Microwave Conductivity of Magnetic Field Induced Insulating Phase of Bilayer Hole Systems

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Abstract

This thesis presents studies of the magnetic field induced insulating phase of bilayer hole systems. This insulating phase terminates the quantum Hall state series at sufficiently small total Landau filling factor $\nu$, and is understood as bilayer Wigner crystal (BWC), for samples of sufficiently low disorder. For a Wigner crystal in real samples, bilayer or single layer, the disorder not only gives the insulating behavior but also produces a striking microwave or rf conductivity resonance, or pinning mode, which is a collective oscillation of the carriers about their pinned positions. Pinning mode resonances of single layers have been studied experimentally and theoretically and have proven to be valuable for obtaining information about the single layer, pinned Wigner solids. As will be presented in this thesis, for BWC, the pinning modes exhibits features which depend sensitively on magnetic field, interlayer separation $d$, and bilayer densities.

We start from the balanced case, in which the two layers have equal carrier densities. Bilayer effects were studied by comparing the spectra of such balanced states to those of single layers realized in situ by depleting one of the layers. The BWC experiences an enhanced pinning (compared with the pinning of single layer Wigner crystals), only for small enough $d$. We interpreted this enhanced pinning as due to a quantum interlayer correlation, in which the BWC has carrier wave functions that spread coherently and equally between the two layers, and thus each carrier is affected by the disorder of the two layers. The BWC like this would be an easy-plane pseudospin ferromagnet, with pseudospin specifying the layers. Our balanced state studies also show that, only for sufficiently small $d$, development of the resonance shows features around $\nu = 1/2$ or $2/3$, demonstrating the effect, within the low $\nu$ BWC insulator, of correlations present in the fractional quantum Hall states.

We also studied the pinning modes of BWC in imbalanced bilayer states. Under a considerable range of imbalance, the enhanced pinning seen in the balanced state per-
sists, while the resonance broadens as imbalance is increased. For a sufficiently large imbalance, this enhanced pinning disappears abruptly. At this point, the resonance line width has a distinct maximum. We interpret these results as due to changes in the pseudospin magnetic ordering, driven by density imbalance.
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Chapter 1

Introduction and Theoretical Background

1.1 Introduction

In 1934, E. Wigner proposed that crystallization can occur in an electron system when the Coulomb energy dominates over the kinetic energy [Wigner, 1934]. For a two-dimensional (2D) system (2DS) at zero magnetic field, the kinetic energy per electron is \( (\pi h^2/2m^*)n_s \) (\( m^* \) is the effective mass), proportional to \( n_s \), while the Coulomb energy per electron is proportional to \( n_s^{1/2} \). Hence, the crystallization occurs at sufficiently low density. A detailed calculation [Tanatar and Ceperley, 1989] predicted the Wigner crystallization at a density, corresponding to \( r_s \approx 37 \pm 5 \), where \( r_s = (\pi n_s)^{-1/2} \) in units of the Bohr radius \( a_0 = 4\pi\epsilon_0\epsilon_r h^2/m^*e^2 \), and \( \epsilon_r \) is dielectric constant. The first realization of an electron Wigner crystal (WC) was a system of electrons floating on a liquid He surface [Grimes and Adams, 1979]. Experimentally, the vibrational modes of 2D WC, coupled to the ripplon modes of the liquid He surface [Fisher et al., 1979], were identified. This experiment was considered strong evidence for an electron crystal. Another favorable candidate for realization of a WC is the 2DS of electrons
or holes in semiconductor heterostructures. However, the densities of these 2DS are typically not sufficiently low for Wigner crystallization at zero magnetic field.

It was proposed [Lozovik and Yudson, 1975] that Wigner crystallization can occur at higher densities if the 2DS is subjected to a strong perpendicular magnetic field. With a perpendicular field, the continuum of energy states of a 2DS becomes a set of highly degenerate Landau levels (LL) at energies \( E_n = (n + 1/2)\hbar\omega_c \), where \( \omega_c = eB/m^* \). The degeneracy of each LL is \( (eB/2\pi\hbar)S \), where \( S \) is the total area of the 2DS. With density \( n_s \), the number of occupied LL, also known as the filling factor, is \( \nu = 2\pi n_s \hbar/eB \). The magnetic field also changes the wave function of a carrier dramatically, from a plane wave to a wave function with characteristic size \( l_B = \sqrt{\hbar/eB} \). As \( B \) is increased, the carrier is more tightly confined, and the zero-point motion, which favors the gaseous phase, is suppressed.

The quantum Hall effects (QHE) exhibited by 2DS have received much attention. The integer quantum Hall effect (IQHE) [von Klitzing et al., 1980] observed around integer filling factors has been explained as an effect of single particle localization [Laughlin, 1981]. On the other hand, the fractional quantum Hall effect (FQHE) [Tsui et al., 1982], appearing around fractional filling factors, is a many-body effect due to the Coulomb interaction between the carriers [Laughlin, 1983], and is a manifestation of a many-body liquid state. The series of quantum Hall states is always terminated at high \( B \) by an insulating phase, which for low enough disorder has been interpreted as a WC pinned by disorder [Shayegan, 1997]. For high quality single layer electron (hole) samples, the high \( B \) insulating phase takes over as the ground state for \( \nu \lesssim 1/5 \) (\( \nu \lesssim 1/3 \)) [Jiang et al., 1990, Santos et al., 1992]. Theoretical calculations showed the ground state switches from fractional quantum Hall liquid to high \( B \) WC around \( \nu = 1/6 \) for electron 2DS [Lam and Girvin, 1984, Levesque et al., 1984, Yang et al., 2001, Zhu and Louie, 1995], and around \( \nu = 1/3 \) for hole 2DS [Zhu and Louie, 1995], both roughly consistent with experimental results for the low-
est disorder samples.

For 2DS of electrons and holes, the microwave spectroscopy, which measures the frequency dependent real diagonal conductivity, shows resonances in the regime of the high $B$ insulating phases [Chen et al., 2004, Engel et al., 1997, Li et al., 1997, 2000, Ye et al., 2002]. This resonance is understood as the “pinning mode” (of the WC).

1.2 Pinning Mode of Single Layer WC at High B Field

1.2.1 Pinning Mode, and its Dependence on Magnetic Field and Carrier Density

In the absence of disorder, at zero $B$, a perfect 2D WC supports two phonon branches, the longitudinal mode $\omega_L(q) = (n_se^2/2m^*\epsilon_0\epsilon_r)^{1/2}q^{1/2}$ and the transverse mode $\omega_T(q) = C_Tq$, where the dielectric constant $\epsilon_r$ is 13 for GaAs, $C_T = \sqrt{\mu_T/n_em^*}$, $\mu_T = 0.245(e^2n_s^{3/2}/4\pi\epsilon_0\epsilon_r)$ is the shear modulus of 2D triangular lattice [Bonsall and Maradudin, 1977]. Both are gapless in the long wave length limit. At finite $B$ (and $\omega_c \gg \omega_L, \omega_T$), $\omega_L$ and $\omega_T$ are hybridized into a magnetoplasmon mode at $\omega_+ = \omega_c + \omega_L^2(q)/2\omega_c$ and a magnetophonon mode at $\omega_- = \omega_L(q)\omega_T(q)/\omega_c$ [Cote and MacDonald, 1991, Fukuyama and Lee, 1978, Normand et al., 1992]. In the long wave-length limit $q \to 0$, $\omega_-$ is gapless, $\omega_+$ is gapped at cyclotron frequency $\omega_c$. For a magnetic field induced WC, at a typical $B = 10$ Tesla, $\omega_c$ is well above the microwave frequency.

Within a random disorder potential, the WC loses its long-range ordering, and breaks into domains. To understand the effects of disorder, we first consider an oscillator model introduced by Fukuyama and Lee (F-L) in their pioneering work in 1978 [Fukuyama and Lee, 1978]. This model assumes the WC domain oscillates in a harmonic potential $M\omega_0^2r^2/2$, where $M$ is the total mass of the domain, $\omega_0 \ll \omega_c$.
characterizes the weak disorder and high magnetic field. With a perpendicular $B$, F-L predicted two modes. In the long wavelength limit, one mode is at $\omega_c + \omega_0^2/\omega_c$, again, out of the microwave frequency range. The other mode, the pinning mode, is at $\omega_{pk} \approx \omega_0^2/\omega_c$. The pinning mode can couple with a spatially uniform microwave signal, showing as a resonance, with the peak frequency $f_{pk} = \omega_{pk}/2\pi$. The F-L theory also introduced an oscillator strength $S_{FL}$, defined as the integral of $Re(\sigma_{xx})$ over frequency. It was predicted that $S_{FL}/f_{pk} = (e\pi/2B)n_s$, proportional to the average density of the solid phase. The F-L theory predicted $f_{pk}$ decreasing with $B$. This is not consistent with some experimental results, in which $f_{pk}$ can increase with field [Li et al., 1997, Ye et al., 2002].

