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Center for Energy and Environmental Studies
Princeton University, Princeton NJ 08544, USA

*Indoor Environment Program Lawrence Berkeley Laboratory
Berkeley, CA 94720

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Abstract

The technique presently considered most effective for mitigating residences for radon is the subslab depressurization technique. Given that a large number of such mitigation systems designed and installed by the professional community do not perform entirely to satisfaction, there is an urgent need to formulate guidelines to enhance the effectiveness of the engineering design of radon mitigation systems. An important component to this activity is an increased scientific understanding of the factors governing the pressure-induced air flow below the slab, thereby allowing modeling of the pressure field in order to optimize the number and location of suction points.

This paper evaluates and refines an earlier study which proposed a mathematical formulation for modeling the pressure field induced by a single suction point, when air flows radially through a porous bed contained between two impermeable disks. The physical relevance of this model as a logical first step towards sound engineering solutions for professional radon mitigators is discussed. Next it is shown that the air flow is most likely turbulent under actual field situations in houses with subslab gravel beds while remaining laminar when soil is present under the slab. Then a laboratory apparatus built to experimentally determine the model coefficients of the pressure drop versus flow for commonly encountered gravel materials is described. Various aspects of non-linear regression relating to the soundness and stability of the estimated coefficients are also discussed. An engineering model is proposed for the pressure drop associated with the change in direction and with the turbulence at entry i.e., at the throat of the suction hole. The related coefficients for different types of gravel are also determined from experiments on the laboratory apparatus. Finally, logical extensions of this study are suggested in order to make it more relevant to actual field applications.
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1. **Introduction**

The presence of radon 222, a colorless, odorless gas naturally present in trace concentrations in soil gas and underground water has been found to be a serious health hazard in many American houses. Generally, radon-contaminated soil gas enters a home through cracks and openings in the slab being driven by pressure differences between the subsoil and the basement and walls, or between the subsoil and the living area in case of a slab-on-grade house construction. These pressure differences occur naturally, either because of stack (i.e. temperature difference) or wind effects, or due to zonal depressurization due to the effect of the heating and air-conditioning system (HAC). The Environmental Protection Agency (EPA) has set a guideline of 148 Bq/m³ (4 pCi/L) as a radon level beyond which mitigation is recommended. Many homes in New Jersey as well as in other states in the US have average radon levels greatly exceeding this limit, hence the concern relating to the issue of proper design of mitigation systems.

An EPA sponsored workshop was held at Princeton University in order to summarize available knowledge on various radon diagnostic techniques [1]. Four phases of radon diagnostics were defined:

a) **Radon problem assessment** diagnostics involving radon source strength, location, house characteristics, and house occupancy characteristics.

b) **Pre-mitigation** diagnostics entailing the selection of the best mitigation system for the particular building taking into account radon source strength and location, particularly at the substructure.
c) Mitigation Installation diagnostics used during installation of mitigation systems in order to assure proper operation.

d) Post-mitigation diagnostics to assure that the radon guidelines have been met and that the mitigation system is adjusted properly.

The emphasis of the workshop was on diagnostics since each home, housing division and region have different radon characteristics which require that special attention be paid to system design in order to maximize mitigation performance and minimize cost. This issue was of particular importance since it was found that a large number of houses which had been mitigated still had radon levels above the EPA recommended guideline of 148 Bq/m³ (4 pCi/L). In fact, a recent study [2] found that 64% of the homes in New Jersey where post-mitigation radon measurements have been made, remain above the recommended radon level. Diagnostics are therefore crucial for providing information relevant and necessary to the successful design and implementation of a radon mitigation system.

Many participants of the workshop felt that radon mitigation via subslab depressurization was the best approach for houses with a gravel bed. Surveys indicate that systems based on this technique account for more than 50% of all installed systems [3]. (Another promising technique involves subslab pressurization. Since the two techniques are similar in terms of subslab dynamics of induced air flow, they can both be treated in the same scientific framework.) In the pre-mitigation diagnostic phase, the degree of "connectivity" under the slab as well as the permeability characteristics of the subslab medium must be determined before appropriate depressurization conditions can be determined. Proper attention to these aspects will ensure that reasonable flows, and hence the desired degree of depressurization,
will prevail at all points under the slab. Lowering the pressures at all points of the subslab to values below those of the basement/crawl-space/living area will subsequently reduce radon-rich soil-gas from moving or seeping into the building.

Parallel with the above aspect is the concern that presently mitigators tend to over-design subslab depressurization systems (SSD) in order to err on the safe side. In so doing, more radon from the soil is removed and vented to the ambient air than what would have occurred naturally. There is thus the need to downsize current overly-robust SSD mitigation systems and decrease emission exhaust quantities of radon while simultaneously assuring that indoor radon levels do not exceed the recommended value.

The Center for Energy and Environmental Studies of Princeton University is currently involved in the formulation and verification of a rapid diagnostic protocol for subslab and wall depressurization systems designed to control indoor radon levels [4]. It is hoped that the protocol would lead not only to the ability to distinguish between homes that are hard or easy to mitigate, but also to the articulation of a more rational and scientific approach which would be especially useful to the ever-increasing body of professional radon mitigators.

Our approach to the formulation of the diagnostics protocol consists of:

(i) a practical component, in that specific guidelines would be suggested so that the effectiveness of the engineering design of the radon mitigation system would be enhanced, and

(ii) scientific studies at a more fundamental level which would lend both credence and also enable defining the guidelines more rationally.
The scientific component would involve improved understanding of those factors governing the pressure-induced air flow beneath the concrete slab and the ability to first predict and then optimize the pressure fields induced by single and multiple suction points. The present study specifically addresses the former aspect, while the latter would be dealt with in a subsequent study.

2. **Specification of the Problem**

In terms of modeling the induced subslab pressure fields, one could conceptually divide the present housing stock construction into broadly three groups: (i) those with a gravel bed under the concrete slab, (ii) those without, in which case soil is the medium under the slab, and (iii) those houses which have both. In the case of (ii), the subslab permeabilities are much lower than for case (i) requiring more careful design of the mitigation system. In New Jersey, houses less than about 30 years old typically have gravel beds of about 0.05-0.1 m (2"-4") under the slab. However other states seem to have very different construction practices: for example, houses in Florida are built directly on compacted fine-grained soil which offers high resistance to air flow.

Figure 1 schematically depicts the type of construction and the expected air flow paths one would typically expect in a house with a gravel bed when a single suction point through the slab is used to induce a pressure field. In case of a radon mitigation system using subslab pressurization, one could, to a good approximation, simply assume similar hydrodynamic effects with the direction of air flows reversed. Since the permeability of the gravel bed is usually very much higher than that of the soil below, one could assume, except for the irregular pattern around the footer which would
occur over a relatively small length, that the subslab air flow is akin to radial flow between two impermeable circular disks with a spacing equal to the thickness of the gravel bed. Note that this model equally accounts for the leakage of air from the basement which essentially occurs from the perimeter tracks or through the basement wall.

In case of a house without a gravel bed, suction applied at a simple penetration through the slab (as in Fig. 1) is no longer practical in low permeability soils since the area of depressurization is small. In order to enhance mitigation effectiveness, the contemporary thinking is to increase this area of depressurization by either digging a gravel pit below the concrete slab as shown in Fig. 2, or more simply by hollowing out a hemisphere of about 0.3-0.45 m (12"-18") radius underneath the suction hole. Even under such conditions, and provided the soil underneath is free of major obstructions like concrete footers, duct work, piping and large rocks, one could view air flow as occurring between two impermeable circular disks with a spacing equal to either the depth of the gravel pit or to the radius of the hollow hemisphere.

The above discussion was intended primarily to suggest that flow underneath the slab be visualized as occurring in radial streamlines terminating at the central suction point. Note that such a representation would perhaps be too simplistic or even incorrect for a house with a partial-basement (case (iii) above). In the present study, we shall limit ourselves to understanding the flow and pressure drop characteristics through a homogeneous bed (of either gravel or sand) with uniform boundary conditions, the obvious case to start with being a circular configuration.
The first question to be addressed relates to the nature of the flow, i.e., whether the flow is laminar or turbulent, and whether there is a transition from one regime to another.

As outlined in Appendix A, where a brief overview of the basic scientific theory of flow through porous beds is given, the Reynolds number gives an indication of the flow regime. Though there is an inherent ambiguity in the definition of the quantity characterizing the length dimension, we shall adhere to the following definition:

\[ \text{Re} = \frac{q}{A} \cdot \frac{l}{\gamma_a} \cdot \frac{d_v}{\phi} \]  

(1)

where

- \( q \) - total volume flow rate,
- \( A \) - cross sectional area of the flow (in case of radial flow through a circular bed of radius \( r \) and thickness \( h \), the area = \( 2\pi rh \)),
- \( \gamma_a \) - kinematic viscosity of air,
- \( d_v \) - equivalent diameter of pebbles,
- \( \phi \) - void fraction or porosity of the gravel bed.

