Throughput Performance in Networks with Linear Capacity Contraints

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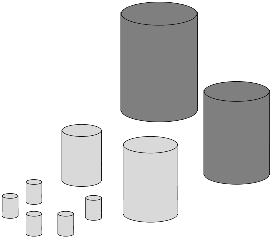
France Telecom & Ecole Normale Supérieure

CISS 2006

Scope of the talk

- **elastic traffic**



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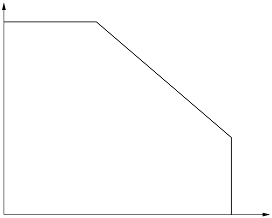

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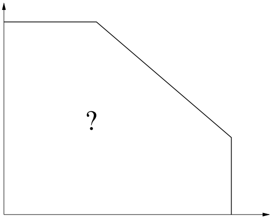
- throughput performance ?



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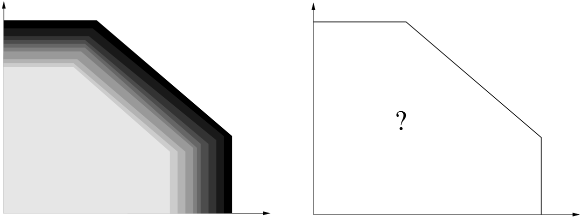

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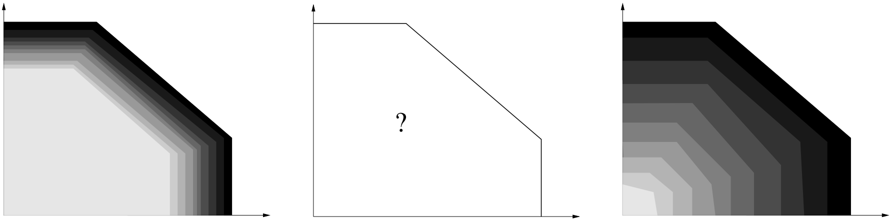

Outline

- model
- stability issues
- balanced fairness
- throughput performance
- conclusion

Model

- L resources

Model

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- N flow classes

Model

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- **there are** C_l **resource-**l **units**

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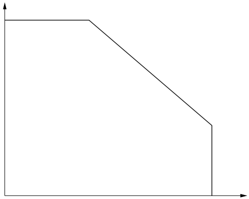
Model

- L resources
- N flow classes
- there are C_l resource-l units
- class-i flows require A_{il} resource-l units per bit/s
- let ϕ_i be the total bit rate of class-i flows

$$\phi A \leq C$$

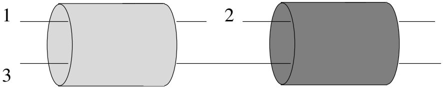


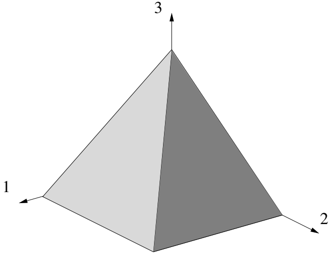
A linear network

- **capacity constraints**

$$\phi_1 + \phi_3 \leq 1$$

$$\phi_2 + \phi_3 \leq 1$$



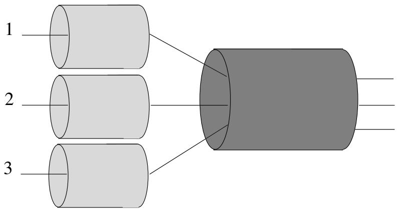


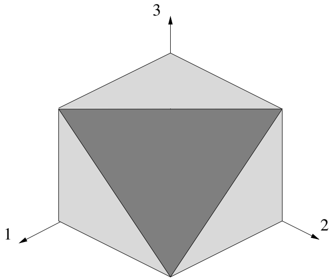
A tree network

- **capacity constraints**

$$\phi_1 + \phi_2 + \phi_3 \leq 2$$

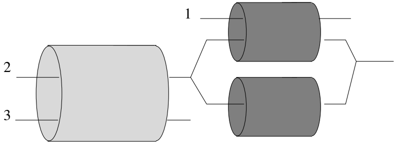
$$\phi_1 \leq 1, \quad \phi_2 \leq 1, \quad \phi_3 \leq 1$$