In the F-L theory, the decreasing $f_{pk}$ with $B$ is due to the assumption of a $B$ independent disorder. Generally, the disorder felt by a carrier, referred as the “effective” disorder, depends on the quantum state of the carrier [Chitra et al., 1998, 2001, Fertig, 1999, Fogler and Huse, 2000]. This quantum effect modifies the “bare” disorder potential $V_b(r)$, which is the disorder felt by a point particle, into the “effective” disorder $V_e(r)$, via a convolution with the form factor (the expectation of the density function) of the carrier wave function. To address the pinning mode with the “effective” disorder, we use a formula, $\omega_{pk} \propto \Delta_e/\epsilon\mu_T\xi_e^6B$, developed by Chitra et al. [Chitra et al., 1998, 2001]. In this formula, $\Delta_e$ and $\xi_e$ are strength and correlation length of the “effective” disorder potential $V_e(r)$. They are defined by a correlation function $\langle V_e(r), V_e(r') \rangle = \Delta_eD_{\xi_e}(r - r')$, where $D_{\xi_e}(r)$ decays rapidly for $r$ larger than $\xi_e$. To derive the “effective” disorder, Chitra et al. assumed the “bare” disorder $V_b(r)$ to be a short range Gaussian random disorder, $\langle V_b(r), V_b(r') \rangle = \Delta_bD_{\xi_b}(r - r')$, where $D_{\xi_b}(r) \sim (1/\xi_b^2)e^{-r^2/\xi_b^2}$, $\Delta_b$ and $\xi_b$ are strength and correlation length of the “bare” disorder, both are taken to be $B$ independent. For a carrier in the lowest LL, a Gaussian form factor $F(r) = (\pi l_B^2)^{-1}\exp(-r^2/l_B^2)$ is used. For $l_B$ larger than $\xi_b$ (at low $B$), the “effective” disorder has been taken to possess correlation length $l_B$ and
\[ \Delta_e = \Delta_b [\text{Chitra et al., 2001, Fogler and Huse, 2000}], \text{leading to} \]
\[ \omega_{pk} \propto \frac{e^2 \Delta}{\mu_T \hbar^3} B^2 \]  
(1.1)

This formula qualitatively explains those experimental results in which \( f_{pk} \) increases with \( B \). For \( l_B \) less than \( \xi_b \) (at high \( B \)), the carriers can be reasonably treated as classical point particles. Hence, \( \Delta_e = \Delta_b, \xi_e = \xi_b \)
\[ \omega_{pk} \propto \frac{\Delta_e}{e \mu_T \xi_b^6 B} \]  
(1.2)
consistent with the F-L theory.

For a single layer WC, the resonance at a constant \( B \) depends on the carrier density. Figure 1-1(a) shows the spectra of a single layer hole system (in this case, realized by depleting the bottom layer of a bilayer sample M465) with several densities, measured at \( B = 10 \) Tesla. \( f_{pk} \) always decreases with density. Both the earlier F-L theory and the recent theories [Chitra et al., 2001, Fertig, 1999, Fogler and Huse, 2000] based on the “effective” disorder give \( f_{pk} \propto n_s^{-\gamma}, \) where \( \gamma = 1.5 \). The experimentally measured \( \gamma \) typically shows different values for different density ranges. In Figure 1-1(b), the \( f_{pk} \) vs. \( n_s \) approximately follows \( f_{pk} \propto n_s^{-\gamma} \), with \( \gamma \approx 0.5 \). In an earlier study [Li et al., 2000] of a single layer hole sample, \( \gamma \approx 0.5 \) was observed for low density, reminiscent of the behavior in Figure 1-1(b), but for high density range, \( \gamma \) switched to 1.5. The density dependence of \( f_{pk} \) can be qualitatively understood by considering the carrier-carrier interaction effect. With increasing density, the carrier-carrier interaction increases, and the WC becomes stiffer. The carrier positions are statistically less associated with the disorder potential, giving rise to lower \( f_{pk} \). In references [Li et al., 1997, 2000], as in Figure 1-1(a), the resonance sharpens as density is increased, also due to the carrier-carrier interaction effect [Fogler and Huse, 2000].
Figure 1.1: (a) Spectra of a single layer 2D hole system, with different densities, at $B = 10$ Tesla and $T \approx 65$ mK. This single layer hole system is realized by depleting the bottom layer of a bilayer hole sample M465. Spectra are vertically displaced for clarity. The area between each spectrum and its “zero” is shaded. (b) $f_{pk}$ vs. $p$ in "log – log" scale. The fitting suggests that $f_{pk}$ vs. $p$ approximately follows a power law, $f_{pk} \sim p^{-\gamma}$, $\gamma \approx 0.5$. 
1.2.2 Disorder Potential due to Interface Roughness

Compared with a 2D WC in a narrow quantum well (QW), one in a wide QW with similar density generally has a pinning mode with much lower $f_{pk}$. This was obtained by comparing $f_{pk}$ measured from QW samples with different well widths [Chen, 2005]. Specifically, for similar densities and magnetic fields, a 650Å QW sample shows $f_{pk} \approx 100$MHz, while a 150Å QW sample shows $f_{pk} \approx 6 - 8$GHz. The carrier-carrier interactions would be nearly the same, with similar density. Hence, for a narrower QW, the higher $f_{pk}$ suggests stronger carrier-disorder interaction, possibly due to stronger vertical confinement.

Interface roughness was first proposed by Fertig [Fertig, 1999] to be the relevant disorder potential for the pinning mode frequency, after comprehensive consideration of different types of disorder in a 2DS. Although the GaAs/AlGaAs interface of 2DS is of high quality, small features as pits and terraces still cannot be eliminated. Two possible mechanisms for the interface roughness disorder potential will be discussed as follows.

1. Due to the interface roughness, the effective distance between QW and its ionized dopant layer varies, which gives fluctuation of electrostatic potential. By assuming reasonable size and depth of the pit, Fertig calculated $f_{pk}$ to be of the right order of magnitude [Fertig, 1999].

2. Another mechanism was proposed by Sakaki et al. [Sakaki et al., 1987]. Due to the interface roughness, effective width of the QW varies, leading to fluctuation of the subband energy. By simplifying the QW as an infinite square well, the lowest subband energy is estimated to be $V = \pi^2 \hbar^2 / 2mW^2$, hence its fluctuation due to width variation $\Delta W$ is $\Delta V = (\pi^2 \hbar^2 / mw^3) \Delta W$, where $W$ is the average width of the quantum well. In this picture, the strength of interface roughness disorder decreases rapidly with $W$, consistent with experimental results [Chen,
1.3 QHE in Bilayer Systems

The bilayer electron (hole) systems have been the subject of intense theoretical and experimental studies for more than ten years. For high quality bilayer samples, in their balanced states (for which the two layers have equal densities), quantum Hall states with no counterpart in single layer systems were observed. These include the FQHE at total filling factor \( \nu = 1/2 \) [Eisenstein et al., 1992, Halperin, 1983, He et al., 1991, Suen et al., 1992, Yoshioka et al., 1989] and the QHE at \( \nu = 1 \) [Boebinger et al., 1990, Manoharan et al., 1996, Murphy et al., 1994, Suen et al., 1991, Tutuc et al., 2003a]. In this section, we will focus on the \( \nu = 1 \) QHE.

The QHE at \( \nu = 1 \) in the bilayers was first observed in samples with strong interlayer tunneling [Boebinger et al., 1990, Suen et al., 1991]. In the limit of strong interlayer tunneling, the \( \nu = 1 \) QHE was explained as a single particle effect [MacDonald et al., 1990]. The tunneling breaks degeneracy between the symmetric and anti-symmetric double quantum well (DQW) states, elevates the anti-symmetric state by a symmetric-antisymmetric gap \( \Delta_{SAS} \). At \( \nu = 1 \), the excitation gap is just \( \Delta_{SAS} \) (at strong \( B \) and in the single particle picture), which stabilizes the QHE.

For sufficiently small interlayer separation \( d \), a QHE at \( \nu = 1 \) was predicted even for the case of zero interlayer tunneling [Chakraborty and Pietilainen, 1987, Fertig, 1989, He et al., 1991, MacDonald et al., 1990, Yoshioka et al., 1989]. This \( \nu = 1 \) QHE (with zero tunneling) was later confirmed by experiment, and was found to be sensitive to the relative strength of the intralayer and interlayer Coulomb interactions. This relative strength is conveniently measured by \( \tilde{d} \equiv d/l_B \) [Murphy et al., 1994], where \( l_B = (4\pi n)^{-1/2} \) (\( n \) is the density in each layer) is the magnetic length at total filling factor \( \nu = 1 \). A critical value of \( \tilde{d} \approx 1.8 \), above which the \( \nu = 1 \) QH state was
not observed, was obtained from DC transport [Murphy et al., 1994, Spielman et al., 2000, 2001, 2004].

The activation energy that stabilizes a $\nu = 1$ QHE can generally contain the contribution from the interlayer tunneling, plus a contribution from the exchange energy. This can be better understood using a pseudospin language, in which an electron in the top (bottom) layer corresponds to the $| \uparrow \rangle$ ($| \downarrow \rangle$), eigenstate of the pseudospin Pauli matrix $\sigma_z$, with eigenvalue $+1$ ($-1$). With finite tunneling, all electrons like to stay in the symmetric DQW state $1/\sqrt{2}(| \uparrow \rangle + | \downarrow \rangle)$, with all pseudospins polarized in the $+\hat{x}$ direction. The exchange energy also tends to keep the pseudospins polarized. The reason is, with all the pseudospins polarized, the pseudospin part of the wave function is symmetric under particle exchange. This makes the spatial wave function antisymmetric, so that the system could gain the exchange energy.