Let us first look at flow through a gravel bed. Some typical values of the above parameters could be assumed:

- \( h = 0.1 \text{ m (4")} \), \( d = 0.0125 \text{ m (1/2")} \), \( \phi = 0.4 \), \( \gamma_a \) (at 15°C) = 14.6x10^{-6} \text{ m/s} \).

The values of \( q \) encountered in practice range from 9.4 x 10^{-3} to 47.2 x 10^{-3} \text{ m}^3/\text{s} (20-100 cfm). Under these conditions, the resulting Reynolds number for radial flow at different radii can be determined from Fig.3. From Appendix A, we note that a safe lower limit for turbulent flow is when \( \text{Re} > 10 \) and a safe upper limit for laminar flow is when \( \text{Re} < 1 \). Since basements do not generally exceed 6 m (20 feet) in radius, we note
from Fig. 3 that subslab flow would tend to be largely turbulent when a gravel bed is present. This by itself is an important finding since explicit recognition does not seem to have been made of this aspect in earlier studies.

The usefulness of Fig. 3 can be extended to cover other types of circular configurations. Thus if one would like to estimate Re numbers for radial flow through a slice with impermeable sides and with an opening angle $\alpha$ instead of an entire pie-configuration, one needs simply to use the following correction:

$$Re_\alpha = Re_{360}(360/\alpha)$$

(2a)

where $Re_{360}$ is read from Fig. 3.

Also if the disk spacing is not 0.1 m (4") but say $h'$, the Reynolds number can be obtained from Fig. 3 corresponding to an effective radius $r'$ in meters given by:

$$r' = r(0.1/h')$$

(2b)

Similar corrections can be made to other parameters as well. Thus, in conclusion, we should expect turbulent flow conditions to prevail through subslab gravel beds during normal operation of mitigation systems using the subslab depressurization (or pressurization) technique. On the other hand, in a house with soil as the subslab medium, this observation is no longer valid. Grain diameters of sand range from 0.06 to 2 mm (0.0024-0.08") [6] while volume flow rates in corresponding mitigation systems are typically lower, about $0.83 \times 10^{-3} - 6.2 \times 10^{-3} \text{ m}^3/\text{s}$ (2-15 cfm). Assuming some typical values of $h = 0.1 \text{ m (4")}$, $\phi = 0.4$ and $q = 2.4 \times 10^{-3} \text{ m}^3/\text{s}$ (5 cfm), the corresponding Reynolds numbers for air flow through sands of different grain diameters have been calculated from eq.(1) and are shown in Fig. 4. We note
that the flow is likely to be laminar in most cases.

Finally, we present Figs. 5 and 6 in order that the reader acquire a feel for the range of air flow velocities which would induce the values of Reynolds numbers seen in Figs. 3 and 4. These have been computed from eq.(1) for different values of particle diameters corresponding to both gravel and sand beds. We note that even with very small velocities, of the order of a few millimeters per second, flow is likely to be turbulent.

3. Mathematical Model for Radial Flow

At the onset, let us mention that there are essentially two different problems involved with modeling the inflow of radon-enriched air into a residence. One problem is associated with flow through openings in the slab resulting from small pressure differences impressed across the concrete slab due to environmental driving forces (stack effect, wind and HVAC operation). The other problem involves modeling the air flow under the concrete slab when pressure differentials are applied at one, or several, perforations through the slab (conditions that arise when houses are being mitigated). Though both these problems essentially involve modeling the air flow through the ground under the slab, the difference is that the dynamics and entry paths are entirely different: flow paths and flow regimes (both in soil and into the basement) and external factors causing flow will be widely different in both. The pressure differences in the former (i.e., flow occurring naturally in the absence of a suction pressure) are so small that the assumption of a laminar flow is valid, and effects like diffusion through soil and via small capillary cracks into the basement have to be considered. In the case of the latter, the flow regime in gravel beds will
most probably be turbulent and the flow paths of radon enriched air will be predominantly towards the suction hole.

As a result of these differences, studies or models pertaining to one problem cannot be applied as such to the other. There are a number of studies which have addressed the first problem (e.g., [7-10]) while studies relating to the latter are very few. We could only find two relevant studies; one numerical study involving a 3-D finite element computer model [11], and the other an empirically based study [5]. The principal drawbacks of Ref. [11] are that laminar flow is assumed and the large computer code is difficult to use and to transfer to other research groups. Moreover, a certain amount of effort is required in order that the results of such codes be useful to practitioners.

The core of any model is the formulation of the structure of a correlation between pressure drop and Reynolds number. There is an abundance of literature relating to flow through porous media as evidenced by the large number of books and monographs on this topic (e.g., [12-17]). A major portion of this literature relates to laminar flow where Darcy’s Law holds (see Appendix A for explanation). This fact is not too surprising since flow characteristics in petroleum and gas fields, or water flow in subsoils, or decontamination of subsoil aquifers, were some of the problems which historically led to the scientific treatment of flow through porous media. Subsequently, this was extended to various other problems such as flow through fluidized and packed beds, both in chemical kinetics and in areas involving drying of grains and sensible heat storage. We have made a preliminary search through such literature and find that one cannot directly
adopt a particular correlation as such to the present problem though insight into the type of needed model structure can be gained.

Consequently, (this is further discussed in Appendix A), we have adopted in the present study the following simple model structure for the onset of turbulent flow.

\[ \text{Re. } f^n = c \] (3a)

where \( f \) is the friction factor of the porous bed and \( c \) and \( n \) are empirical dimensionless coefficients.

In Appendix A (see eqs. (A16 and A17)), we show that this model structure is identical to the following model:

\[ \frac{1}{\rho_f g} \cdot \frac{dp}{dx} = a \left( \frac{q}{A} \right)^b \] (3b)

where the left hand-side is the pressure drop in head of the flowing fluid per unit bed length. The parameter \( a \) can be loosely interpreted as the resistivity of the porous bed to the flow of the particular fluid.

Theory predicts a value of unity for the exponent \( b \) when the flow is laminar (we then have the Darcy Law), and a value nearer to 2 when the flow is turbulent. Given the irregularities both in shape and size prevalent in gravel beds below the concrete slab, the coefficients should be determined from regression to actual data obtained by experiments on the specific bed material. The real objective of this study is thus to gauge the accuracy of such a model structure when applied to house-like conditions. Note also
that the effective permeability* of a porous bed (see eqs. (A9-A12) of Appendix A) can be easily deduced from the coefficient $a$ of eq. (3b), since

$$k = \frac{\nu_a}{g} \cdot \frac{1}{a}$$

(4)

where $\nu_a$ is the kinematic viscosity of air.

Ref. [5] also uses a model like that of eq. (3) with, however, the parameters transposed, i.e. $(q/A)$ expressed as a function of the pressure drop gradient. It is trivial to go from one form to another but we find it more convenient to work with an equation like eq. (3b).

We shall now seek to derive a mathematical expression for the pressure field when air flows radially through a circular homogeneous gravel bed when suction is applied at the center of the circle. The laws of conservation of matter and of energy must be satisfied in any hydrodynamic system. By setting viscosity terms to zero in the Navier-Stokes equations we get the Euler equation of motion. For the suction pressures encountered in this particular problem, air can be assumed to be an incompressible fluid and we have Bernoulli’s equation, which in differential form is:

$$\frac{d}{dr} \left[ \frac{1}{2g} \left( \frac{v^2(r)}{g \cdot \rho_a} \right) + \frac{p(r)}{g \cdot \rho_a} \right] = 0$$

(5)

*: We use the term ‘effective permeability’ instead of permeability, only because of the uncertainty involved in the extrapolation from the present experimental data (done under turbulent flow) to Darcy flow (i.e., laminar flow conditions) under which permeability is conventionally defined. This aspect is further discussed in Section 5.
where \( \rho_a \) is the density of air, \( V \) the superficial velocity and \( p(r) \) the pressure of air at a radial distance \( r \) from the center. Strictly, the distance should be taken from the outer edge of the suction pipe (\( r' \) in Fig. 7). The diameter of the suction pipe is typically so small that one could neglect this difference without any error in the subsequent analysis.