Multi-path routing

- **some flows may use several routes simultaneously**



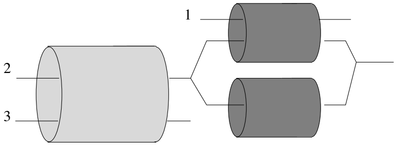
Multi-path routing

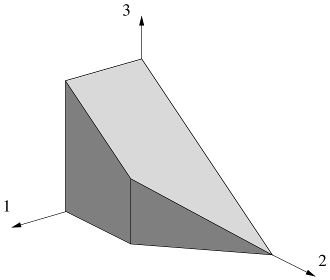
- some flows may use several routes simultaneously
- capacity constraints

$$\phi_1 \leq 1$$

$$\phi_1 + \phi_2 \leq 2$$

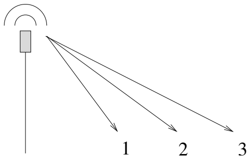
$$\phi_2 + \phi_3 \leq 2$$





Wireless network

- a single access point
- time-shared downlink, adaptive modulation and coding



Wireless network

- a single access point
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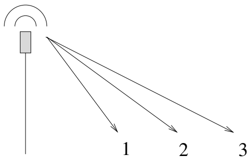
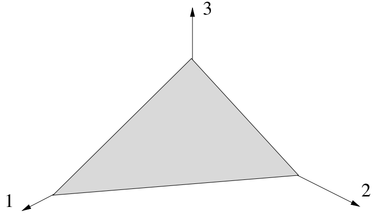
- capacity constraints

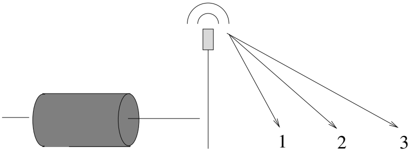
$$\frac{\phi_1}{c_1} + \frac{\phi_2}{c_2} + \frac{\phi_3}{c_3} \leq 1$$





Wireless network

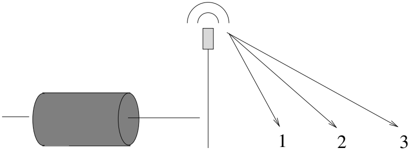
- **impact of wireline backhaul link**

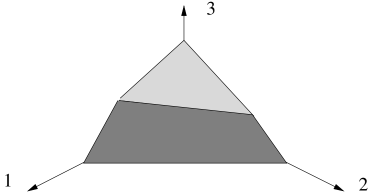


Wireless network

- **impact of wireline backhaul link**
- **capacity constraints**

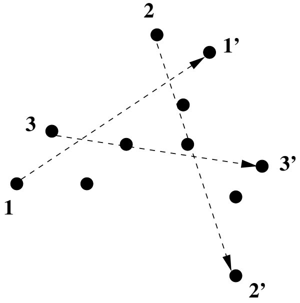
$$\frac{\phi_1}{c_1} + \frac{\phi_2}{c_2} + \frac{\phi_3}{c_3} \leq 1, \quad \phi_1 + \phi_2 + \phi_3 \leq C$$





Ad-hoc network

- synchronous slotted system



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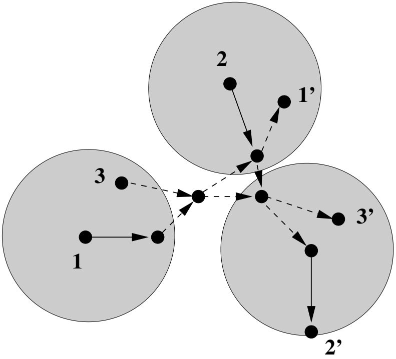
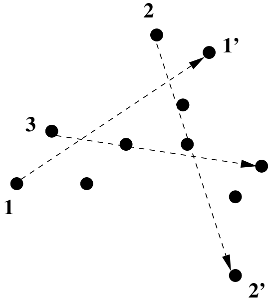