Within the case of zero tunneling, we will discuss three situations. (1): In the limit of $d \to 0$, the polarization direction of pseudospins is arbitrary in the 3-dimensional pseudospin space. (2): For finite but small $d$, the pseudospins still like to be polarized, so that the system could take advantage of the exchange energy. Unlike the $d = 0$ case, all the pseudospins would be polarized in the $x-y$ plane. This would mean the carrier wave function spreads equally and coherently between the two layers, so that the capacitive energy is minimized. In the pseudospin space, all the carriers stay in the state $1/\sqrt{2}(| \uparrow \rangle + e^{i\phi}| \downarrow \rangle)$, with phase angle $\phi$ indicating the direction of polarization in the $x-y$ plane. We refer this bilayer system as having interlayer phase coherence, or easy-plane pseudospin ferromagnetism. At $\nu = 1$, the excitation gap which stabilizes the QHE originates from the exchange energy. (3): For large $d$, the carriers like to stay in the individual layers, to reduce the direct Coulomb energy. The bilayer becomes a pseudospin anti-ferromagnet. At $\nu = 1$, the exchange energy induced excitation gap disappears, so the QHE is suppressed.
1.4 Magnetic Field Induced Insulating Phase in Bilayer Systems

At sufficiently small $\nu$, bilayer systems evolve into a high $B$ insulating phase, which has been interpreted as a bilayer WC (BWC) pinned by disorder [Manoharan et al., 1996, Suen et al., 1994, Tutuc et al., 2003a]. Theories [Esfarjani and Kawazoe, 1995, Goldoni and Peeters, 1996, Narasimhan and Ho, 1995, Zheng and Fertig, 1995] have predicted a number of distinct BWC phases, in the absence of disorder. In this section, we go over main results of these theories, but focus on the case of zero interlayer tunneling, which is more relevant to the samples studied in this thesis.

In the absence of tunneling, the relative importance of interlayer and intralayer interaction is crucial in these theories, and is measured by the ratio $d/a$, where $a = (2\pi p)^{-1/2}$ is the in-plane carrier spacing and $p$ is the carrier density per layer. Related by a factor of $\sqrt{2}$ to $d/a$, $\tilde{d}$ also measures this relative importance, and so will be used in the rest of this thesis, to establish possible relation between $\nu = 1$ QHE and small $\nu$ BWC. A one-component triangular lattice is expected at small enough $\tilde{d}$, and is an easy-plane pseudospin ferromagnetic BWC (FMBWC), with one carrier evenly and coherently spreading between the two layers, at each lattice site. Interlayer-staggered two-component lattices occur at larger $\tilde{d}$, and without interlayer tunneling, are pseudospin antiferromagnetic BWCs (AFMBWC), with carriers essentially completely in one layer alternating with those completely in the other. Among these, a two-component square lattice was predicted to cover the widest range of $\tilde{d}$ [Goldoni and Peeters, 1996, Narasimhan and Ho, 1995, Zheng and Fertig, 1995]. But other phases, with rectangular or rhombic lattices, are also possible [Goldoni and Peeters, 1996, Narasimhan and Ho, 1995]. For the sufficiently large $\tilde{d}$, the intralayer interaction dominates, so the layers are simply triangular lattices like single layer Wigner crystals, but interlayer-staggered.
1.5 Structure of This Thesis

The thesis will be organized as follows. Chapter 2 briefly describes the bilayer hole samples, and experimental setup for the microwave conductivity measurement. Chapter 3 describes the methods we use to define and control the bilayer densities. In Chapter 4, we will focus on the microwave spectra in the balanced state. The effects from the density imbalance (between the two layers) will be discussed in Chapter 5. Conclusions and possible future research are presented in Chapter 6.
Chapter 2

Samples and Experimental Setup

2.1 Hole Double Quantum Well Samples

The samples studied in this thesis are high quality GaAs/AlGaAs/GaAs DQW’s, grown by molecular beam epitaxy (MBE) [Tutuc, 2004]. Table 2-1 summarizes the main features of these bilayer samples, in their balanced states. All the samples are designed to suppress the interlayer tunneling. M465 and M453 (with “∗” mark) are asymmetrically doped on both sides of the DQW. All the other samples have dopants only at the front of the DQW. The interlayer separation \( d = w + b \) is the distance between the centers of the two QW’s, where \( b \) is the barrier width, \( w = 150\,\text{Å} \) is the width for each QW in all these samples. As will be discussed latter, the balanced state is produced only by tuning the backgate voltage and can for each cooldown be produced at only one per-layer density \( p \). From \( d \) and \( p \), we calculate \( \tilde{d} \equiv d(4\pi p)^{1/2} \).

For small \( \tilde{d} \) samples (M440, M465, M417), the \( \nu = 1 \) interlayer phase coherent QHE was observed in both our microwave studies and the DC transport measurements on pieces of the same wafers [Tutuc, 2004].
Table 2.1: All bilayer samples studied in this thesis are listed, for their balanced conditions. All these samples are designed to have negligible interlayer tunneling. M465 and M453 (with “*” ) have silicon dopants at both the front and the back of the DQW. All the other samples only have dopants at the front of the DQW. For each wafer, interlayer separation $d$ is defined as the distance between centers of two quantum wells. $p$ is the density in each layer, $\tilde{d} \equiv d/l_B$, where $l_B = (4\pi p)^{-1/2}$ is the magnetic length at total filling factor $\nu = 1$. For M440, M417 and M433, two cool downs (labeled as “1st” and “2nd”) give different balanced states. The states showing a feature in magnetoconductivity due to $\nu = 1$ interlayer phase coherent QHE are labeled.
2.2 Microwave Conductivity Measurement in 2D System

The technique of using a coplanar wave guide (CPW) to measure the real diagonal conductivity \( Re(\sigma_{xx}) \) of a 2DS was first developed by Engel [Engel et al., 1993]. A schematic of the microwave measurement setup is shown in Figure 2-1. A metal film CPW, which consists of a center strip and two side planes, is deposited on the surface of the sample. The geometry of the CPW is designed to work on GaAs wafers, so that its characteristic impedance is \( Z_0 = 50\Omega \). The CPW matches the microwave coaxial cables (also with 50\( \Omega \) impedance) coming into and out of the measurement setup, hence, the microwave reflection is minimized. The weakly conductive 2DS capacitively couples to the CPW, and affects its propagation constant. From the attenuation of the microwave power, the real diagonal conductivity \( Re(\sigma_{xx}) \) of the 2DS can be derived.

In our measurement, the center strip is driven by a microwave source, and the two side planes are grounded. For sufficiently high frequency, the in-plane electric field is mainly confined within the CPW slot [Gillick et al., 1993], and the geometric capacitor between the CPW and the 2DS serves as an AC contact. The CPW that couples with a 2DS can be modeled as a uniformly distributed-circuit [Liao, 1990] shown in Figure 2-2. \( L \) is the inductance (per unit length) of the center strip \(^1\), and \( C \) is the capacitance (per unit length) between the center strip and the side planes. The transmission line is loaded by the admittance \( Y \) (per unit length), where \( Y \approx 2\sigma_{xx}/W \), and is essentially from the 2DS. The propagation constant is \( \gamma \approx \sqrt{j\omega L(Y + j\omega C)} \).

For a weakly conductive 2DS, \( Y \ll \omega C \), \( \gamma \approx \sqrt{L/C}\sigma_{xx}/W \approx Z_0\sigma_{xx}/W \), where \( Z_0 \approx \sqrt{L/C} = 50\Omega \) is the characteristic impedance of the transmission line (CPW). The power attenuation along a CPW with the center line length \( L \) is

\(^1\)We have neglected the sheet resistance (per unit length) of the center strip, since it is much smaller than \( \omega L \) for thick CPW metal and high frequency.
Figure 2.1: Schematic microwave measurement setup. (a) Top view of a sample with a coplanar wave guide (CPW) (the black region) on its surface. (b) Side view of sample. Neither view represents the real scale. The center strip is driven by a microwave source, the two side planes are grounded, and the transmitted power is measured by a detector. The 2DS is also grounded through the contacts that are put on the edges. $L$ is the length of the transmission line, $W$ is the width of CPW slots.
Figure 2.2: Distributed-circuit of the transmission line (CPW) loaded by a weakly conductive 2DS. $L$ is inductance (per unit length) of the center strip, $C$ is the capacitance (per unit length) between the center strip and the side plane. The transmission line is loaded by admittance (per unit length) $Y$. For sufficiently high frequency and small $|\sigma_{xx}|$, $Y \approx 2\sigma_{xx}/W$, where $\sigma_{xx}$ is diagonal conductivity of the 2DS, $W$ is the width of the CPW slots.

\[
\frac{P}{P_0} = \exp(-2\text{Re}(\gamma)L) \approx \exp\left(-Z_0 \frac{2\text{Re}(\sigma_{xx})L}{W}\right)
\]

(2.1)

where $P$ ($P_0$) is the transmitted power with (without) the attenuation by the 2DS. Equation (2-1) is the basic formula we use to calculate $\text{Re}(\sigma_{xx})$.

In our experiment, a network analyzer essentially measures the transmission coefficient of microwave power $P_R/P_E$, where $P_E$ and $P_R$ are emitted and received microwave power respectively. In general, $P_R/P_E = \tau \times P/P_0$, where $\tau$ is the transmission coefficient along the coaxial cables and reflection is neglected.

We usually do two types of measurement.