Assuming that energy or pressure drop (in units of head of air) lost as a result of viscous drag by the gravel bed can be simply treated as an additive term, we have

\[
\text{Total pressure drop} = \text{Pressure drop due to changing cross-section} + \text{Pressure drop due to viscous drag}
\]

Assuming a simple model such as eq.(3b) yields

\[
\frac{d}{dr} \left[ \frac{p(r)}{\rho_a g} \right] = - \frac{d}{dr} \left[ \frac{1}{2g} \left( \frac{q}{2\pi h} \right)^2 \frac{1}{r^2} \right] + a \left( \frac{q}{2\pi h} \right)^b \frac{1}{r^b} \tag{5}
\]

Integrating eq.(6), we have

\[
\frac{p(r)}{\rho_a g} = a \left( \frac{q}{2\pi h} \right)^b \frac{r^{1-b}}{(1-b)} - \frac{1}{2g} \left( \frac{q}{2\pi h} \right)^2 \frac{1}{r^2} + \text{constant} \tag{7}
\]

The constant of integration can be found from the boundary conditions. At the outer fringe of the cylindrical disk:

\[
r = r_o \quad , \quad p = p_a \quad (\text{the atmospheric pressure}).
\]

Introducing this into eq.(7), the following expression for the pressure drop is obtained

\[
\frac{p(r)-p_a}{\rho_a g} = a \left( \frac{q}{2\pi h} \right)^b \frac{1}{(1-b)} \left( r^{1-b}-r_o^{1-b} \right) - \frac{1}{2g} \left( \frac{q}{2\pi h} \right)^2 \left( \frac{1}{r^2} - \frac{1}{r_o^2} \right) \tag{8}
\]
Note that the expression \([p(r) - p_a]\) is a negative quantity which represents the suction pressure, i.e., the pressure below the ambient pressure.

Since the pressure drop is often measured in units of head of water, it is more convenient to modify the above equations into:

\[
\frac{p(r) - p_a}{\rho_w g} = a \cdot \frac{\rho_a}{\rho_w} \left( \frac{q}{2\pi h} \right)^b \cdot \frac{1}{(1-b)} \cdot (r^{1-b} - r_0^{1-b}) - \frac{1}{2g} \cdot \frac{\rho_a}{\rho_w} \left( \frac{q}{2\pi h} \right)^2 \left( \frac{1}{r^2} - \frac{1}{r_0^2} \right) \quad (9)
\]

We note that the second term on the R.H.S. of eq.(9) relates to the pressure drop arising simply as a result of decreasing cross-sectional area in the direction of flow (which is radially inward) while the first term accounts for the viscous drag due to the gravel bed. In case (and this will normally be so, as we shall see in the next section) the viscous drag term is very much larger than the former effect, the above equation simplifies into:

\[
\frac{p(r) - p_a}{\rho_w g} = \frac{1}{k} \cdot \frac{\gamma_a}{g} \cdot \frac{\rho_a}{\rho_w} \left( \frac{q}{2\pi h} \right)^b \cdot \frac{1}{(1-b)} \cdot (r^{1-b} - r_0^{1-b}) \quad (10)
\]

The above equation predicts the pressure field for a pre-specified total air flow rate \(q\). In case one wishes to predict the resulting flow rate for an imposed suction pressure, the above equation can still be used by simply rearranging the appropriate terms.

It is clear that the derivation is easily modified in case one wishes to either assume another model structure for the pressure drop correlation (i.e., eq.(3)), or even when the flow is through a homogeneous circular bed with impermeable sides and segmented into an angle \(\alpha\) as against \(360^\circ\). The expression \((q/2\pi h)\) simply needs to be modified appropriately.
For the special case of $b=2$, eq. (9) transforms into:

$$\frac{p(r) - p_a}{\rho_w g} = - \left[ a \left( \frac{1}{r} - \frac{1}{r_0} \right) + \frac{1}{2g} \left( \frac{1}{r^2} - \frac{1}{r_0^2} \right) \right] \frac{\rho_a}{\rho_w} \cdot \left( \frac{q}{2\pi h} \right)^2$$  \hspace{1cm} (11)

On the other hand, during laminar flow, Darcy's Law holds and the exponent $b=1$. Moreover during such cases, the pressure drop due to changing cross-section is essentially negligible as compared to the pressure drop due to viscous drag offered by the particles of the porous bed (see for example, [12]). Under these circumstances, the expression for the radial pressure drop in a circular porous bed is given by

$$\frac{p(r)}{\rho_a g} = a \cdot \frac{q}{2\pi h} \int \frac{1}{r} \, dr$$  \hspace{1cm} (12)

which, on integration and on introducing the boundary conditions yields

$$\frac{p(r) - p_a}{\rho_w g} = a \cdot \frac{\rho_a}{\rho_w} \cdot \frac{q}{2\pi h} \cdot \frac{r}{r_o} \cdot \ln \frac{r}{r_o}$$  \hspace{1cm} (13a)

$$= \frac{\nu a}{g} \cdot \frac{1}{k} \cdot \frac{\rho_a}{\rho_w} \cdot \frac{q}{2\pi h} \cdot \frac{r}{r_o} \cdot \ln \frac{r}{r_o}$$  \hspace{1cm} (13b)

It is easy to modify these equations to apply to outward radial flow as one would encounter in houses where the subslab pressurization technique is used. The boundary conditions are still the same but now the pressure at the throat of the suction pipe is higher than ambient pressure and the quantity $(p(r) - p_a)$ is positive and represents the pressure above the ambient pressure. The final expression analogous to eq. (9) is:
\[ \frac{p(r) - p_a}{\rho_w \cdot g} = -a \cdot \frac{\rho_a}{\rho_w} \left( \frac{q}{2\pi h} \right)^b \left( \frac{1}{r^{1-b}} \right) \left( \frac{1}{r_0^{1-b}} \right) - \frac{1}{2g} \frac{\rho_a}{\rho_w} \left( \frac{q}{2\pi h} \right)^2 \left( \frac{1}{r^2} \right) - \frac{1}{r_0^2} \] (14)

Another instance where our approach is directly applicable is when the porous bed consists of two or more types of porous material. For example, one could come across a house construction where the subslab gravel bed consists of two horizontally distinct layers of gravel of different sizes. The above equations can be easily modified to apply to such cases as well.

The practical implications of the parameters \( k \) and \( b \) are that if they are really constant for a given bed material and can be determined by actual experiments in the field, they will serve as indices by which a mitigator will be able to assess how much of the area from the suction hole he can hope to access for a given suction pressure.

The irregular boundary conditions that arise in practice are however not easily tractable with a simple expression such as eq.(9), and resorting to a numerical computer code may be the only proper way of proceeding in order to predict resulting pressure fields. Another problem in applying an approach such as the above to practical situations may be the drastic departure from homogeneity in certain subslab gravel beds (as also in the case of subslab soil beds). We propose to address these problems in the framework of a subsequent study.

4. **Laboratory Apparatus**

One needs to evaluate the soundness of the mathematical derivation presented above and also to determine the numerical values of the empirical coefficients of eq.(3b). To this end, a laboratory model consisting of a 2.4 m (8 ft) diameter circular section and 0.15 m (6") deep was constructed as is schematically shown in Fig. 7. The top and bottom impermeable disks

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were made from 0.02 m (3/4") thick plywood, and a wire mesh at the outer periphery of the disks was used to contain the gravel between the two disks (Fig. 8). The apparatus allowed experiments to be conducted with a maximum disk spacing (or depth of gravel bed) of 0.095 m (3.75"). An open-cell foam sheet 0.025 m (1") thick was glued to the underside of the top plywood disk. It was hoped that during experiments heavy weights placed on top of the plywood disk would effectively eliminate gaps that may exist between the disk and the gravel top that could cause short-circuiting of the air flow. In so doing, we hoped to guarantee that air flow occurs through the bed and not over it.

A 0.038 m (1.5") diameter hole was drilled at the center of the top disk to serve as the suction hole. Nine holes, whose layout is shown in Fig. 9, were drilled on three separate rays of the top disk and a PVC pipe of 0.012 m (1/2") inner diameter with chamfered ends was tightly squeezed into these holes. Pressure measurements at these nine holes would then yield an accurate picture of the pressure field over the entire bed.

The total volume of the packed bed is approximately 0.43 m$^3$ (16 ft$^3$) which for river-run gravel translates into a net weight of about 700 kg (1530 lbs).