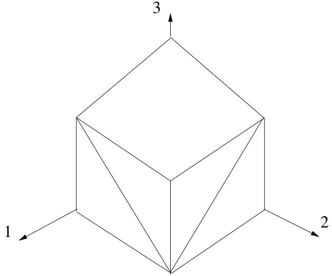
Ad-hoc network

- synchronous slotted system
- capacity constraints

$$4\phi_1 \leq 1, \quad 4\phi_2 \leq 1$$

$$3\phi_1 + \phi_2 + \phi_3 \leq 1, \quad \phi_1 + 3\phi_2 + \phi_3 \leq 1, \quad \phi_1 + \phi_2 + 3\phi_3 \leq 1$$





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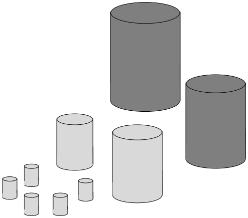

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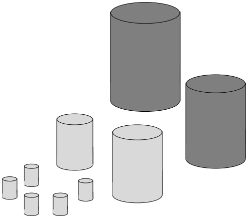
- traffic intensity = arrival rate $\times$ mean flow size (in bit/s)



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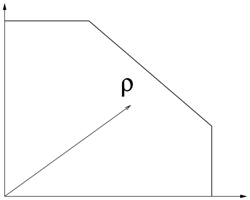
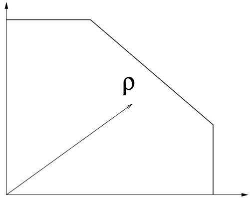


Stability issues

- **necessary condition**

$$\rho A \leq C$$



Stability issues

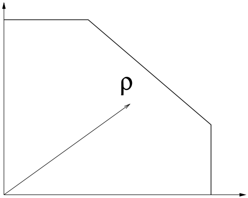
- **necessary condition**

$$\rho A \leq C$$

- **sufficient condition**

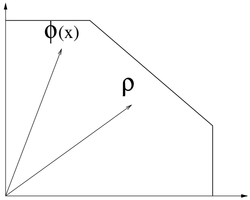
$$\rho A < C$$

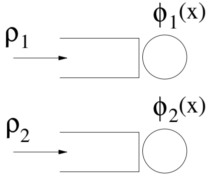
proved for max-min fairness and proportional fairness under some restrictive traffic assumptions (de Veciana et. al. 99, B-Massoulié 01)



Balanced fairness

- a queueing network with state-dependent service rates

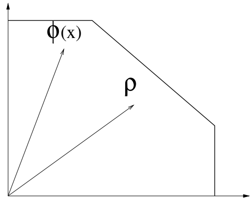


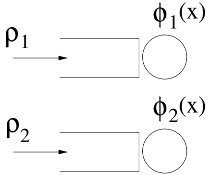


Balanced fairness

- **a queueing network with state-dependent service rates**
- **a Kelly-Whittle network iff**

$$\phi_i(x)\phi_j(x - e_i) = \phi_j(x)\phi_i(x - e_j) \qquad (1)$$





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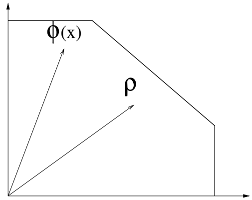
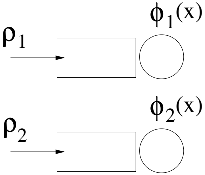
- no resource wasting iff

$$\text{in all states } x, \quad (\phi(x)A)_l = C_l \quad \text{for some } l \qquad (2)$$





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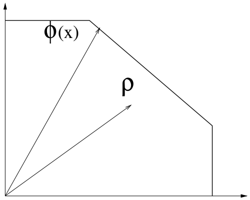

