Distributed Approaches for Proportional and Max-Min Fairness in Random Access Ad-Hoc Networks

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OUTLINE

- Introduction
  - Motivation and System model
- Proportional Fairness
  - At the link layer
  - For end-to-end sessions
- Max-min Fairness
  - At the link layer
- Open Issues

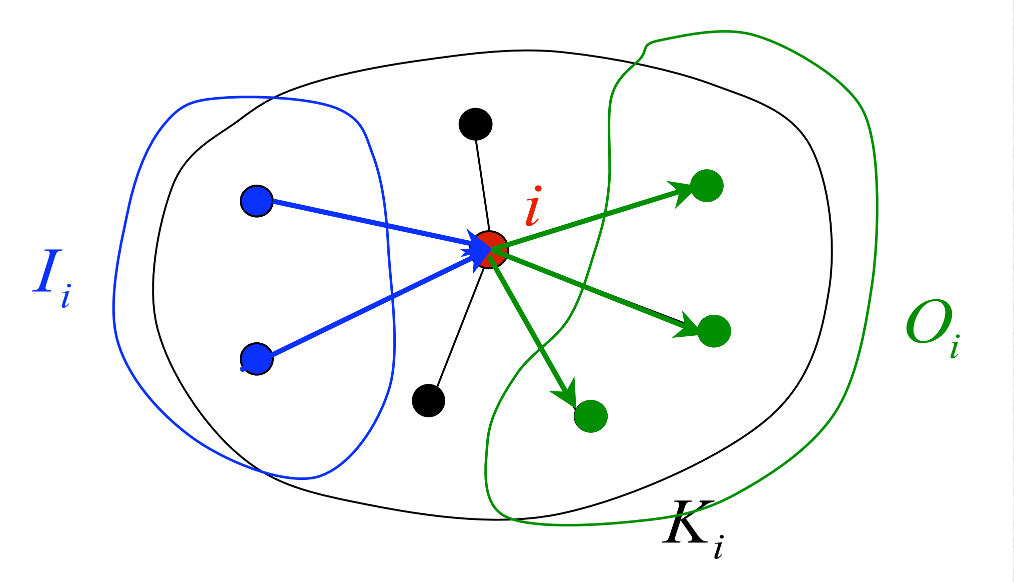
MAC Rate Optimization (1/2)

- Bandwidth optimization question at the MAC layer
  - How to achieve global fairness or throughput guarantees (in a multi-hop network)
  - Using distributed approaches (only local information/ coordination)
- We focus on random access networks
  - Aloha model
  - We have some results for a CSMA/CA model as well
    - Much more complex model
    - Weaker results
    - Not the focus of this talk

MAC Rate Optimization (2/2)

- We also consider a cross-layer rate optimization question
  - MAC + Transport Layer
  - More complex question
- Fairness metrics:
  - Proportional fairness: maximize the sum of the logarithms of the link rates
  - Max-min fairness: maximize the minimum of the link rates (in a “lexicographic” manner)
- Mostly focus on dual-based algorithms

System Model & Assumptions



- Assumptions:
  - Symmetric hearing matrix
  - Single transceiver per node
  - No capture

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Proportional fairness: S-Aloha

- Rate equations:

$$x_{ij}(\mathbf{p}) = p_{ij}(1 - P_j) \prod_{k \in K_j \setminus \{i\}} (1 - P_k)$$

$$\sum_{j \in O_i} p_{ij} = P_i$$

  - p_{ij} : attempt probability on link (i, j)

- Proportional fairness:
  - Choose $\mathbf{p}$ so as to maximize

$$\sum_{(i,j) \in L} \log\left(x_{ij}(\mathbf{p})\right)$$

Proportional Fairness: U-ALOHA

- Rate equations:

$$x_{ij}(\lambda) = T\lambda_{ij}\; e^{-T\sum_{k\in K_j\cup\{j\}\setminus\{i\}}\lambda_k} \prod_{k\in K_j\cup\{j\}} \frac{1}{1+T\lambda_k}$$

$$\sum_{j\in O_i} \lambda_{ij} = \lambda_i$$

  - λ_{ij} : attempt probability on link (i, j)

- Proportional fairness:
  - Choose λ so as to maximize

$$\sum_{(i,j)\in L} \log(x_{ij}(\lambda))$$

Optimum Rate Computation

- s-Aloha

$$p^*_{ij} = \frac{1}{|I_i| + \sum_{k \in K_i} |I_k|}$$

- u-aloha

$$\lambda^*_{ij} = \frac{\sqrt{1 + \frac{|O_i|}{\sum_{k \in K_i \cup \{i\}} |I_k| - |O_i|}} - 1}{T|O_i|}$$

- Computing the optimal rates requires only <u>local topology</u> information

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Cross-Layer Proportional Fairness