1. $\text{Re}(\sigma_{xx})$ vs. $B$ at a constant frequency (magnetoconductivity)
Since $\tau$ has negligible $B$ dependence, and the influence of the reflection has been carefully minimized, $-W/(2Z_0L) \ln(P_R/P_E)$ is different from $Re(\sigma_{xx})$ by a constant $-W/(2Z_0L) \ln(\tau)$.

2. $Re(\sigma_{xx})$ vs. frequency at a constant $B$ (spectrum)

$\tau$ is frequency dependent. For spectra, we need a “reference” state to remove the effects of the coaxial cables. The best “reference” state is the fully depleted state ($\sigma_{xx} = 0$), in which no power is absorbed along the CPW. The spectrum of the 2DS is

$$Re[\sigma_{xx}(f)] = -\frac{W}{2Z_0L} \left[ \ln \left( \frac{P_R}{P_E} \right) - \ln \left( \frac{P_{R}^{ref}}{P_{E}^{ref}} \right) \right]$$

(2.2)

where $P_{R}^{ref}$ and $P_{E}^{ref}$ are the received and emitted microwave power in the “reference” state. As an example, Figure 2-3(a) shows $-W/(2Z_0L) \ln(P_R/P_E)$ and $-W/(2Z_0L) \ln(P_{R}^{ref}/P_{E}^{ref})$, both are $f$ dependent. Figure 2-3(b) shows the difference between these two curves, which is the spectrum of 2DS.

### 2.3 Microwave Conductivity Measurement in Bilayers

The CPW technique had been successfully applied in single layer 2DS to study the high $B$ insulating phase [Chen et al., 2004, Engel et al., 1997, Li et al., 1997, Ye et al., 2002], as well as the bubble and stripe phases at high Landau levels [Lewis et al., 2004, 2002]. This thesis is the first application of this technique to bilayer samples. Several points need to be noted.

We refer the geometric coupling capacitance between the CPW and the top (bottom) quantum well as $C'_t$ ($C'_b$), and the capacitance between the two quantum wells
Figure 2.3: (a) The two curve show $-\frac{W}{2Z_0L}\ln\left(\frac{P_R}{P_E}\right)$ for two states. One state (solid line) has the 2DS. For the “reference” state (dashed line), the 2DS is depleted. (b) The spectra of the 2DS, which is the difference between the two curves shown in (a).
as $C'_m$. $C'_t$ is expected to be close to $C'_b = C'_t C'_m / (C'_t + C'_m)$, since $C'_m \gg C'_t$. For sufficiently high frequency, $C'_t$ and $C'_b$ work as good contacts to the top and bottom QW’s respectively, hence, the CPW couples almost equally with both the top and the bottom QW’s.

All the samples for our microwave measurement do not have a front gate over the CPW slot, the active region where the 2DS conductivity is essentially measured. This is due to the following considerations. A metal film too close to the slots would effectively shield the 2DS from the required in-plane microwave field. To avoid such shielding, the distance $h$ (between the front gate and the sample surface) should be at least $100 \mu m$, several times larger than the slot width ($W \approx 40 \mu m$). In addition, the insulating medium between the front gate and the sample surface should have $\epsilon_r \approx 1$. The reason is, large $\epsilon_r$ would change the characteristic impedance of the CPW, and cause large impedance mismatch between the CPW and the coaxial cables. Given the $h$ and $\epsilon_r$, in order to tune the top layer density by $3 \times 10^{10} cm^{-2}$ (the typical density of the top layer for these bilayer samples), a front gate voltage of the magnitude of 800V is required. A high voltage like this could cause leaking problem. An alternative, though still difficult, would be to redesign the CPW geometry to allow smaller $h$ or larger $\epsilon_r$.

Due to these difficulties, in this thesis, the bilayer densities are controlled only by a back gate bias $V_g$. The bilayer densities vs. $V_g$ will be discussed in detail in Chapter 3.
Chapter 3

Identification of Bilayer Densities

As introduced in Chapter 2, the bilayer densities are only controlled by a back gate bias $V_g$. In this chapter, we will describe how we determine $p_T$ and $p_B$ vs. $V_g$. For small $d$ samples (M440, M465, and M417), as will be presented in Section 3-1, the IQHE (including the $\nu = 1$ QHE) are observed at several integer total filling factors. From the positions of these IQH minima, only the total density $p_{TOT}$ can be derived. The balanced condition, and its per-layer density, can be experimentally derived as well. From $p_{TOT}$ vs. $V_g$ and the balanced density, the respective layer densities vs. $V_g$ have to be estimated based on a simple capacitive model (with details in Section 3-1).

For large $d$ samples (M433, M436, M443, and M453), as will be discussed in Section 3-2, the top and bottom densities can be derived from the positions of IQH minima associated with individual layers, but only within a limited $V_g$ range. Beyond these ranges, the densities are estimated by extrapolation, also based on the capacitive model.

In both sections, determination of the individual layer densities vs. $V_g$ heavily relies on a capacitive model. As proposed by several experimental and theoretical studies, density distribution within a bilayer can be more complex. In Section 3-3,
we will discuss possible error bar in the densities derived in Sections 3-1 and 3-2.

### 3.1 Bilayer Densities for Small $d$ Samples

In this section, we will show how we determined bilayer densities for M465. The same procedure worked for M440 and M417 as well. We measured \( Re(\sigma_{xx}) \) vs. \( B \) of M465 under a series of \( V_g \). For each \( V_g \), the total bilayer density \( p_{TOT} \) is calculated from the positions of the QH minima, especially the QH minimum at total filling factor \( \nu = 1 \). Figure 3-1 shows \( p_{TOT} \) vs. \( V_g \), which is easily fitted as \( p_{TOT} = 8.37 - 0.0191 \times V_g \).

We will then identify the balanced state by use of two criteria learned from DC transport studies on other pieces of the same wafer [Tutuc et al., 2003a].

1. In the balanced condition, the sample shows only the even IQHE and the inter-layer phase coherent QHE at \( \nu = 1 \).

2. The weakest \( \nu = 1 \) QHE and the strongest even IQHE are observed in the balanced state.

For the microwave measurement, we identify the balanced state in two steps.

1. We measure \( Re(\sigma_{xx}) \) vs. \( B \) with \( f = 200 \) MHz in a series of \( V_g \). After identifying the QH minima, we re-plot \( Re(\sigma_{xx}) \) vs. total filling factor \( \nu \) as shown in Figure 3-2(a). After this step, we can identify a range of \( V_g \) (\( 10V < V_g < 100V \)), in which the QH minima at \( \nu = 1 \) and \( \nu = 2, 4, 6 \ldots \) are much better developed than those at \( \nu = 3, 5, 7 \ldots \). The balanced state is within this \( V_g \) range.

2. Figure 3-2(b) shows \( Re(\sigma_{xx}) \) vs. \( B \) for a series of \( V_g \) within the range defined in Figure 3-2(a). The balanced bilayer, which shows the weakest \( \nu = 1 \) QHE and strongest \( \nu = 2 \) QHE, corresponds to \( V_g = 55 \pm 5V \).

\[1\text{In this chapter, the unit is } 10^{10} \text{ cm}^{-2} \text{ for densities, and } V \text{ for voltages.} \]
From the $p_{TOT}$ vs. $V_g$ relation, the per-layer density in the balanced state is $p_T = p_B = 3.65 \pm 0.1 \times 10^{10} cm^{-2}$. For imbalanced states, the densities of individual layers can be estimated based on a simple capacitive model.

In this capacitive model, both the 2D layers (when not fully depleted) and the metallic gate are considered as ideal conductors. The two quantum wells (grounded through the contacts) are separated by $d$, and the substrate thickness is $s$. As long as the bottom layer is not depleted, it would perfectly screen the top layer, and the electric field induced by $V_g$ would be terminated at the bottom layer. For this case, $p_T$ would be a constant, while $\Delta p_B = -\Delta V_g \times \varepsilon_0 \varepsilon_r / s$. After depletion of the bottom layer, $\Delta p_T = -\Delta V_g \times \varepsilon_0 \varepsilon_r / (s + d)$. For all the samples studied in this thesis, $d$ is much less than $s$. Hence, the difference between $\varepsilon_0 \varepsilon_r / s$ and $\varepsilon_0 \varepsilon_r / (s + d)$ is negligible.