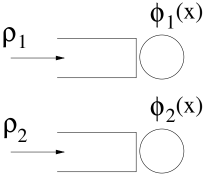

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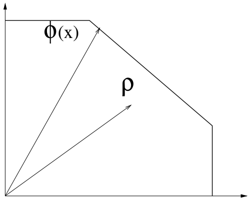
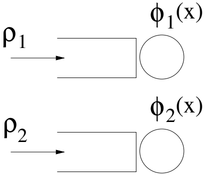
- **balanced fairness = (1) + (2)**





Insensitivity

- **stationary distribution**

$$\pi(x) = \phi(0)\Phi(x)\prod_{i=1}^{N}\rho_i^{x_i}$$

with

$$\Phi(x) = \frac{1}{\phi_{i_1}(x)\phi_{i_2}(x-e_{i_1})\ldots\phi_{i_n}(e_{i_n})}$$

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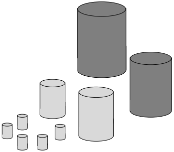
- **insensitive to all traffic characteristics beyond** ρ



Insensitivity

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with

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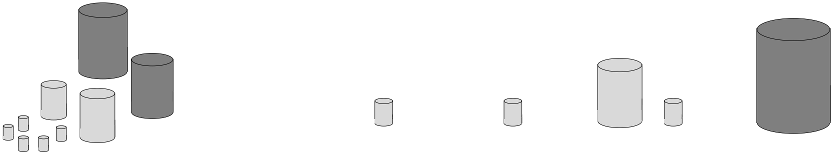
- insensitive to all traffic characteristics beyond ρ
- you only need Poisson session arrivals (B-Proutière 03)



Throughput performance

- **mean bit rate as experienced by users**

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- **class-i flow throughput**

$$\gamma_i = \sum_x \frac{x_i \pi(x)}{\bar{x}_i} \frac{\phi_i(x)}{x_i}$$
$$= \frac{1}{\bar{x}_i} \sum_x \pi(x)\phi_i(x) = \frac{\rho_i}{\bar{x}_i} \quad \textbf{(in bit/s)}$$

Throughput performance

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$$= \frac{1}{\bar{x}_i} \sum_x \pi(x)\phi_i(x) = \frac{\rho_i}{\bar{x}_i} \quad \textbf{(in bit/s)}$$

- by Little's law

$$\gamma_i = \frac{1}{\tau_i}$$

τ_i = mean per-bit delay (in s/bit)
= mean flow duration/mean flow size

Key result

- let $\varrho = \rho A$ be the vector of resource utilizations (the stability condition is $\varrho < C$)

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- for balanced fairness,

$$\max_l \frac{A_{il}}{C_l - \varrho_l} \leq \tau_i \leq \max_l \frac{A_{il}}{C_l} + \sum_l \frac{\varrho_l}{C_l} \frac{A_{il}}{C_l - \varrho_l}$$

Key result

- let $\varrho = \rho A$ be the vector of resource utilizations (the stability condition is $\varrho < C$)
- for balanced fairness,

$$\max_l \frac{A_{il}}{C_l - \varrho_l} \leq \tau_i \leq \max_l \frac{A_{il}}{C_l} + \sum_l \frac{\varrho_l}{C_l} \frac{A_{il}}{C_l - \varrho_l}$$

- tight if $\varrho_l \to C_l$ for some resource l

$$\tau_i \sim \frac{A_{il}}{C_l - \varrho_l}$$

Examples

- **focus on the lower performance bound (conservative throughput estimate)**

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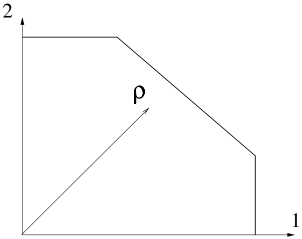
- **assume equal traffic intensities** $\rho_1 = \ldots = \rho_N$