- Objective: Attain the optimal end-to-end session rates
  - Joint optimization question involving the MAC & transport layer
- End-to-end proportional fairness:

$$
\begin{array}{lll}
\max & \sum_{s\in S}\log(y_s), & \\
\text{s.t.} & \sum_{s\in S(i,j)} y_s \le x_{ij}(\mathbf{p}) & \forall (i,j)\in L, \\
 & y_s \ge 0 & \forall s \in S,
\end{array}
$$

$$
x_{ij}(\mathbf{p}) = p_{ij}(1-P_j)\prod_{k\in K_j\setminus\{i\}}(1-P_k)
$$

- The problem couples the link attempt probabilities with the end-to-end session rates

Dual Based Algorithm (1/3)

- A two-timescale algorithm based on "local" coordination converges to the optimal rates of this problem
- Algorithm (primal-dual type):
  - Flow Control at Transport Layer (faster time scale)
    - Assuming the link rates are fixed, the sessions solve the optimal flow rates
  - Link Attempt Probability Adjustment at Link Layer (slower time scale)
    - Each link then adjusts its rate based on its dual price, and the rates of links in its neighborhood

Dual based algorithm (2/3)

- Transport-layer problem:

$$\begin{array}{lll} \max & \sum_s \log(y_s), & \\ \text{s.t.} & \sum_{s \in S(i,j)} y_s \le \tilde{x}_{ij} & \forall (i,j) \in L, \\ & y_s \ge 0 & \forall s \in S. \end{array}$$

- Convex, separable problem
  - Can be solved in a distributed manner, using “local” coordination
  - An approach that yields the optimal dual variables is required (these are required in the link attempt probability adjustment)
  - Can use the dual based approach by Low and Lapsley ’98)

Dual based algorithm (3/3)

- Link-layer rate update:

$$p_{ij}^{(n+1)} = p_{ij}^{(n)} + \alpha \sum_{(s,t)\in L} \lambda_{st}^{*(n)} \frac{\partial x_{st}}{\partial p_{ij}}(\mathbf{p}^{(n)}),$$

$$\frac{\partial x_{st}}{\partial p_{ij}} = \begin{cases} (1-P_t)\prod_{k\in K_t\setminus\{s\}}(1-P_k) & \text{if } t=j, s=i, \\ -p_{st}\prod_{k\in K_t\setminus\{s\}}(1-P_k) & \text{if } t=i, s\in I_t, \\ -p_{st}(1-P_t)\prod_{k\in K_t\setminus\{s\}}(1-P_k) & \text{if } t\in K_i, s\in I_t\setminus\{i\}, \\ 0 & \text{otherwise,} \end{cases}$$

  - $\lambda_{st}^{*(n)}$: optimal dual variables of the transport-layer problem

- Only requires "local" coordination

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Max-Min Fairness

- Maximize the minimum rate *lexicographically*
  - Maximize the minimum rate, then maximize the second minimum rate (subject to the minimum rate being the maximum), and so on
  - A rate cannot be increased further without decreasing a rate of equal or lesser value
- Widely popular fairness metric
  - "Ideal" fairness measure

DIFFICULTIES

- Does not have a “nice” programming formulation
  - Tries to maximize multiple objectives simultaneously
- Can be approximated by a single-objective separable utility maximization problem
  - E.g., define $U_i(x) = (\log(x_i))^n$
  - Maximize the sum of the utilities
  - Close approximation if n is large
  - Convergence problems if a large n is chosen $\Rightarrow$ attaining close approximation is difficult

Basic Approach

- Motivated by the popular "bottleneck-based" max-min rate allocation algorithm in wired networks
  - Maximizes multi-hop session rates subject to link capacity constraints
- Basic idea: Iteratively do the following:
  - Identify bottleneck link(s)
  - Divide the available bottleneck link capacity to all sessions sharing (bottlenecked by) that link
  - Remove the bottleneck link and the bottlenecked sessions from consideration