Equipment needed for the experiments included:

(i) an industrial vacuum cleaner capable of sucking $45 \times 10^{-3}$ m$^3$/s (95 cfm) of air through a 0.05 m (2") diameter orifice under 1.9 m (75") of water static vacuum pressure;

(ii) a speed control and an air by-pass adapter (which is simply a perforated length of plastic pipe). Both these are needed in order to vary the air flow rate through the porous bed;
(iii) a pitot-tube anemometer to measure velocities from 0.05 to 15 m/s (10 to 3000 ft/min). Tables for different pipe diameters (as described in Ref. [4]) enabled the corresponding volume flow rate to be deduced; (iv) an electric digital micromanometer (EDM) which can measure pressures with a resolution of $0.025 \times 10^{-3}$ m $\left(10^{-3}''\right)$ of water or 0.25 Pa, and having a maximum range of up to 0.5 m (20") of water.

Other apparatus included two mounting devices: a 0.038 m (1.5") outer diameter brass pipe in order to connect the suction hole to the vacuum hose with arrangements to fix the pitot tube (the device has been termed Flow Pressure Tube (FPT)), and also the EDM, and a 0.019 m (0.75") stainless steel pipe to mount the EDM in order to measure the pressure at each of the nine different taps. These devices have already been described in detail in a previous report[4].

The experimental procedure for the apparatus filled with a certain type of porous medium entailed fixing the 0.038 m (1.5") FPT device (inner diameter of 0.035 m or 1.36") above the central suction hole of the top plywood disk and connecting it to the suction hose of the vacuum cleaner. A pressure tap at this pipe (see Fig. 7) placed 5 diameters above the suction hole permitted the static vacuum pressure to be measured as well, which was then used to get an estimate of pressure losses due to changing direction and cross-section, and also due to turbulence at entry into the pipe.

An experimentation run consisted first of selecting a certain total air flow rate and gradually changing the speed of the vacuum cleaner in order to achieve this flow rate. The pressure measurements (representative of the corresponding static pressure inside the porous bed) at each of the nine
taps were then taken in turn with all other taps closed. This completed a 
series of readings pertaining to one run. In subsequent runs, the total air 
flow rate was set to another predetermined value and the series of readings 
was repeated.

5. Experimental Results and Analysis of Radial Flow

Table 1 summarizes the different experiments performed using the 
laboratory apparatus. For example, Exp.A involved river run gravel of 
nominal diameters of 0.012 m and 0.019 m (1/2" and 3/4") which we shall 
refer to as small and large gravel, respectively. Experiments A1 and A2 
differ only in the spacing between the plywood disks, i.e. the thickness of 
the gravel bed was altered. Experiment A1 involved three separate runs each 
with a different total volume flow rate, the specific values of which are 
also given in Table 1. Finally, the last column has been included in order 
to give us an indication of the flow regime (based on the corresponding 
Reynolds number).

The specific values of the mean gravel diameter and the porosity of the 
bed are required for computing the Reynolds number (given by eq.(1)). We 
have performed porosity measurements and also computed the mean equivalent 
diameter of the various porous bed materials chosen in the present study; 
these are described in detail in Appendix B.

Also, in all the analyses involving regressions which are discussed 
below, the values of the static pressure at hole 10 were not included since 
it is very likely that the turbulence due to entrance effects would still be 
present that close to the outer periphery of the disk.

From Table 1, we note that the flow is likely to be turbulent. Hence 
we start by regressing the observed data following eq.(11). Table 2
assembles the subsequent results. Since caution is recommended when the soundness of a regression model is to be judged according to its coefficient of determination (i.e. the $R^2$ value) [18], we have also run the model with an intercept and found no appreciable discord between the two sets of $R^2$ values (see Table 2). Consequently, in our present study, we can assume the $R^2$ value to be indicative of the goodness-of-fit of the no-intercept model. We also note from Table 2 that the estimated values of the constant 'a' differ only by about 5%. This is how one would expect it to be since permeability of the bed (inversely related to the coefficient 'a' as given by eq.(4)) should not alter with change in the thickness of the porous bed. This suggests that the foam attached to the top plywood disk of our laboratory apparatus does seem to do a satisfactory job of eliminating short-circuiting of air flow. In order to get average estimates, we have treated the observations of Exps.A1 and A2 together and estimate the 'mean' parameters. The corresponding value is also shown in Table 2. In general, the Standard Error of the Mean (SEM) values are 5% or less of the mean value, a gratifying observation.

We note that $R^2$ values are very high despite which, as depicted by Figs. 10 and 11, the fits could be improved. Consequently, we have rerun the regression using eq.(9) wherein both the constant 'a' and the exponent 'b' are to be determined by least square errors. Instead of simply determining the optimal value of the exponent b we have performed several runs the results of which are summarized in Table 3. Such an approach yields insight into the sensitivity of the parameters a and b on the regression. We note that $R^2$ values have improved significantly and one cannot realistically expect better fits (given the measurement errors in our
readings, we may in fact be overfitting in the sense that we are trying to assign physical meaning to random errors), as compared to assuming a quadratic exponent (see Table 2). This is also seen in Figs. 12 and 13 where we note that the fit between model predictions and observed pressure drop has improved.

The value of the exponent $b$ that yields the highest $R^2$ value is underlined in Table 3. Note also that the SEM values for the estimate of 'a' are relatively small.

Other aspects need specific mention as well:

(a) We find that for the type of river run gravel used and for the range of flow conditions investigated, the effect of pressure drop in the flow arising as a result of changing cross-sectioned area (i.e. the second term on the RHS of eq.(9)) corresponds to less than a percent of the total pressure drop. Thus eq.(10) is the appropriate one to use in order to predict pressure fields in porous gravel beds under flow conditions akin to those encountered during operation of practical radon mitigation systems.

(b) We note that the optimal value of $b$ is not too well determined, since $R^2$ values only change in the third decimal point when $b$ is varied in steps of 0.10 (Table 3). It is highly unlikely that the experimental accuracy of our readings can lead us to place this much faith in the exact or best value of $b$ identified by regression. Consequently, one should rather think in terms of a certain range for the $b$ values and not try to attach undue importance to physical interpretation of the exact value of the exponent $b$.

(c) The study referred to earlier [5] found values of the exponent $b$ to be 1.56 for the cylindrical disk model. This is generally borne out in the present study where we find $b=1.6$ for the small river-run gravel and $b=1.4$
for the large gravel.

(d) There is however a serious drawback in our ability to accurately determine the values of the bed permeabilities from the present experiments. This is because the present data were collected during turbulent flow regimes, and though the estimate of 'a' and 'b' identified by regression are perfectly satisfactory for predicting pressure drops at flow regimes in the range within which the present experiments were performed, these estimates may yield misleading and erroneous predictions when used outside this range of flow conditions. In other words, parameter sets identified by regression are not generally accurate for extrapolation purposes.

In order to make this point clearer, let us rewrite eq.(10) as follows:

\[
\ln \left[ \frac{p(r) - p_a}{\rho_w \cdot g} \right] = \ln \left( \frac{1}{k} \cdot \frac{\gamma_a}{g} \right) + \ln \left[ \frac{\rho_a}{\rho_w \cdot (2\pi h)} \cdot \frac{1}{(1-b)} \cdot (r^{1-b} - r_0^{1-b}) \right]
\] (15)

If we were to plot the term on the left-hand side on the y-axis and the second term on the right-hand side of the above equation on the x-axis, the intercept of such a line would give us an estimate of the permeability. Figure 14 shows such a representation for the experimental data using small gravel (Exps.A1 and A2) and b=1.6. It is now clear that since the data points essentially lie in a region far away from the x-axis, the intercept term is bound to be ill-defined from a subsequent statistical regression.

The values of effective permeability of the porous bed calculated following eq.(4) are included in Table 3 and show a three fold difference between small and large gravel sizes. The numerical values do seem to correspond to those cited in the radon literature [6,19]. If more accurate values of permeability are to be determined, the experimental design of our
apparatus needs to be suitably modified.

(d) There is another purely statistical limitation in identifying parameters from a least square regression such as the present, where a mathematical model without an intercept term is fitted to quantities which vary drastically like the pressure drop quantities do as one moves radially outward (Figs. 10-13 indicate an order of magnitude variation). The regression would favor the larger values of pressure drop since it tries to capture as much of the variation in observed values of pressure drop in terms of absolute variation from zero. Consequently the model parameters estimated will not be very sound because the regression is unduly influenced by a relatively small number of observations. One possibility is not to evaluate goodness-of-fit between model and observed data based on the $R^2$ statistic but rather on the Chi-square statistic [18]. Though this test would overcome the above mentioned limitation, other problems (beyond the scope of this report) are likely to arise. Another possibility could involve performing more measurements at higher pressure drops in order to avoid such uncertainty in the estimated parameters. These issues will have to be addressed in subsequent studies.