Within the capacitive model, for M465, $p_T$ and $p_B$ vs. $V_g$ are

$$p_T = \begin{cases} 3.65, & \text{for } V_g < 245, \\ 3.65 - 0.0191 \times V_g, & \text{for } V_g > 245. \end{cases}$$ (3.1)

$$p_B = \begin{cases} 4.71 - 0.0191 \times V_g, & \text{for } V_g < 245, \\ 0, & \text{for } V_g > 245. \end{cases}$$ (3.2)

### 3.2 Bilayer Densities for Large $d$ Samples

For large $d$ samples (in this thesis: M433, M436, M443, and M453), the QHE is not necessarily observed at integer total filling factors, when the bilayer is not balanced. Determination of bilayer densities requires a different approach. In this section, we will show this approach, and use M453 as an example. We measure $Re(\sigma_{xx})$ vs. $B$ of M453 for a series of $V_g$. For $V_g \lesssim 30V$, QH minima are identified, which move with $V_g$. These QH states are taken to be associated with the bottom layer. $p_B$ is calculated from the positions of these QH states. Figure 3-3 shows $p_B$ vs. $V_g$ (with
Figure 3.1: Total bilayer density $\rho_{TOT}$ vs. back gate bias $V_g$ for M465. For each $V_g$, $\rho_{TOT}$ is estimated from the positions of QH states, which are observed at total filling factors.
Figure 3.2: (a) $\text{Re}(\sigma_{xx})$ vs. total filling factor $\nu$ for M465 at three $V_g$. (b) $\text{Re}(\sigma_{xx})$ vs. $B$ for M465 at a series of $V_g$. From 1 to 9, $V_g$ decreases from 95V to 15V with a step of 10V. The balanced bilayer shows the weakest interlayer phase coherent QHE at $\nu = 1$ and the strongest QHE at $\nu = 2$. Data are measured at $T \approx 65\text{mK}$, using a $f = 200\text{MHz}$ signal.
“□” symbol), well fitted as \( p_B = 6.2 - 0.032 \times V_g \), consistent with the capacitive model. For \( 30V \lesssim V_g \lesssim 190V \), the positions of QH minima are not accurate. Within this range, based on the capacitive model, we extrapolate the above linear relation till \( p_B \) drops to zero. Hence, \( p_B \) vs. \( V_g \) would be

\[
p_B = \begin{cases} 
6.2 - 0.032 \times V_g, & \text{for } V_g < 192, \\
0, & \text{for } V_g > 192.
\end{cases}
\] (3.3)

For \( V_g \) higher than about 190\( V \), QH minima are identified again, and move with \( V_g \). These QH minima are taken to be associated with the top layer. The calculated \( p_T \) vs. \( V_g \) is also shown in Figure 3-3 (with “□” symbol). \( p_T \) also decreases linearly with \( V_g \), consistent with the capacitive model for a single layer. Fitting \( p_T \) vs. \( V_g \) with a straight line, we obtain \( p_T = 11.1 - 0.03 \times V_g \), for \( V_g > 192V \). For \( V_g \lesssim 192 \), \( p_T \) is taken to be a constant. Hence, \( p_T \) vs. \( V_g \) would be

\[
p_T = \begin{cases} 
5.3, & \text{for } V_g < 192, \\
11.1 - 0.03 \times V_g, & \text{for } V_g > 192.
\end{cases}
\] (3.4)

Based on \( p_T \) and \( p_B \) vs. \( V_g \), the sample would reach the balanced state for \( V_g = 28V \). For M453 (as well as M433, M436, M443), \( \rho_{xx} \) exhibits a hysteresis in DC transport measurements [Tutuc, 2004, Tutuc et al., 2003b], for imbalanced cases. The hysteresis is likely caused by an instability in the charge distribution of the two layers [Tutuc, 2004, Tutuc et al., 2003b], which will be mentioned in Section 3-3. In the AC measurement, we use the disappearance of this hysteresis, combined with quantum Hall features, to identify the balanced state.

Figure 3-4 shows \( \text{Re}(\sigma_{xx}) \) vs. \( B \) for several \( V_g \), with \( f = 200\text{MHz} \). The solid (dashed) lines indicate the traces taken when \( B \) is swept down (up). In the balanced state, the hysteresis is minimized, and \( \text{Re}(\sigma_{xx}) \) vs. \( B \) shows a FQH state at total filling factor \( \nu = 4/3 \), accurate relative to IQH states. After a measurement like this,
Figure 3.3: $p_T$ (○) and $p_B$ (□) vs. $V_g$ for M453. $p_T$ and $p_B$ are calculated from the positions of QH states. The two solid straight lines very well fit the data points. For the region, in which measured data points are not available, $p_T$ and $p_B$ vs. $V_g$ are estimated based on the capacitive model, as plotted by dashed lines.

The balanced state is identified for $V_g = 30 \pm 10V$, consistent with the previous result.

### 3.3 Discussion

The estimation of the bilayer densities (in the earlier sections) are based on a simplified capacitive model, in which the carriers are considered as independent particles, and the kinetic energy and the carrier-carrier interaction are neglected. As the kinetic energy and the carrier-carrier interaction are “turned on”, in a state with $p_T$ and $p_B$, a carrier transfer $\delta p$ may take place (through the DC contacts).

This carrier transfer was observed at $B = 0$ [Eisenstein et al., 1994, Papadakis et al.,
Figure 3.4: $Re(\sigma_{xx})$ vs. $B$ for M453 at several $V_g$. The solid (dashed) lines indicate the traces taken when $B$ is swept down (up). Data are measured at $T \approx 65\text{mK}$, using a constant $f = 200\text{MHz}$ signal. In the balanced state, the hysteresis is minimized, some of the QH states are labeled with total filling factors $\nu$. 
1997, Ying et al., 1995], and in the high $B$ insulating regime [Eisenstein et al., 1994]. For a bilayer hole system with a back gate, due to the carrier transfer, for $V_g$ not large enough to deplete the bottom layer completely, the top layer density increases slightly with $V_g$, while the bottom layer density decreases faster than linearly. In other words, the depletion of the bottom layer occurs at lower $V_g$, or higher $p_{TOT}$, compared with those in the capacitive model. Based on a rough calculation that will be shown in Appendix A, the $p_{TOT}$ at which the bottom layer is completely depleted, as estimated using the capacitive model, may be too small, by as much as $1 \times 10^{10} cm^{-2}$.

As the field is swept up, in the IQHE regime, the chemical potentials of both layers oscillate due to the Landau quantization, leading to carriers transferring back and forth between the two layers. This effect was indeed observed in experiments [Tutuc et al., 2003b, Zhu et al., 2000], in which the carrier transfer shows as hysteresis.
Chapter 4

Microwave Conductivity of Balanced Bilayers

This chapter presents a systematic study of the microwave pinning mode resonances of low $\nu$ insulating phase for a series of bilayer samples (see Table 2-1), in their balanced states. To isolate the effects of interlayer interaction or correlation, we compare spectra of balanced states (layer carrier densities $(p, p)$) to those of single layer states (layer carrier density $(p, 0)$, realized by depleting one of the layers). Denoting the resonance frequencies in these states by $f_{pp}$, $f_{p0}$, we focus on $\eta = f_{pp}/f_{p0}$ vs. $\tilde{d}$, where $\eta$ has a distinct minimum at $\tilde{d} \approx 1.8$. The interpretation is in terms of two competing effects, which respectively tend to lower and raise $\eta$ as $\tilde{d}$ decreases. (1) turn-on of the interlayer interaction on going from the $(p, 0)$ single layer to the $(p, p)$ BWC and (2) enhancement of the effective pinning disorder in the $(p, p)$ state relative to that in the $(p, 0)$ state only when the $(p, p)$ state is an interlayer correlated pseudospin ferromagnetic BWC (FMBWC).
4.1 Magnetoconductivity of Balanced Bilayer

Figure 4-1 shows $Re(\sigma_{xx})$ vs. $B$ for M465 and M433, in their balanced states, measured at $f = 200$ MHz. For M465, there is a strong minimum at $\nu = 1$. This minimum is ascribed to the interlayer phase coherent state [Tutuc et al., 2003a]. M433, on the other hand, shows no such minimum, though its $\nu = 2, 4, \text{and} 6$ minima are comparably developed to those of M465, indicating its disorder is not substantially larger than M465. The interlayer phase coherent $\nu = 1$ state is known to require a sufficiently small $\tilde{d}$, below about 1.8 [Murphy et al., 1994, Spielman et al., 2000, 2001, 2004]. M465 has $\tilde{d} \approx 1.56$, well below this threshold, while M433 in the state for which the trace was measured has $\tilde{d} \approx 1.8$. Hence, the absence of a $\nu = 1$ QH minimum in the M433 trace is understood as an effect of its larger separation, which suppresses the interlayer phase coherent state. In our studies of all the samples, we find minimum in the magnetoconductivity at $\nu = 1$ only for $\tilde{d} \lesssim 1.7$, as noted in Table 2-1.

The magnetoconductivity measurement confirms that the microwave experimental setup allows us to see the same features as demonstrated in DC transport measurements. As discussed in Chapter 2, microwave measurement uses capacitive coupling instead of direct coupling (via Ohmic contacts) in DC transport measurements. The difference in coupling, the presence of CPW metal and the lack of a front gate do not obscure the main features of these bilayer systems.

4.2 Spectra of Balanced Bilayer

Figures 4-2 illustrates the evolution with $B$ (and $\nu$) of the spectra of M465 in its balanced state. For clarity, the spectra are vertically displaced, proportional to the total filling factor. The area between each spectrum and its “zero” is shaded. Parameters from the spectra, $f_{pp}$ and $Q$ ($f_{pp}$ divided by full width at half maximum), are plotted vs. $\nu$ in Figure 4-4. For M465, and two other samples M440 and M417 (see
Figure 4.1: $Re(\sigma_{xx})$ vs. $B$ of M465 and M433 measured at $f = 0.2\,\text{GHz}$, in their balanced states. M465 has $d = 230\,\text{Å}$, and density $p = 3.65 \times 10^{10}\,\text{cm}^{-2}$/layer, which give $\tilde{d} \equiv d/l_B = 1.56$, where $l_B = (4\pi p)^{-1/2}$ is the magnetic length at total filling factor $\nu = 1$. M433 has $d = 300\,\text{Å}$, density $2.85 \times 10^{10}\,\text{cm}^{-2}$/layer, and $\tilde{d} = 1.8$. The numbers used to label the QH states are total filling factors. All Data are measured at $T \approx 65\,\text{mK}$. 
Appendix B), with the smallest $d$, the resonance is present for $\nu$ just below $\nu = 1$ QH minimum, and sharpens dramatically as $\nu$ decreases below about 0.5, so that the $Q$ vs. $\nu$ curve in Figure 4-4 shows a change of slope at that filling. For M465 and M417, another slope change also occurs at $\nu = 2/3$. For M465, the resonance weakens as $\nu$ approaches $2/3$ and $1/2$, showing as dips of $f_{pp}$ around these $\nu$'s. These features in the development of the resonance, are evidence that some of the correlations related to the FQHE are present in the pinned BWC, and will be discussed in Section 4-4-1. Possibly due to their lower $p$ compared with M465, M417 and M440 only have inflections in $f_{pp}$ at $\nu = 2/3$ or $1/2$.