Differences

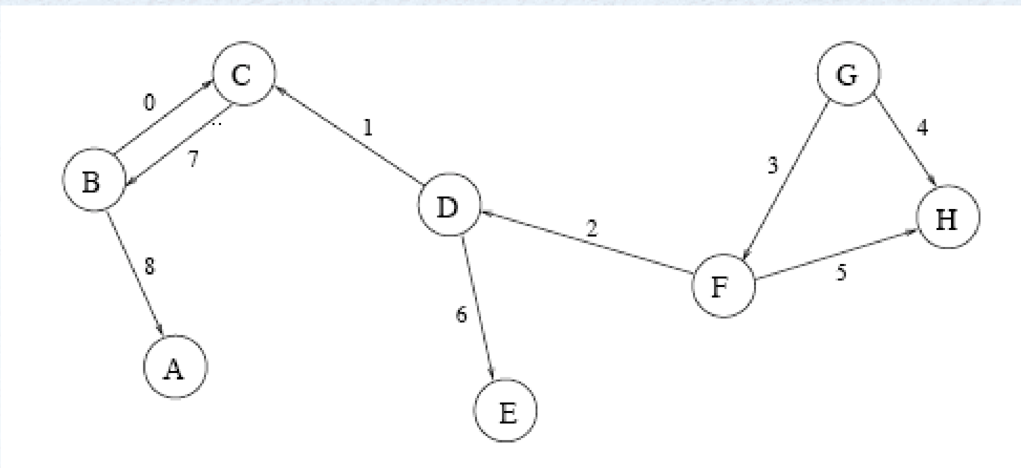
- No multi-hop sessions in our case
  - We consider link layer only
- Constraints are much more complex in wireless
  - Unlike the wired case, constraints are not linear anymore

Directed Link Graph (1/2)

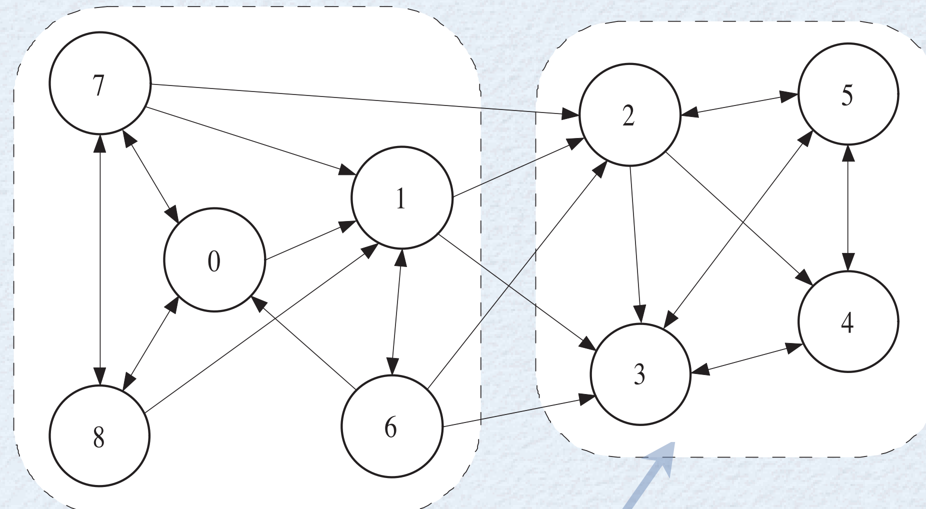
- Nodes represent links
- Edge from i to j if link corresponding to i *interferes* link corresponding to j
  - When i transmits, j cannot transmit successfully
  - Occurs of receiver of j is in the range of the transmitter of i
  - Interference is in general an asymmetric relationship

DIRECTED LINK GRAPH (2/2)

original graph



directed link graph (DLG)



strongly connected components

- Strongly connected component (SCC)
  - All nodes in a SCC must have a path to each other

Approach Outline (1/3)

- Property:
  - Max-min fair rates of all links in the same SCC must be equal
- Max-min fairness in graph with a single SCC (s-max-min):

$$\begin{aligned} \max \quad & x, \\ \text{s.t.} \quad & x \le x_{ij}(\mathbf{p}), \qquad \forall (i,j) \in L, \end{aligned}$$

$$x_{ij}(\mathbf{p}) = p_{ij}(1 - P_j) \prod_{k \in K_j \setminus \{i\}} (1 - P_k)$$

- Can be solved in a distributed manner
  - Simple transformation makes the problem separable & convex

