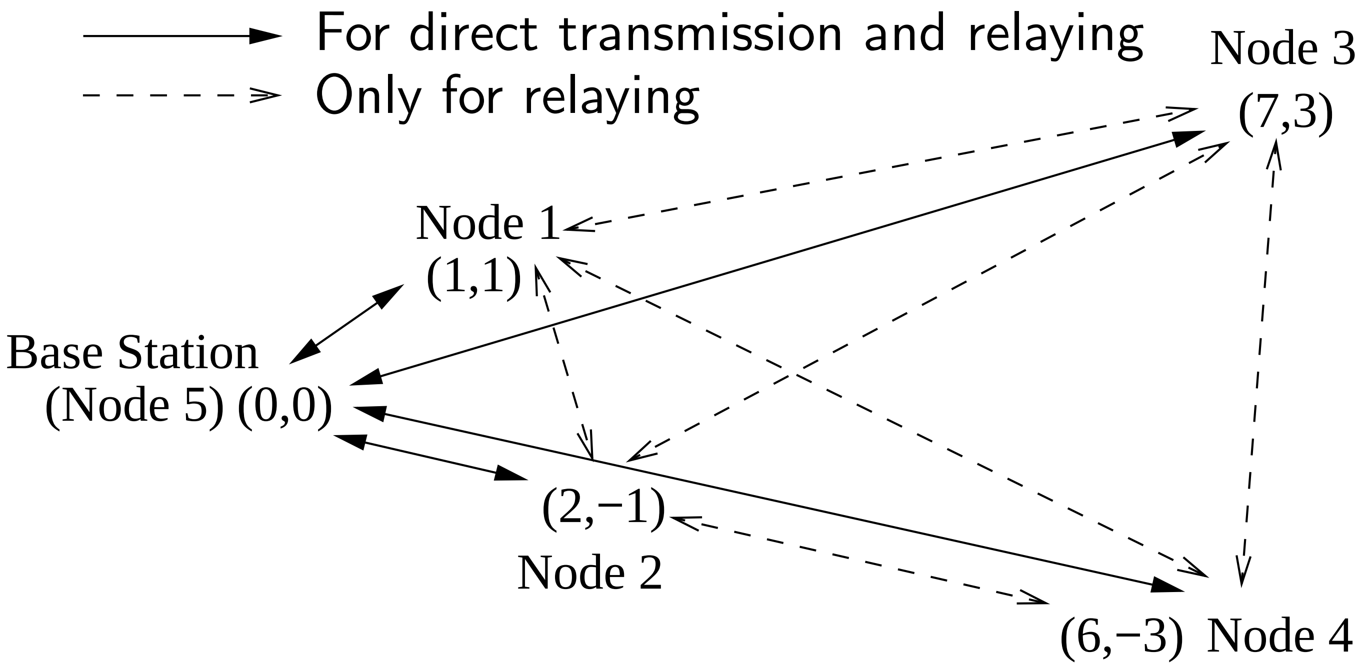


Which Relaying Strategy is the Best?



Dominating Relay Mode

Simulation: 4-User Example



Simulation: 4-User Example

Stream	No Relay	Allow Relay	Percentage Change
$(1, d)$	152.8Mbps	148.4Mbps	-2.9%
$(2, d)$	135Mbps	129.4Mbps	-4.1%
$(3, d)$	46.6Mbps	71.3Mbps	53.0%
$(4, d)$	54.1Mbps	80.5Mbps	48.8%
$(1, u)$	18.8Mbps	16.6Mbps	-11.7%
$(2, u)$	18.8Mbps	16.3Mbps	-13.3%
$(3, u)$	11.9Mbps	13.8Mbps	16.0%
$(4, u)$	14.1Mbps	13.4Mbps	-5.0%

Node	Percentage of power spent as relay
1	94.9%
2	92.2%
3	0%
4	0%

Conclusion

- We have presented an optimization technique to maximize the system sum utility in a cellular network with relays.
  - A dual decomposition framework that separates the application- and physical-layer problems
  - A physical-layer model for different relay strategies

- The dual technique makes the problem computationally feasible.

- Our technique allows a characterization of optimal relaying mode and optimal power allocation.