For samples with $\tilde{d} \gtrsim 1.7$ (in this thesis: M433, M436, M443, and M453), as shown in Figure 4-3 for M453 (and in Appendix B for other samples), the resonance starts to develop for $\nu$ less than about 0.6. This resonance development is typical of that seen at the same per-layer filling, previously in low density $p$ type single layers [Li et al., 1997], and in the present samples in their single layer $(p, 0)$ states. $f_{pp}$ and $Q$ for M453 are plotted vs. $\nu$ in Figure 4-4 as well. The evolution of the resonance with $B$ (or $\nu$) is gradual. No features such as dips of $f_{pp}$, are observed.

4.3 Comparison of Microwave Spectra between Balanced Bilayer and Single Layer

The main results of this section follow from direct comparison, for each sample, of the balanced $(p, p)$ and single layer $(p, 0)$ states. We realize a single (top) layer state by reducing the total density by 50% from the balanced state \(^1\).

Spectra from these pairs of states are shown for each sample, at $B = 10$ T in Figure 4-5. The bilayer spectra are shown as solid lines, and the single layer spectra

\(^1\)In the capacitive model, reducing the density by 50% from state $(p, p)$ would give a single top layer state $(p, 0)$. This is still the case, even with consideration of carrier transfer effect, as described in Chapter 3 and Appendix A.
Figure 4.2: The spectra of M465 in the balanced state, measured at several total filling factors $\nu$ (magnetic fields $B$). M465 has $d = 230\text{Å}$, density per layer $p = 3.65 \times 10^{10} \text{cm}^{-2}$, which give $\tilde{d} = 1.56$. For clarity, the spectra are vertically displaced, proportional to $\nu$. For each spectrum, $\nu$ and $B$ are labeled. The resonance sharpens dramatically as $\nu$ goes below about $1/2$. When $\nu \to 2/3$ and $1/2$, the resonance (dashed line) weakens and shifts to lower $f$. All the data are measured at $T \approx 65 \text{mK}$. 
Figure 4.3: The spectra of M453 in its balanced state, measured at several total filling factors $\nu$ (magnetic fields $B$). M453 has $d = 2170\, \text{Å}$, and density per layer $p = 5.25 \times 10^{10} \, \text{cm}^{-2}$, which give $d = 18$. For clarity, the spectra are vertically displaced, proportional to $\nu$. For each spectrum, $\nu$ and $B$ are labeled. All the data are measured at $T \approx 65 \, \text{mK}$. 
Figure 4.4: $f_{pp}$ and $Q$ vs. total filling factor $\nu$ for M465 and M453, in their balanced states, at $T \approx 65$ mK. $f_{pp}$ is peak frequency of the resonance, $Q$ is $f_{pp}$ divided by full width at half maximum. For M465, around $\nu = 1/2$ and $2/3$, the $Q$ vs. $\nu$ curve changes slope around, and $f_{pk}$ exhibits minima. For M453, both $f_{pk}$ and $Q$ increase monotonically on reducing $\nu$. 
are dashed. $Re(\sigma_{xx})$ from the single layer states is doubled to facilitate comparison. The M453 spectra in Figure 4-5(g) are nearly identical spectra, as expected for independent layers, and not surprising considering the large $\tilde{d} = 18$ for that sample. Important for the interpretation in subsequent section, the nearly identical spectra also indicate that the disorder statistics relevant to the pinning mode are nearly the same in the top and bottom wells. This symmetry of the disorders of the layers should hold for all the samples, since they all had similar growth characteristics such as asymmetrical doping and interfacial compositions.

Hence we interpret the differences between the $(p,p)$ and $(p,0)$ spectra in Figure 4-5(a)-(g) as due to changes in interlayer interaction and correlation. Relative to the $(p,0)$ spectra, the $(p,p)$ spectra shift slightly to lower $f$ as $\tilde{d}$ decreases down to 2.47, as shown in Figures 4-5(e) and (f). At $\tilde{d} \approx 1.8$, for M433 as shown in Figure 4-5(d), the $(p,p)$ resonance is markedly shifted downward in $f$, and is stronger and sharper. But decreasing $\tilde{d}$ further (even through a different cooldown of the same M433 sample) tends to reduce the downward shift of $f_{pp}$ relative to $f_{p0}$, though the $(p,p)$ resonance remains much sharper than the $(p,0)$ resonance, as seen in Figures 4-5(a)-(c). To summarize, Figure 4-6 shows the ratio $\eta \equiv f_{pp}/f_{p0}$ vs. $\tilde{d}$; this curve has a striking minimum at $\tilde{d} \approx 1.8$. Figure 4-7 shows $\eta$ vs. $\nu$, the total filling factor of the $(p,p)$ state, for several samples. This graph shows that the minimum in $\eta$ vs. $\tilde{d}$ would remain, whether $B$ or $\nu$ is fixed, as long as $\nu \lesssim 0.5$. For $\tilde{d} \lesssim 1.7$, as shown for M465 in Figures 4-2 and 4-4, resonances become well developed below about that $\nu$. 
Figure 4.5: For each bilayer sample, we compare the spectrum of the balanced state \((p, p)\) (solid line) with the spectrum of the single layer state \((p, 0)\) (dashed line). The single layer spectra are multiplied by a factor of 2, to facilitate comparison. For each wafer, \(\tilde{d} \equiv d/l_B\) is labeled, where \(l_B\) is the magnetic length at \(\nu = 1\).
Figure 4.6: The ratio $\eta \equiv f_{pp}/f_{p0}$ vs. $\tilde{d}$, at $B = 10$ Tesla and $T \approx 65$ mK, where $f_{pp}$ and $f_{p0}$ are peak frequencies for the balanced state $(p, p)$ and single layer state $(p, 0)$ respectively.
Figure 4.7: The ratio $\eta \equiv \frac{f_{pp}}{f_{p0}}$ is plotted vs. total filling factor $\nu$ ($\nu \lesssim 0.5$), for several samples with different $d$. 
4.4 Discussion

4.4.1 FQH Correlation in the Solid Phase

The features (such as the dips in $f_{pk}$ vs $\nu$ and the slope change in $Q$ vs. $\nu$) around $\nu = 1/2$ and $2/3$ can be attributed to quantum correlations like those responsible for the FQHE. Such features are not present in M453, which has large $\tilde{d}$. The quality of M453 is about the same as that of the small $\tilde{d}$ samples, so the absence of features around $\nu = 1/2$ is not due to disorder. Hence, the fractional filling features in the high $B$ Wigner solid appear to be an effect of the reduced $\tilde{d}$. This is consistent with DC transport studies, in which the $\nu = 1/2$ FQH state was observed only at sufficiently small $\tilde{d}$ [Eisenstein et al., 1992, Suen et al., 1992].

The effects of FQH correlation on the Wigner crystal have been observed in many earlier works [Buhmann et al., 1991, Chen et al., 2004, Pan et al., 2002]. In a similar microwave measurement on single layer electron systems [Chen, 2005], as filling factor $\nu$ approaches $1/5$, $f_{pk}$ of the resonance also drops. In weak pinning, a decrease of $f_{pk}$ can be due to an increase of lattice stiffness or to a reduction of pinning. Upon approaching these fractional filling factors ($\nu = 1/5$ for single layer electrons and $\nu = 1/2$ and $2/3$ for bilayer hole sample M465), the WC is predicted to soften [Narevich et al., 2001, Normand et al., 1992] \(^2\). Hence, the drop of $f_{pk}$ around these fillings would be due to a reduction of pinning. Chen (2005) reported that $S_{FL}/f_{pk}$ (see Chapter 1), as well as $f_{pk}$, drops as a well developed $\nu = 1/5$ FQH state is approached from within the solid. In that case, coexistence of liquid and solid was speculated to cause the decreasing in $f_{pk}$, and would also explain the drop in $S_{FL}/f_{pk}$ as due to partial melting (or decreasing of solid phase fraction). However, this interpretation does not apply for our bilayer cases. We note that, for $\nu$ below 1, M465 bilayer is an insulator, and no FQHE was observed in DC transport studies [Tutuc et al., 2003a].

\(^2\)Although the theory considered single layer cases, the softening would be reasonably expected for bilayer as well.
importantly, around total filling factors $\nu = 1/2$ and $2/3$, $S_{FL}/f_{pk}$ shows no dips (see Appendix B), suggesting partial melting of the solid is not the cause of the $f_{pk}$ drop. Hence, the observed drops of $f_{pk}$ appear to be caused by some underlying changes of the solid phase, due to the FQH correlations.