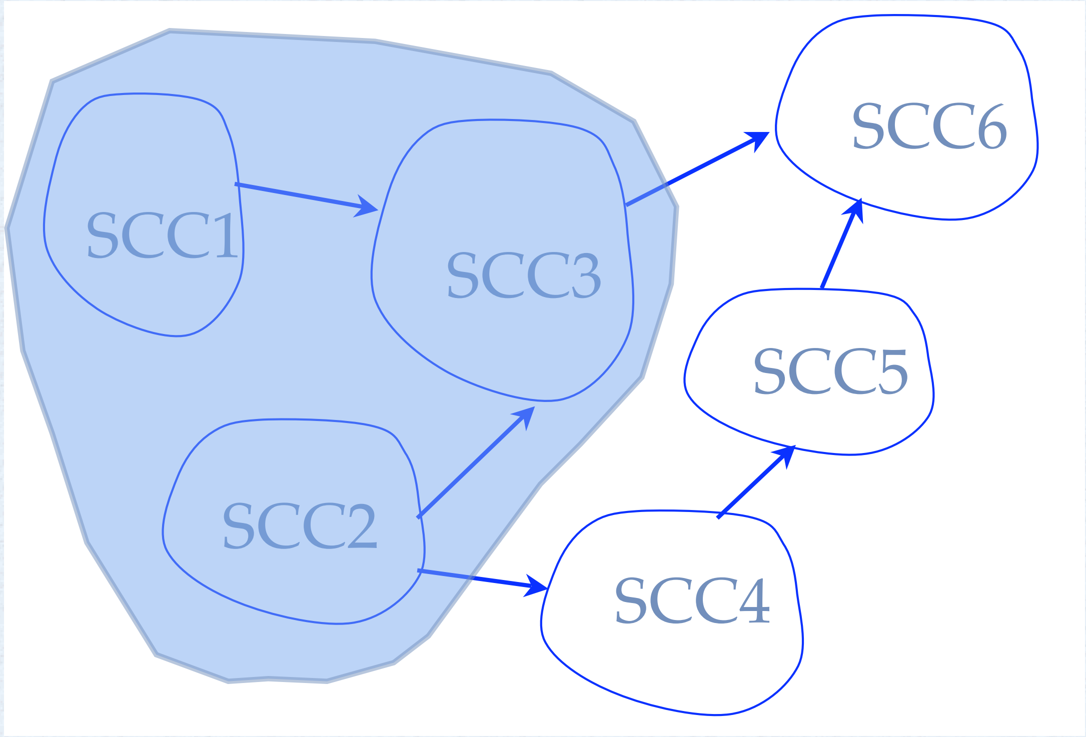
Approach Outline (2/3)

- What if the graph consists of multiple SCCs ?
  - Rates of different SCCs may be different
  - The (lexicographic) max-min fairness problem is difficult
  - However, maximizing the minimum rate ("simple" max-min) is easy $\Longrightarrow$ solve the s-max-min problem

- Bottleneck SCC = SCC that attains this minimum "robustly"

- Algorithm: Iteratively do the following:
  - Solve the s-max-min rate allocation problem
  - Identify all bottleneck SCCs (or any set of bottleneck SCCs that do not have any predecessor SCC)
  - Set the link attempt probabilities of all links in the SCCs to the optimal values of the s-max-min rate

APPROACH OUTLINE (3/3)



Number of iterations
= Number of SCCs
(in worst case)

- Valid choices of the bottleneck SCC set
  - {SCC1}, {SCC2}, {SCC1,SCC2}, {SCC1,SCC2,SCC3}, ...
- Incorrect choices of the bottleneck SCC set
  - {SCC3}, {SCC2,SCC3}, ...

Bottleneck SCCs & Links

- Properties:
  - A link is bottleneck if it corresponds to a singularity constraint in the s-max-min problem
  - In any SCC, either all links are bottleneck, or none is a bottleneck
  - A SCC is bottleneck if any (or all) of its links are bottlenecks
- Notes:
  - If we can identify at least one bottleneck link, we are done
  - Ideally (to minimize number of iterations), we want to identify all bottleneck links

Bottleneck Link Identification (1/2)

- How can we identify bottleneck links (SCCs) easily ?
  - In general, can identify singularity constraints by solving another convex optimization problem for each link (too expensive)
- Dual based approach
  - A link is bottleneck iff its optimal dual variable is positive
  - Optimal dual variables of all non-bottleneck links is zero
  - Not true in general (hold due to the special structure of the s-max-min problem)

Bottleneck Link Identification (2/2)

- How about a primal based approach ?
  - Just looking at the active (binding) constraints does not work (non-bottleneck links can be active)
  - Applying a barrier penalty pushes "redundant" or "non-robust" constraints away from the boundary
  - In the limit, constraints are active iff they are singular
  - In practice, however, can only identify bottleneck links "approximately"

Comparisons

Session-level max-min fairness in wired networks	Link-level max-min fairness in wireless random access networks
Optimize rates of end-to-end sessions	Optimize rates of links
Linear packing-type constraints	Non-linear constraints
Links	Strongly Connected Components (SCCs)
Bottleneck links easy to identify	Bottleneck SCC (link) identification requires an optimal dual solution
Optimal rates in any iteration easy to find	Finding optimal rates requires solving a convex optimization problem

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Open Questions (1/2)

- Max-min Fairness for end-to-end sessions ?
  - Cross-layer (Transport + MAC) rate optimization question
- Can we attain max-min fairness with local topology information ?
  - At least approximately (with a “good” approximation ratio)
- How about more general fairness metrics ?

Open Questions (2/2)

- Fairness questions in CSMA (CSMA/CA) models
  - Throughput expressions are very different
  - Expressed in terms of independent sets in the network
  - “Global” dependencies
  - The case of a general ad-hoc network appears very difficult

THANK YOU!

Related papers/technical reports available at:

http://www.ecse.rpi.edu/~koushik/