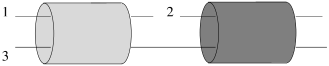


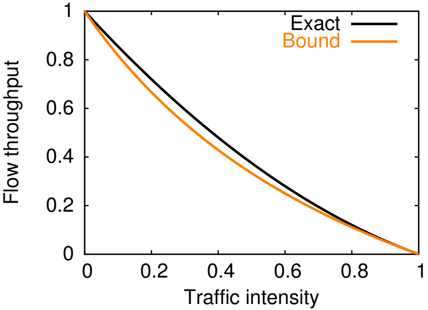
A linear network

- **flow throughput**

$$\gamma_1 = 1 - \varrho_1, \quad \gamma_2 = 1 - \varrho_2$$

$$\gamma_3 \geq \frac{(1-\varrho_1)(1-\varrho_2)}{1-\varrho_1\varrho_2}$$

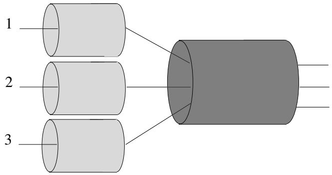


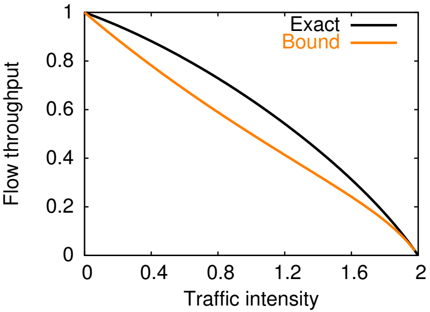


A tree network

- **flow throughput**

$$\gamma_i \geq \frac{2(2-\varrho_1)(3-\varrho_1)}{12-3\varrho_1-\varrho_1^2}$$



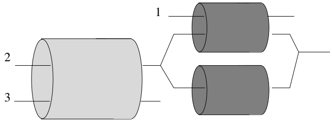


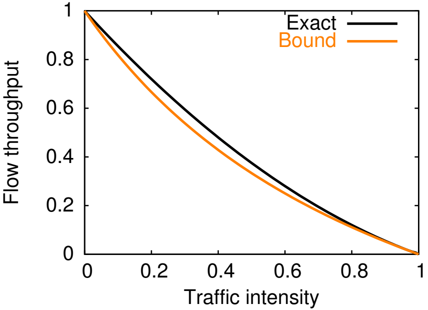
Multi-path routing

- **flow throughput**

$$\gamma_1 \geq \frac{2(1-\varrho_1)(2-\varrho_2)}{4-\varrho_1\varrho_2-\varrho_2}$$

$$\gamma_2 \geq \frac{2(2-\varrho_2)(2-\varrho_3)}{4-\varrho_1\varrho_2}$$



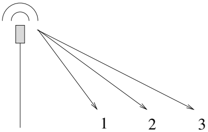


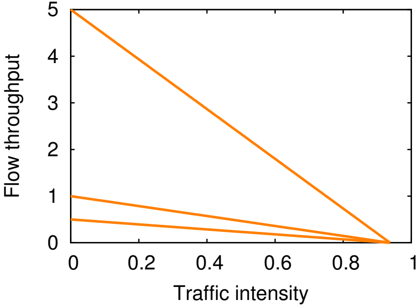
Wireless network

- **flow throughput**

$$\gamma_i = c_i(1 - \varrho)$$

ϱ, **cell load**

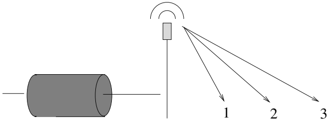




Wireless network

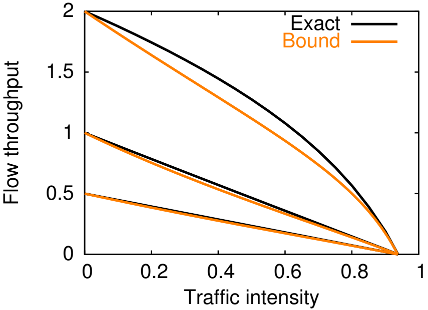
- **flow throughput**

$$\gamma_1 \geq \left(\frac{\varrho_1}{c_1(1-\varrho_1)} + \frac{1}{C-\varrho_2} \right)^{-1}$$