4.4.2 Quantum Interlayer Coherence in the Bilayer WC

We interpret the $\eta$ vs. $\tilde{d}$ curve as a result of two competing effects. The first effect is driven by carrier-carrier interaction and must be present in any weakly pinned WC, bilayer or single layer. In this effect, when carrier-carrier interaction is increased, by decreasing carrier spacing (or increasing their overall density), the resonance frequency decreases. In single layers at fixed $B$, at which the resonance is well developed, as carrier density $n_s$ is increased, the peak frequency $f_{pk}$ always decreases and the resonance sharpens. Typically [Li et al., 2000], $f_{pk} \propto n_s^{-\gamma}$, with $\gamma \approx 3/2$ for higher $n_s$ giving way to $\gamma \approx 1/2$ at lower $n_s$; the present samples in single layer states all have $\gamma \approx 1/2 \pm 10\%$. As introduced in Chapter 1, the interpretation in weak pinning of the decrease of $f_{pk}$ with increasing $n_s$ is that increasing of carrier-carrier interaction (i.e., the crystal stiffness) cause the carriers to adjust to positions which essentially fall less into the impurity potential. This decreases the pinning energy per carrier and the restoring force on the carriers, hence $f_{pk}$.

We interpret the decrease of $\eta$ with decreasing $\tilde{d}$, for $\tilde{d} > 1.8$, as due to this carrier-carrier interaction effect, within an AFMBWC. In the limit of small $\tilde{d}$, the result of this interlayer interaction, on going from $(p,0)$ to $(p,p)$, is analogous to doubling the areal density of a single layer, and for $\gamma \sim 1/2$ gives $\eta = 2^{-\gamma} \approx 0.71$. This agrees with the $\eta$ we measure for $\tilde{d} = 1.8$. The sharp increase in $\eta$ as $\tilde{d}$ goes below 1.8 is not readily explainable in terms of the carrier-carrier interaction effect. Transitions between different types of AFMBWC are predicted by the theories [Goldoni and Peeters, 1996, Narasimhan and Ho, 1995, Zheng and Fertig, 1995] and if $\tilde{d}$ is near a transition,
the BWC can conceivably soften (multiple low energy arrangements possible), which
would produce some increase of \( \eta \) around particular \( \tilde{d} \). It is not likely though, that
even around a transition between AFMBWC phases, \( \eta \) would be as close to unity as
it is in Figures 4-7 and 4-8 at the smallest \( \tilde{d} \).

The second of the competing effects that we use to explain \( \eta \) vs. \( \tilde{d} \) is driven by
interlayer correlation, is present only in the easy-plane FMBWC, and was considered
theoretically by Chen [Chen, 2006]. Chen found that in an easy-plane FMBWC pin-
nning is enhanced when there is disorder that is spatially correlated in the planes of
the top and bottom layers. At sites where impurities or interfacial features induce
local interlayer tunneling, such spatial correlation would naturally result. When this
disorder enhancement is considered, along with the competing carrier-carrier interac-
tion effect, \( \eta \) as large as \( 2^{1-\gamma} \) is possible, so the transition to an easy-plane FMBWC
is sufficient to explain the increase of \( \eta \) with decreasing \( \tilde{d} \) seen in Figures 4-7 and
4-8. Even within the FMBWC, an increase of \( \eta \) as \( \tilde{d} \) decreases is expected, since
the smaller \( \tilde{d} \) would increase the interlayer-spatially correlated component of effective
disorder.

The data then indicate that \( \tilde{d}^* \), the critical \( \tilde{d} \), below which the FMBWC is present,
is around 1.8. Theories with small but finite tunneling predict smaller \( \tilde{d}^* \), around
0.4 [Narasimhan and Ho, 1995, Zheng and Fertig, 1995]. A possible explanation of
the discrepancy lies in the increased pinning experienced by the FMBWC, since the
pinning energy can stabilize the FMBWC against the more weakly pinned AFMBWC
phases that succeed it at larger \( \tilde{d} \). Such stabilization has been considered [Chen et al.,
2006, Price et al., 1993], in the context of transitions between FQHE liquids and single
layer Wigner crystals. A possible clue to the presently estimated \( \tilde{d}^* \) is that it is close
to the maximal \( \tilde{d} \) value (\( \approx 1.8 \)) below which the interlayer correlated QH states at
\( \nu = 1 \) exist [Murphy et al., 1994, Spielman et al., 2000, 2001, 2004].
Chapter 5

Imbalance Effects on the Pinning Mode of Bilayer WC

Introducing a density imbalance (between the two layers) can affect interlayer correlation or interaction. Imbalance effects have been experimentally observed around the $\nu = 1$ quantum Hall liquid state [Spielman et al., 2004, Tutuc et al., 2003a, Wiersma et al., 2005]. In this chapter, we study the imbalance effects on the small $\nu$ BWC.

Our experimental setup has been described in Chapters 2. Within the capacitive model introduced in Chapter 3, for back gate voltage $V_g$ too small to deplete the bottom layer, $p_T$ is constant, and $p_B$ decreases on increasing $V_g$. For $V_g$ large enough that $p_B = 0$, $p_T$ decreases on increasing $V_g$. We study a series of samples, with different $d$, $p_T$, and $\tilde{d}$ (at balance), summarized in Table 5-1. As long as $p_B$ is larger than a threshold, $p_B^*$, the imbalanced BWC spectra have two resolved resonances. For M453 and M443, with the largest $d$, two resonances are observed for all imbalanced bilayer states. For M433 and M417, $p_B^*/p_T \sim 1.7$. For M465 and M440, the smallest $d$ samples, there is no sign of two resonances up to the largest $p_B$ we looked at, as large as $p_B \sim 1.9 \times p_T$. When resolved, the development of the resonances vs $B$ or
vs bilayer densities is consistent with associating each resonance mainly with one of the layers. For each layer to have its own pinning mode, the intralayer interaction would dominate the interlayer interaction. The intralayer interaction of the top layer is mainly determined by the fixed $p_T$. The intralayer interaction of the bottom layer is likewise determined by $p_B$, and would increase on increasing $p_B$, while the interlayer correlation weakens on increasing $d$. Hence, the threshold $p_B^*$ would decrease as $d$ increases, consistent with experimental results.

<table>
<thead>
<tr>
<th>Wafer Name</th>
<th>$d$ (Å)</th>
<th>$p_T$ ($10^{10}cm^{-2}$)</th>
<th>$d$</th>
<th>$p_B^*$ ($10^{10}cm^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M440</td>
<td>225</td>
<td>3.0</td>
<td>1.39</td>
<td></td>
</tr>
<tr>
<td>M440</td>
<td>225</td>
<td>3.85</td>
<td>1.57</td>
<td>&gt; 7</td>
</tr>
<tr>
<td>M465</td>
<td>230</td>
<td>3.65</td>
<td>1.56</td>
<td>&gt; 7</td>
</tr>
<tr>
<td>M417</td>
<td>260</td>
<td>3.05</td>
<td>1.61</td>
<td>~ 4.8</td>
</tr>
<tr>
<td>M433 1st cool down</td>
<td>300</td>
<td>2.52</td>
<td>1.7</td>
<td>~ 4.8</td>
</tr>
<tr>
<td>M433 2nd cool down</td>
<td>300</td>
<td>2.85</td>
<td>1.8</td>
<td>~ 4.7</td>
</tr>
<tr>
<td>M443</td>
<td>650</td>
<td>2.85</td>
<td>3.9</td>
<td>~ 0</td>
</tr>
<tr>
<td>M453</td>
<td>2170</td>
<td>5.25</td>
<td>18</td>
<td>~ 0</td>
</tr>
</tbody>
</table>

Table 5.1: All the samples studied in this chapter. For each wafer, interlayer separation $d$, as cooled $p_T$, and $\tilde{d}$ are labeled. For $p_B > p_B^*$, the BWC has two pinning modes. For $p_B < p_B^*$, the BWC has a single pinning mode. $p_B^*$ is given for each wafer.

For $p_B < p_B^*$, the BWC has a single pinning mode, whose development vs. $p_{TOT}$ is the main subject of this chapter. This is found in four samples, M433, M417, M465, and M440. Development of the single pinning mode of the BWC depends on $\tilde{d}$. For $\tilde{d} = 1.8$, both $f_{pk}$ and $\Delta f$ decrease monotonically on increasing $p_{TOT}$. For $\tilde{d} \lesssim 1.7$, the development of the BWC pinning mode with $p_{TOT}$ is more complex. We found the enhanced pinning at balance remains, under a density imbalance $|p_T - p_B|/(p_T + p_B)$ as large as 20%. At different ranges of bilayer states ($p_T, p_B$), the BWC would be expected to have different pseudospin magnetic orderings. Features in $f_{pk}$ and $\Delta f$ will be interpreted in the framework of changes in pseudospin magnetic orderings, driven by density change.
5.1 $p_B > p_B^*$: the BWC Has Two Pinning Modes

In M453 and M443, the BWC has two pinning modes \(^1\), as long as there is significant $p_B$. In M433 and M417, with smaller $d$, the BWC has two pinning modes only when $p_B$ is above about $1.7 \times p_T$. In this section, we focus on the pinning modes of M453 and M443, since they are resolved over wide ranges of the experimentally available bilayer densities $(p_T, p_B)$.

M453 has the largest $d = 2170\text{Å}$ and $\tilde{d} = 18$, and is closest to the independent bilayer case. Figure 5-1 shows the evolution of spectra with $B$ for M453, in an imbalanced state, with $p_T > p_B$. As $B$ is increased, resonances “B” and “T” develop as the respective filling factors of the bottom and top layers go below about 1/3. For single layer hole systems (with comparable density and mobility) including the present samples with $p_B = 0$, a resonance appears for $\nu$ just below the fractional quantum Hall liquid state around 1/3 [Li et al., 1997]. Hence, for M453, the fillings at which the resonances appear are consistent with associating the resonances “T” and “B” with the pinning modes of WC’s in the top and bottom layers.