6. Model and Analysis of Entrance Effects into the Suction Pipe

The pressure losses at the entrance to the suction pipe need to be estimated properly since this is an important contribution to the total pressure drop. This involves accounting for the three following effects: (i) change in flow direction, (ii) change in cross-sectional area, (iii) entrance effects at the throat of the suction pipe. From an engineering viewpoint, it is more convenient to treat these together as one effect. In accordance with actual practice [20], we propose to use the following
simplified empirical equation for the head loss:

\[
\frac{\Delta p}{\rho_w g} = K_p \left(1 - \frac{A_p}{A_b}\right) \cdot \frac{V_p^2}{2g}
\]  

(16)

where \(K_p\) is the dimensionless pressure loss coefficient which should not depend on either the velocity or the bed thickness, and is a constant for a specific type of bed material,

\(A_p\) is the cross-sectional area of the suction pipe,

\(A_b\) is the surface area of a cylinder of diameter equal to that of the suction pipe and height equal to the depth of the porous bed,

and \(V_p\) is the velocity of air in the suction pipe.

If \(d_p\) is the diameter of the suction pipe, then

\[
\frac{A_p}{A_b} = \frac{\pi d_p^2}{4} \cdot \frac{1}{\pi d_p h} = \frac{d_p}{4h}
\]  

(17a)

and

\[
V_p = \frac{q}{A_p} = \frac{4q}{\pi d_p^2}
\]  

(17b)

Note that we have defined the pressure drop as that occurring from the outer edge of the suction hole to a point in the suction pipe above the entry point at which entrance effects would not be present. This pressure drop is estimated by first computing the pressure drop at a radial distance \(r = d_p/2\) corresponding to the outer edge of the suction hole (using eq.(9) with the values of the parameters estimated by regression as described in
the previous section), and subtracting the above from the static pressure measured in the suction pipe.

The object of our present analysis is to estimate the values of $K_p$ for different types of porous media from experiments performed on the laboratory apparatus. Using these values of $K_p$ to estimate associated pressure drops under actual field conditions would be more accurate than simply selecting arbitrary values from engineering handbooks which unfortunately do not cover such specialized applications.

Table 4 assembles the values of the pressure drops at different values of $q$ for both the small gravel (Exps.A1 & A2) and the large gravel (Exp.A3). We note that the values of $K_p$ are essentially constant - a gratifying result. These results enable us to place a certain amount of confidence in both our modeling approach and in the mean estimated values of $K_p$. We suggest that these values be used until better and more accurate experiments are performed in order to quantify entrance effects.*

7. Summary and Future Work

Table 5 summarizes the various laboratory experiments performed and the physical parameters deduced in the framework of the present study. Specifically, the important features of this study are as follows:

(a) We have outlined the general problem of radon mitigation system design and discussed the scope and limitations of prior studies in both this aspect

*NB: The values of $K_p$ deduced from the laboratory model are strictly accurate for a 0.038 m (1.5") central suction pipe. It still remains to be verified whether they will be equally pertinent for a 0.1 m (4") diameter pipe, which is the size most widely used.
and also at a more fundamental hydrodynamical level. The first problem should be to determine the nature of air flow below the concrete slab and next to see how this is likely to affect the pressure drop versus flow correlation for given subslab conditions.

(b) Next we give arguments to support the suggestion of a prior study [5] that flow under the slab of a house during mitigation using the subslab depressurization (and the pressurization) technique be likened to radial flow between two impermeable disks.

(c) We point out that subslab air flow under actual operation of mitigation systems is likely to be turbulent if a gravel bed is present and laminar in the presence of soil.

(d) We then present a mathematical treatment to analytically predict the pressure field of turbulent flows in homogeneous circular porous beds.

(e) A laboratory apparatus constructed so that it can specifically duplicate conditions which occur in practice under slabs of real houses being mitigated for radon using the depressurization (or the pressurization) technique is then described. The experimental procedure followed in order to measure the pressure field of turbulent air flow is outlined. Data analysis methods are also discussed in order not only to validate or substantiate the basic correlation between pressure drop and flow, but also to determine the regression coefficients of the model and the range of porous bed permeabilities which actual construction materials may exhibit.

(f) Proper pressure extension fields require that test hole measurements be made in order to first detect presence of subslab flow barriers and short-circuit paths, and then to determine the required model parameters by non-linear regression. Various aspects of non-linear regression relating to
the soundness and stability of the estimated parameters are pointed out in order to impart a better appreciation to readers who are not well versed in such aspects.

(g) We also suggest a model framework for estimating pressure drop due to entrance effects at the suction point and deduce the pressure drop coefficients from experimental observations made with the laboratory apparatus.

Logical extensions of this study would involve applications of this methodology to houses with (i) homogeneous beds but with irregular boundaries, and (ii) non-homogeneous porous beds. One approach is to develop a simplified computer program using numerical methods (either finite element or finite difference could be used) to solve the basic set of hydrodynamic and mass conservation equations. Pressure fields under the slab for practically any configuration could be thereby predicted. An optimization algorithm could then be attached to the above program in order to obtain the optimal layout and the number of mitigation suction points for the particular subslab conditions such that certain well-defined and physically relevant constraints are satisfied.

Our present line of thinking is that though the above approach offers great flexibility, it is not easy to use by non-experts. Developing engineering guidelines for practitioners based on such a code demands a certain amount of effort and practical acumen. It would be wiser to define a few "standard" cases of basement shape, subslab conditions and mitigation pipe locations; try to develop simplified closed-form solutions of these cases; and then see how well these solutions fare with respect to actual measurements taken in the field. If such an approach does give satisfactory
engineering accuracy, it's subsequent use to mitigation system design would be relatively simple and straightforward.

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Nomenclature

A  cross-sectional area of flow
a  parameter representative of the resistivity to flow of the porous bed
b  exponent appearing in eq.(3a)
c  dimensionless constant appearing in eq.(3a)
d  diameter
d_v  equivalent diameter of pebbles
f  friction factor
g  acceleration due to gravity
h  thickness of the porous bed
K  parameter representative of the conductivity to flow of the porous bed
K_p  pressure loss coefficient at entry to suction pipe
k  permeability of porous bed
n  exponent appearing in eq.(3a)
\Delta p  pressure drop
p  pressure
P_a  atmospheric pressure

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q  total volume flow rate
\( R^2 \)  coefficient of determination of regression
Re  Reynolds number
r  radial distance from center of the suction hole
r_o  outer radius of the laboratory apparatus
SEM  standard error of the mean of the regression estimate
V  air velocity
x  distance along flow
\( \alpha \)  opening angle of porous bed through which radial flow occurs
\( \rho \)  density
\( \mu \)  dynamic viscosity
\( \gamma \)  kinematic viscosity
\( \phi \)  porosity of porous bed

Suffix
a  air
b  porous bed
p  pipe
w  water

References


Appendix A: Brief Review of Scientific Theory

Relating to Flow through Porous Media [12-17]

A porous medium is defined as a solid containing holes or voids, either connected or unconnected, dispersed within it in either a regular or random manner such that holes occur relatively frequently within the solid [13]. In this study, we are specifically interested in unconsolidated isotropic beds such as sand or gravel and our discussion will be limited to such material. Since the structure of such beds is so irregular and random that it can be described only in statistical terms, the prevalent approach is to treat such beds on a macroscopic basis, analogous to the approach followed in the kinetic theory of gases. Thus, on a macroscopic scale the system can be defined in terms of a few determinable quantities from which phenomena like fluid flow or heat transfer can be accurately predicted for engineering purposes.

A1. Definitions of geometrical quantities

All the following properties are bulk properties in that they pertain to a unit total volume of the bed. Note that as such they have significance only for samples of porous beds containing a relatively large number of pores.

(a) Porosity ($\phi$)

The porosity or void fraction of a material is the fraction of the bulk volume of the total material occupied by voids. Thus

$$
\phi = \frac{\text{Volume of pores or void volume}}{\text{Total or bulk volume}} = \frac{V_p}{V_T} \quad (A1)
$$
The void fraction for randomly packed beds of uniformly sized spheres in containers of diameters about 50 times the particle diameter is in the range of 0.36-0.43 [17].

(b) **Equivalent diameter** (d<sub>v</sub>)

Porous unconsolidated material such as gravel beds are made up of pebbles with varying sizes and of irregular shape. The equivalent diameter is defined in terms of a mean spherical particle having the same volume. Thus

\[ d_v = \left( \frac{6V_s}{n\pi} \right)^{1/3} \]  

(A2a)

where \( V_s \) is the volume of \( n \) particles selected randomly.