ϱ_1, **wireless link load**

ϱ_2, **wired link load**

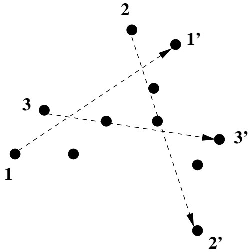


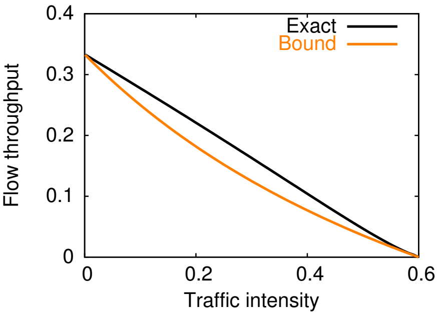
Ad-hoc network

- **flow throughput**

$$\gamma_3 \geq \frac{3 - 5\varrho}{9 + 10\varrho}$$

ϱ, **total traffic intensity**





Impact of flow rate limits

- a_i, class-i flow rate limit

Impact of flow rate limits

- a_i, **class-i flow rate limit**
- **then,**

$$\max\left\{\frac{1}{a_i}, \max_l \frac{A_{il}}{C_l - \varrho_l}\right\} \le \tau_i \le \frac{1}{a_i} + \sum_l \frac{\varrho_l}{C_l}\frac{A_{il}}{C_l - \varrho_l}$$

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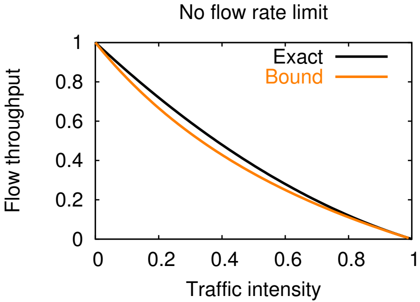
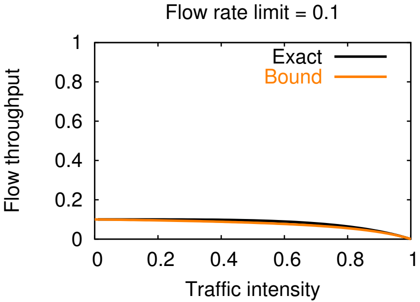
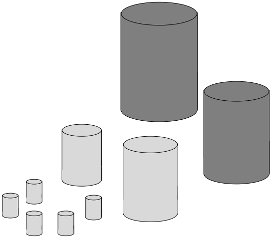
- example of the linear network





Conclusion

- **flow throughput in the presence of random arrivals**



Conclusion

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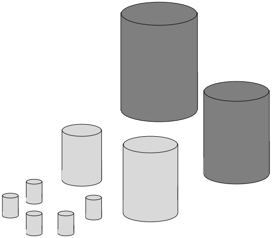
- a complex issue for max-min fairness and proportional fairness



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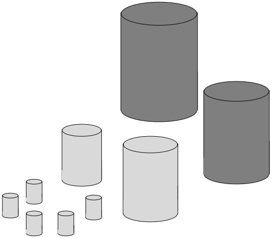

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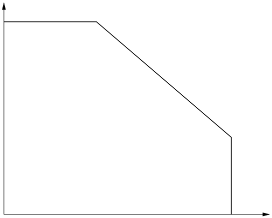

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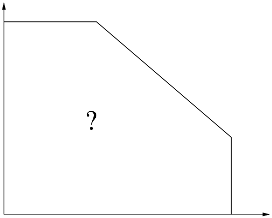

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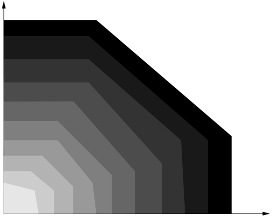

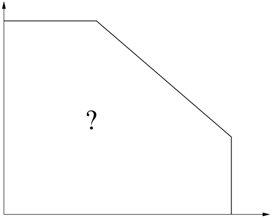

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