Alternatively, since \( V_s \) is not an easily measurable quantity, we can use the following expression to estimate \( d_v \):

\[ d_v = \left( \frac{6}{n\pi} \cdot (1-\phi) \cdot V_T \right)^{1/3} \]  

(A2b)

Note that \( d_v \) is the mean diameter. For a more accurate treatment, the distribution of the particle diameters have to be determined experimentally for which purpose sieving is done using different sizes of screens.

(c) **Particle shape factor** (s)

The shape factor is important as it affects the surface area per unit volume and is usually defined in terms of a spherical particle which has the minimum surface area per unit volume. Thus
\( s = \frac{\text{surface area of a sphere per unit volume}}{\text{surface area of the particle}} \)  

(A3)

(d) **Effective diameter** \((d_S)\)

For purposes of friction drop or heat transfer calculations, it is the surface area \((A_S)\) of the particles per unit volume of material which is the influencing parameter. Thus

\[
A_S = \frac{n \pi d_Y^2}{V_T} = \frac{\pi d_Y^2 n}{6} \cdot \frac{6}{V_T d_S} = (1-\phi) \cdot \frac{6}{d_S} \quad \text{(A4)}
\]

Following the hydraulic radii theory, the effective diameter can be computed as:

\[
d_S = \frac{4 \text{ (void volume)}}{\text{wetted area}} = \frac{2 \phi}{3 (1-\phi)} \cdot d_Y \quad \text{(A5)}
\]

A2. **Reynolds number**

The Reynolds number, which is the ratio of inertia force to viscous force, gives an indication of the type of flow: whether laminar or turbulent. This is of crucial importance since pressure drop as well as heat transfer characteristics of the porous bed are greatly influenced by the regime under which flow occurs.

The Reynolds number is defined as

\[
Re = \frac{q \rho}{\frac{V.1}{A\mu}} = \frac{q \rho}{V.1 \gamma} \quad \text{(A6)}
\]

where \(V\) is the apparent or surface velocity, \(q\) the volumetric flow rate, \(\rho\) the fluid density, \(\mu\) the dynamic viscosity, \(\gamma\) the kinematic
viscosity, \( A \) the cross-sectional area of the porous bed and \( l \) a characteristic length dimension.

Note that \( V \) is not the actual velocity in the pores but is a velocity obtained by measuring the discharge \( q \) through an area \( A \) in the absence of the porous bed. It is often referred to as the superficial velocity.

The question that arises is what to choose as the length dimension of the bed which should determine the nature of the flow. Different researchers seem to have chosen this dimension differently leading to a certain inconsistency in the corresponding Reynolds numbers thereby computed \([12-14]\). The most common definitions of the characteristic length for porous beds are:

(i) \( l = d_s \)

(ii) \( l = d_v \)

(iii) \( l = (d_v/\phi) \)

It must be pointed out that usually in flow through pipes, a dimension of the flow channel is chosen for \( l \). However for porous beds it is difficult to measure pore or void diameter and consequently the particle or pebble diameter is preferred. However, this by itself does not explain all the characteristics of an actual porous bed with heterogeneous and irregular particles. Consequently in the present study we have opted to follow definition (iii). By including the porosity as well, the definition of the Reynolds number will actually correspond to the fluid velocity in the pores since this is given by \((V/\phi)\).

For the purposes of this study, we shall assume the following safe limits for the flow: laminar for \( Re < 1 \) and turbulent for \( Re > 10 \). Since the particles are irregular and of different sizes, the transition from laminar
to turbulent flow is not abrupt at a single critical Reynolds number as is the flow through pipes. Instead the transition is rather gradual and therefore there exist studies which report laminar flows at Re values close to 5 and above, while others report the onset of turbulence at Re numbers close to 5. Thus the critical values stated above should be viewed as indicators for establishing the regime in an approximate manner rather than strict numerical cut-off values.

A3 Flow dynamics

The flow of fluids through porous media is complicated by the fact that the flow is highly irregular and tortuous. Despite this, the analogy to flow through pipes is used to study both laminar and turbulent regimes by starting explicitly with a correlation between Reynolds number and the friction factor. Consequently for porous media, the friction factor $f$ can be defined as \cite{13}:

$$f = dV \cdot g \cdot \left( \frac{\phi A}{q} \right)^2 \cdot \frac{dp}{dx} \tag{A7}$$

where dp is the pressure drop differential (Pa in SI units) across a porous bed length differential dx.

During the laminar regime, i.e. $Re < 1$, the product $(Re \times f)$ is assumed to be essentially equal to a constant $c$. Thus, from eq. (A7) and the definition of Re, we have

$$\frac{q}{A} = g \cdot \frac{\phi \cdot dV^2}{c \cdot \gamma} \cdot \frac{dp}{dx} \tag{A8}$$
where $\gamma$ is the kinematic viscosity of the fluid flowing through the porous bed.

This equation is referred to as Darcy's law following the experimental scientist who originally proposed it. We can recast eq.(A8) as

$$\frac{q}{A} = K \frac{dp}{dx} \quad (A9)$$

where $K = g\frac{\phi \cdot d\nu^2}{c \cdot \gamma} \quad (A10)$

and is a coefficient representative of the conductivity of the porous bed to the particular fluid.

The parameter $K$ depends on characteristic parameters of the porous bed as also (provided of course that the flow remains laminar) on those of the fluid (because of the inclusion of $\gamma$). In order to separate these, we define the permeability (sometimes also referred to as the 'intrinsic permeability') of the porous bed as

$$k = \frac{\phi \cdot d\nu^2}{c} \quad (A11)$$

The conductivity $K$ and the permeability are correlated as follows:

$$k = \frac{K \cdot \gamma}{g} \quad (A12)$$

Thus the permeability can be defined as the volume of a fluid per unit viscosity passing through a unit cross section of the medium in unit time under the action of a unit pressure gradient [12]. It has units of area and is determined only by the structure of the porous bed and is entirely independent of the nature of the fluid. It is thus a constant for a bed
made up of a specific porous material.

Finally, the following aspects need to be spelled out explicitly:

(i) The above derivation is intended more as a heuristic guide to understanding flow behavior rather than a formal proof of Darcy's law which most text books derive from the classical hydrodynamical equations of Navier-Stokes [16].

(ii) Equation (All) is actually an operational definition of \( k \). This is because the heterogeneity and irregularities of commonly encountered porous beds do not permit \( k \) to be accurately computed from basic properties of the bed which have to be deduced experimentally. Thus the form of the equation suggests that a statistical averaging as against strict accounting for variation in flow in the individual pores has been adopted. Consequently, only on a macroscopic sense is the velocity of a fluid flowing through a porous medium directly proportional to the pressure gradient acting on the fluid.

Darcy's law is no longer valid when the flow becomes partially or completely turbulent. Under such conditions, the literature contains several empirical models proposed by different researchers to treat flow in porous media. These are addressed briefly below.

(a) Linear dependence of (Re.f) on Re [12]:

This approach starts with the assumption that

\[
\text{Re} \cdot f = c_1 + c_2 \cdot \text{Re} \quad \quad \text{(A13)}
\]

where \( c_1 \) and \( c_2 \) are dimensionless constants which depend on the properties of both fluid and porous media.

From the above, we obtain the relation
\[
\frac{dp}{dx} = c_1' \left( \frac{q}{A} \right) + c_2' \left( \frac{q}{A} \right)^2
\]  
(A14)

where \( c_1' \) and \( c_2' \) are given by

\[
c_1' = \frac{c_1 \cdot \gamma}{g \cdot d_v \phi} \quad \text{and} \quad c_2' = \frac{c_2}{g \cdot d_v \phi^2}
\]  
(A15)

Such a treatment originally proposed by Forchheimer, has a certain amount of appeal since it can simultaneously account for different types of flow while yielding the relative contributions of each on the total pressure drop. With \( c_2' = 0 \), we get back Darcy's law while with \( c_1' = 0 \), we obtain the quadratic exponent found for turbulent flow in pipes according to Fanning's equation. Thus we can view this approach as treating actual pressure drop as consisting of a pressure drop resulting from laminar flow added to a pressure drop occurring from turbulent flow.

(b) White and later Missbach [14] have suggested the following model:

\[ \text{Re. } f^n = c \]  
(A16)

which is analogous to the following

\[
\frac{dp}{dx} = a \left( \frac{q}{b} \right)
\]  
(A17)

where

\[
a = \frac{1}{g} \left[ \frac{c \cdot \gamma}{d_v \cdot n+1 \cdot \phi^{2n-1}} \right]^{1/n} \quad \text{and} \quad b = \frac{2n-1}{n}
\]  
(A18)
It is clearly seen that for laminar flow $b$ would be equal to 1 while for turbulent flow it would be close to 2. For mixed flow, the exponent would be between 1 and 2, the exact value being dependent on the circumstances specific to the particular case. Unlike the Darcy equation (eq.(A9)) where the interpretation of the constant $k$ is unambiguous, it is difficult to assign a rigorous interpretation to the coefficients of eqs.(A13 & A16).

However loosely speaking the coefficient '$a$' of eq.(A17) can be considered to represent the resistivity to flow offered by the porous media to the particular fluid.

Several other studies have proposed either variants of the above two approaches or more complex empirical correlations, either between the Reynolds number and the friction factor, or directly for the pressure drop in the porous bed against parameters describing both material and flow conditions. We shall not discuss these here given the more advanced nature of these models and the inappropriateness of resorting to such models in the framework of the type of practical application we have in mind.
Appendix B: Experiments to Determine Porosity and Equivalent Diameters of Gravel

The porosity of a gravel bed ($\phi$) and the equivalent diameter ($d_v$) of porous materials have been defined in Appendix A. Though these parameters do not explicitly figure in the pressure drop versus flow models outlined in the main portion of this report as also in Appendix A, they do implicitly influence such models through the permeability term of the porous bed. Thus a knowledge of these parameters would indeed be useful. Consequently we have undertaken an experimental determination of $\phi$ and $d_v$, the results of which are presented and discussed below.

B1. Determination of bed porosity ($\phi$)

Experimental techniques to determine porosity of porous beds are well known (see for example, Refs. [12,13]. Perhaps the simplest technique is to choose a certain volume of the bed material and then measure the volume of the voids by measuring the volume of a liquid (for example, water) needed to completely saturate the porous bed. Either the volume of liquid poured in or drained out (or both) could then be used to directly estimate the porosity.

Tables B1 and B2 present the experimental observations relating to the volume of water both poured in and then drained out from a total volume of the porous material of 1 liter. Note that the observations of the first run have to be discarded due to errors arising as a result of initial wetting of the gravel. Repetitions both in number of samples and runs for each sample assure the determination of a sounder and more representative value of $\phi$. 

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We note that within-sample variance is smaller than that across samples for larger gravel which suggests that the errors of our experimental procedure are lower than the variations associated with taking different samples. Though this is not so for the smaller gravel, the magnitude of these values are small and can be confidently overlooked for our purpose.

We summarize the values of the porosity estimated for river run gravel:

- 0.012 m (1/2") nominal diameter:  = 0.374
- 0.019 m (3/4") nominal diameter: = 0.424.

B2. Determination of equivalent diameter ($d_y$)

The most convenient way of deducing $d_y$ is from eq. (A2b). Since we already have an estimate of the porosity, all that remains is to determine the number of gravel stones in a given volume.

Again we chose a total volume of 1 liter and counted the number of stones, the results of which are shown in Table B3. It would have been better to get an estimate of the particle size distribution as well, but this could not be done due to lack of appropriate screening sieves. Thus only an estimate of the mean diameter has been obtained in this study.

The mean number of pebbles in both sizes of gravel are given in Table B3 along with the SEM values. We note that for large gravel the SEM is almost 5% while for smaller gravel it is less: something which is to be expected given that experimental errors are larger for larger gravel.

The mean values of $d_y$ computed from the experimental readings of Table B3 are:

<table>
<thead>
<tr>
<th>River-run gravel</th>
<th>Nominal diameter</th>
<th>Estimated equivalent diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large</td>
<td>0.019 m (3/4&quot;)</td>
<td>0.018 m (0.71&quot;)</td>
</tr>
<tr>
<td>Small</td>
<td>0.012 m (1/2&quot;)</td>
<td>0.009 m (0.36&quot;)</td>
</tr>
</tbody>
</table>
We note that deviation between nominal and equivalent diameters is small for the large river-run gravel. As for the smaller sized gravel, the important difference is likely to be as a result of the fact that this gravel type contained a relatively large number of very small pebbles thereby decreasing the estimated effective mean value. Thus we attribute this departure from the nominal diameter to have arisen as a result of improper or non-rigorous sieving separation process adopted by the supplier rather than as a result of the pebbles being systematically smaller. A fact to be retained is that in practical situations, the large variation in the distribution of particle diameters even when a specific nominal diameter is specified would lead to a loss of scientific predictability or reproducibility in the pressure drop versus flow relationships when these are estimated from actual experimental measurements.
Table 1. Summary table of the different experiments performed with the laboratory apparatus using river run gravel

<table>
<thead>
<tr>
<th>Set</th>
<th>Experiment</th>
<th>Gravel size (nominal diameter)</th>
<th>Disk spacing</th>
<th>No. of runs</th>
<th>Total flow rate (cfm)</th>
<th>1/s</th>
<th>Calculated flow regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1/2&quot; (0.012 m)</td>
<td>3&quot; (0.075 m)</td>
<td>3</td>
<td>43.5</td>
<td>20.5</td>
<td>Turbulent</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>63.8</td>
<td>30.1</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>79.0</td>
<td>37.3</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1/2&quot; (0.012 m)</td>
<td>3.75&quot; (0.10 m)</td>
<td>2</td>
<td>46.8</td>
<td>22.1</td>
<td>Turbulent</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>66.5</td>
<td>31.4</td>
<td>&quot;</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>3/4&quot; (0.019 m)</td>
<td>3.75&quot; (0.10 m)</td>
<td>4</td>
<td>23.7</td>
<td>11.2</td>
<td>Turbulent</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>32.2</td>
<td>15.2</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>37.4</td>
<td>17.6</td>
<td>&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>44.0</td>
<td>20.8</td>
<td>&quot;</td>
</tr>
</tbody>
</table>
Table 2. Results of the regression using a quadratic model for pressure drop (eq.(11))

<table>
<thead>
<tr>
<th>Experimental run</th>
<th>No. of data points</th>
<th>$R^2$ Model without intercept</th>
<th>$R^2$ Model with intercept</th>
<th>Parameter 'a' Mean</th>
<th>Parameter 'a' SEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>24</td>
<td>0.964</td>
<td>-</td>
<td>344.7</td>
<td>13.82</td>
</tr>
<tr>
<td>A2</td>
<td>16</td>
<td>0.975</td>
<td>-</td>
<td>358.2</td>
<td>14.84</td>
</tr>
<tr>
<td>A1+A2</td>
<td>40</td>
<td>0.968</td>
<td>0.953</td>
<td>350.1</td>
<td>10.13</td>
</tr>
<tr>
<td>A3</td>
<td>28</td>
<td>0.967</td>
<td>0.914</td>
<td>303.4</td>
<td>10.72</td>
</tr>
</tbody>
</table>

Table 3. Results of the regression using an exponential model for pressure drop (eq.(9)). The underlined values of b correspond to those yielding the highest $R^2$.

<table>
<thead>
<tr>
<th>Experimental run</th>
<th>No. of data points</th>
<th>b</th>
<th>$R^2$</th>
<th>Parameter 'a' Mean</th>
<th>Parameter 'a' SEM</th>
<th>Effective permeability of porous bed (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1+A2</td>
<td>40</td>
<td>1.3</td>
<td>0.984</td>
<td>86.71</td>
<td>1.78</td>
<td>9.4 x 10^-9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.4</td>
<td>0.991</td>
<td>106.8</td>
<td>1.59</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.5</td>
<td>0.996</td>
<td>130.6</td>
<td>1.30</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.6</td>
<td>0.998</td>
<td>158.5</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.7</td>
<td>0.997</td>
<td>191.2</td>
<td>1.75</td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>28</td>
<td>1.2</td>
<td>0.994</td>
<td>23.60</td>
<td>0.34</td>
<td>34 x 10^-9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.3</td>
<td>0.996</td>
<td>32.35</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.4</td>
<td>0.997</td>
<td>44.22</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.5</td>
<td>0.996</td>
<td>60.26</td>
<td>0.74</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.6</td>
<td>0.994</td>
<td>81.90</td>
<td>1.22</td>
<td></td>
</tr>
</tbody>
</table>
Table 4. Determination of the pressure loss coefficient $K_p$ at entry to the suction pipe (eq. (16)).

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Total flow rate (1/s)</th>
<th>(1)* (cm-water)</th>
<th>(2)** (cm-water)</th>
<th>(3)*** (cm)</th>
<th>$K_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>20.5</td>
<td>5.08</td>
<td>2.17</td>
<td>2.30</td>
<td>1.26</td>
</tr>
<tr>
<td></td>
<td>30.1</td>
<td>10.16</td>
<td>3.99</td>
<td>4.95</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>37.3</td>
<td>15.24</td>
<td>5.61</td>
<td>7.59</td>
<td>1.27</td>
</tr>
<tr>
<td>A2</td>
<td>22.1</td>
<td>5.08</td>
<td>1.72</td>
<td>2.80</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td>31.4</td>
<td>10.16</td>
<td>3.00</td>
<td>5.66</td>
<td>1.27</td>
</tr>
<tr>
<td>A3</td>
<td>11.2</td>
<td>3.94</td>
<td>0.21</td>
<td>0.72</td>
<td>5.19</td>
</tr>
<tr>
<td></td>
<td>15.2</td>
<td>7.24</td>
<td>0.31</td>
<td>1.33</td>
<td>5.20</td>
</tr>
<tr>
<td></td>
<td>17.7</td>
<td>9.91</td>
<td>0.39</td>
<td>1.79</td>
<td>5.31</td>
</tr>
<tr>
<td></td>
<td>20.8</td>
<td>13.56</td>
<td>0.49</td>
<td>2.48</td>
<td>5.27</td>
</tr>
</tbody>
</table>

NB: * Measured static pressure in the suction pipe 5 diameters from the pipe entrance

** Estimated pressure drop in the bed at outer edge of the suction pipe (from eq. (9) using the values of $a$ and $b$ determined by regression.)

$$***\left(1 - \frac{A_p}{A_b}\right)^2 \frac{v_p^2}{2g}$$
Table 5. Summary of various laboratory experiments performed and the physical parameters deduced in the framework of the present study using river run gravel.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Diameter of particles nominal measured (m)</th>
<th>Measured porosity</th>
<th>Exponent of eq.(3b)</th>
<th>Permeability of bed (m²)</th>
<th>Pressure loss coeff. at entry to suction hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1+A2</td>
<td>0.012 0.011</td>
<td>0.374</td>
<td>1.60</td>
<td>9.4x10⁻⁹</td>
<td>1.21-1.27</td>
</tr>
<tr>
<td>A3</td>
<td>0.019 0.022</td>
<td>0.424</td>
<td>1.40</td>
<td>34x10⁻⁹</td>
<td>5.2-5.3</td>
</tr>
</tbody>
</table>
Table B1. Experimental observations in order to determine porosity of the river-run gravel of 3/4" (0.019 m) nominal diameter. Numbers indicate volume of water in cubic centimeters poured in (1) and drained out (2) from a total volume of 1 liter.

<table>
<thead>
<tr>
<th>Run</th>
<th>Sample A</th>
<th>Sample B</th>
<th>Sample C</th>
<th>Sample D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2</td>
<td>1 2</td>
<td>1 2</td>
<td>1 2</td>
</tr>
<tr>
<td>1*</td>
<td>439 414</td>
<td>440 420</td>
<td>454 435</td>
<td>462 440</td>
</tr>
<tr>
<td>2</td>
<td>424 425</td>
<td>415 415</td>
<td>447 447</td>
<td>427 426</td>
</tr>
<tr>
<td>3</td>
<td>417 420</td>
<td>410 413</td>
<td>437 440</td>
<td>420 425</td>
</tr>
<tr>
<td>4</td>
<td>412 415</td>
<td>414 416</td>
<td>434 439</td>
<td>422 425</td>
</tr>
<tr>
<td>5</td>
<td>409 410</td>
<td>416 418</td>
<td>438 436</td>
<td>422 424</td>
</tr>
</tbody>
</table>

Mean 416.5 414.6 439.7 423.9 423.7
SEM 2.16 0.84 1.71 0.83 5.72

(*) The values of this run are discarded due to initial wetting of the gravel.

Table B2. Experimental observations in order to determine porosity of the river-run gravel of 1/2" (0.012 m) nominal diameter. Numbers indicate volume of water in cubic centimeters poured in (1) and drained out (2) from a total volume of 1 liter.

<table>
<thead>
<tr>
<th>Run</th>
<th>Sample A</th>
<th>Sample B</th>
<th>Sample C</th>
<th>Sample D</th>
<th>Sample E</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2</td>
<td>1 2</td>
<td>1 2</td>
<td>1 2</td>
<td>1 2</td>
</tr>
<tr>
<td>1*</td>
<td>442 465</td>
<td>433 376</td>
<td>435 389</td>
<td>418 365</td>
<td>419 365</td>
</tr>
<tr>
<td>2</td>
<td>378 384</td>
<td>376 381</td>
<td>394 392</td>
<td>375 374</td>
<td>374 375</td>
</tr>
<tr>
<td>3</td>
<td>370 374</td>
<td>376 385</td>
<td>385 385</td>
<td>370 373</td>
<td>362 363</td>
</tr>
<tr>
<td>4</td>
<td>368 370</td>
<td>372 376</td>
<td>376 377</td>
<td>371 374</td>
<td>366 368</td>
</tr>
<tr>
<td>5</td>
<td>370 366</td>
<td>374 375</td>
<td>380 380</td>
<td>365 373</td>
<td>365 368</td>
</tr>
</tbody>
</table>

Mean 372.5 375.4 383.6 371.9 367.6 374.2
SEM 2.10 0.92 2.35 1.14 1.14 2.66

(*) The values of this run are discarded due to initial wetting of the gravel.
Table B3. Results of the experiment to determine the mean equivalent particle diameter of the river-run gravel. Total volume of the sample was 1 liter.

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Large pebbles</th>
<th>Small pebbles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nominal diameter</td>
<td>Nominal diameter</td>
</tr>
<tr>
<td></td>
<td>= 3/4&quot; (0.019 m)</td>
<td>= 1/2&quot; (0.012 m)</td>
</tr>
<tr>
<td>1</td>
<td>209</td>
<td>1458</td>
</tr>
<tr>
<td>2</td>
<td>179</td>
<td>1688</td>
</tr>
<tr>
<td>3</td>
<td>174</td>
<td>1529</td>
</tr>
<tr>
<td>4</td>
<td>181</td>
<td>-</td>
</tr>
<tr>
<td>Mean</td>
<td>185.75</td>
<td>1558.3</td>
</tr>
<tr>
<td>SEM</td>
<td>7.89</td>
<td>68.0</td>
</tr>
</tbody>
</table>
Fig. 1 Schematic of subslab air flows in a house with a gravel bed when the house is being mitigated following the subslab depressurization technique. (Note that part of the air flowing through the gravel bed originates from the basement and the rest from the ambient air.)
Fig. 2. Schematic of subslab air flow in a house without a gravel bed when the house is being mitigated following the subslab depressurization technique.
Fig. 3 Expected Reynolds number for air flows through subslab gravel beds of houses being mitigated for radon. A radial cylindrical disk flow with impermeable boundaries is assumed with disk spacing = 0.1m (4"), diameter of gravel = 0.012m (1/2") and porosity of bed = 0.4. (Reynolds numbers above 10 indicate turbulent flow while those below 1 correspond to laminar flow.)
Fig. 4  Expected Reynolds number for air flows through subslab soil beds of houses being mitigated for radon. A radial cylindrical disk flow with impermeable boundaries is assumed with disk spacing = 0.1m (4''), flow rate = 2.36 l/s (5 cfm) and porosity of bed = 0.4. (Reynolds numbers below 1 indicate laminar flow while those between 1 - 10 correspond to transition range.)
Fig. 5 Variation in Reynolds number of air flow through a porous media for different superficial air velocities. The porous media is assumed to have a porosity of 0.4, and the three different values of particle diameter relate to gravel beds.
Fig. 6  Variation in Reynolds number of air flow through a porous media for different superficial air velocities. The porous media is assumed to have a porosity of 0.4, and the three different values of particle diameter relate to soil beds.
Fig. 7 Schematic of a model to duplicate flow conditions occurring beneath the concrete slab of a residence when induced by a single suction point. The air flow is assumed to be radial flow through a homogenous porous bed of circular boundary.
Fig. 8 Cross-section of the experimental laboratory apparatus

Fig. 9 Layout of the test holes to measure static pressures in the porous bed.
Fig. 10 Data from Exps. A1 and A2 versus model with exponent $b = 2$ (eq. (11)).

Fig. 11 Data from Exp. A3 versus model with exponent $b = 2$ (eq. (11)).
Fig. 12 Data from Exps. A1 and A2 versus model with exponent $b = 1.6$ (eq. (9)).

Fig. 13 Data from Exp. A3 versus model with exponent $b = 1.4$ (eq. (9)).
Fig. 14 Log-log plot of the observed pressure drop values of Exps. 1 and 2, in meters of water, versus those of the second term on the left hand side of eq. (15). This figure serves to illustrate the fact that the intercept, which corresponds to the permeability of the porous bed, cannot be estimated very accurately from regression of the data points at hand.