Figure 5-2 shows the spectra of M453 in several bilayer and single layer states, at 14 T. In the balanced state, two resonances are not resolved. When $p_B$ is too small, resonance “B” is not observed. As $p_B$ is reduced, the peak frequency of the resonance “B”, $f_{pk}^B$, increases, but the peak frequency of the resonance “T”, $f_{pk}^T$, does not. In the region with $p_B = 0$ (labeled as “(0)” in the figure), $f_{pk}^T$ shifts up on reducing $p_T$. As summarized in Figure 5-3, the complete independence of $f_{pk}^T$ on $p_B$ suggests that the two layers are independent, not surprising considering the large $\tilde{d}$ for this sample.

For M443, with its smaller $d$, the two resonances also develop as the filling factors

\(^1\)It is possible particularly for large imbalance to observe only one pinning mode at high $B$. At sufficiently small individual layer filling factors, peak conductivities and oscillator strengths of these resonances decrease as $B$ is increased. This behavior is consistent with earlier studies on some single layer systems [Chen, 2005], and also with the F-L theory [Fukuyama and Lee, 1978]. At high $B$, with large imbalance, the resonance associated with the lower density layer may not be evident, and particularly may not be resolved from the much stronger resonance of the higher density layer.
of the two layers go below $1/3$. Unlike M453, $f_{pk}^T$ shifts slightly on changing $p_B$, as shown in Figure 5-4. Though the shift demonstrates the layers are coupled to each other, this coupling is not strong, so the two pinning modes can be thought of as predominantly due to one layer or the other.

### 5.2 $p_B < p_B^*$: the BWC Has a Single Pinning Mode

For $p_B$ less than $p_B^*$, the spectra of imbalanced states show only one resonance, which is the pinning mode of the whole BWC. We will study the development of this single pinning mode vs. $p_{TOT}$, for four samples (M433, M417, M465, and M440, all with $p_B^*$ well above $p_T$).

Figure 5-5 shows spectra of M465 with $\tilde{d} = 1.56$, for several bilayer and single layer states, at 14 T. In the figure, all the spectra for the bilayer states show a single resonance. Figure 5-6 summarizes $f_{pk}$ and $\Delta f$ vs. $p_{TOT}$ on “log-log” scale, at two magnetic fields (14 and 8 T). Within the region with $p_B = 0$ (marked as “(0)” in the figure), $f_{pk}$ vs. $p_{TOT}$ ($p_{TOT} = p_T$) fits $f_{pk} \propto p_{TOT}^{-\gamma}$, producing the solid fit lines shown. $\gamma$ is field dependent, but varies within a typical range of $0.5 \pm 10\%$, consistent with earlier microwave studies on p-type single layers [Li et al., 2000]. In this region, $\Delta f$ decreases on increasing $p_{TOT}$, also consistent with the earlier studies on single layer hole systems. For $p_B > 0$, we define three regions in Figure 5-6, according to the slope of $f_{pk}$ vs. $p_{TOT}$. Within region (1), $f_{pk}$ decreases on increasing $p_{TOT}$. The power law $f_{pk} \propto p_{TOT}^{-\gamma}$, extrapolated from region (0) and shown as dotted lines in Figure 5-6, applies to region (1) as well. In region (2), $f_{pk}$ increases with increasing $p_{TOT}$, and $\Delta f$ vs $p_{TOT}$ exhibits a local maximum. Upon further increasing $p_{TOT}$, in region (3) which includes the balance, $f_{pk}$ decreases. In region (3), $f_{pk}$ is markedly above the curve extrapolated from region (0), and $\Delta f$ shows a minimum just at balance. Similar $f_{pk}$ and $\Delta f$ vs. $p_{TOT}$ traces are observed for M440 and M417 (see Appendix C), with
Figure 5.1: Spectra of M453, in an imbalanced state, at several magnetic fields ($B$). For clarity, the spectra are vertically displaced by 4$\mu$S from each other. For each spectra, filling factors of top and bottom layers, $\nu_T$ and $\nu_B$, are also marked. At $\nu_T = 1/3$, or $\nu_B = 1/3$, the spectra are plotted as dashed lines. We clearly identify two resonances, “T” and “B”. Resonance “T” (“B”) starts to develop when $\nu_T$ ($\nu_B$) goes below about 1/3.
Figure 5.2: Spectra of M453, in several bilayer and single layer states, at 14 T. The spectra are offset by 12µS from each other. For each spectrum, $p_T$ and $p_B$ are labeled. In the imbalanced states, we clearly identify two resonances, “T” and “B”. In the balanced state, the spectrum is plotted as a dashed line, resonances “T” and “B” are too close to resolve.
Figure 5.3: $f_{pk}^T$ and $f_{pk}^B$ (the peak frequencies of resonances “T” and “B”, as identified in Figures 5-1 and 5-2) vs. total bilayer density $p_{TOT}$, at 14 T. In region (0), $p_B = 0$, $p_T = p_{TOT}$. For the region on the right of the dashed line, $p_T \approx 5.3 \times 10^{10} cm^{-2}$, $p_B = p_{TOT} - p_T$. The balanced state is marked as “BAL”. Around the balance, data points are missed, likely because “T” and “B” stay too close to resolve.
Figure 5.4: $f_{pk}^T$ and $f_{pk}^B$ vs. total bilayer density $p_{TOT}$ for M443 ($d = 650\,\text{Å}, \bar{d} = 3.9$), at $B = 9\,\text{T}$ and $T \approx 65\,\text{mK}$. In region (0), $p_B = 0$, $p_T = p_{TOT}$. For the region on the right of the dashed line, $p_T \approx 2.85 \times 10^{10}\,\text{cm}^{-2}$, $p_B = p_{TOT} - p_T$. The balanced state is marked as “BAL”. The inset shows only $f_{pk}^T$ vs $p_{TOT}$, also at $B = 9\,\text{T}$. Around the balance, data points are missed, likely because “T” and “B” stay too close to resolve. For $p_B < p_T$, $f_{pk}^B$ data points are missed, because resonance “B” is dominated by resonance “T”.
respective $\tilde{d} = 1.39$ and 1.61.

For M433, with $\tilde{d} = 1.7$, as presented in Figure 5-7, the development of the pinning mode is slightly different. The positive slope region (2) of $f_{pk}$ vs $p_{TOT}$ is less prominent than it is for M465. $\Delta f$ decreases monotonically on increasing $p_{TOT}$, and does not exhibit any clear feature in region (2) or at balance. In a different cool down of M433, $p_T$ is slightly higher. This leads to larger $\tilde{d} = 1.8$. As shown in Figure 5-8, a region of $f_{pk}$ vs. $p_{TOT}$ with positive slope is not observed. Instead, both $f_{pk}$ and $\Delta f$ decrease monotonically on increasing $p_{TOT}$. The two cool downs of M433, with their different $\tilde{d}$, highlight the sensitivity of $f_{pk}$ vs $p_{TOT}$ to $\tilde{d}$.

5.3 Discussion

The presence of a single pinning mode of the whole BWC is a manifestation of strong interlayer correlation. It is reasonable to expect that the BWC would have different pseudospin magnetic orderings at different bilayer states $(p_T, p_B)$. The features in $f_{pk}$ and $\Delta f$ can be interpreted as due to evolution of these magnetic orderings as the bilayer densities change. The following interpretation of the imbalanced state data is based on the classification of BWC in the balanced state by $\tilde{d}$ as described in Chapter 4. The carrier-carrier interaction effect and disorder enhancement effect, as introduced in Chapter 4, are crucial for the present discussion as well.

In region (3), for example in Figure 5-6, $f_{pk}$ decreases on increasing $p_{TOT}$, consistent with the carrier-carrier interaction effect. Within this region, $f_{pk}$ vs. $p_{TOT}$ is markedly above the curve extrapolated from the single layer region (0). This upward displacement suggests the pinning is stronger, compared with pinning of single layer WC. In the previous chapter, by studying $f_{pp}/f_{p0}$ vs. $\tilde{d}$, we have found the BWC experiences stronger pinning at balance, for $\tilde{d} \lesssim 1.7$. This enhanced pinning (at balance) was there interpreted as a consequence of easy-plane pseudospin ferro-
Figure 5.5: Spectra of M465 ($d = 230\,\text{Å}$), in several bilayer and single layer states, at 14 T. For clarity, the spectra are offset by $30\,\mu\text{S}$ from each other. The spectrum at balance is plotted as dashed line. For each spectrum, the top and bottom layer densities $p_T$ and $p_B$, are labeled. In the imbalanced states, the spectra show single resonance.
Figure 5.6: $f_{pk}$ and $\Delta f$ (full width at half maximum) vs. total bilayer density $p_{TOT}$, for M465 ($\tilde{d} = 1.56$), at 8 and 14 T. In region (0), $p_B = 0$, $p_T = p_{TOT}$. On the right side of the dashed line, $p_T = 3.65 \times 10^{10} cm^{-2}$, $p_B = p_{TOT} - p_T$. The balanced state is marked by a vertical line “BAL”. In region (0), both $f_{pk}$ and $\Delta f$ decreases on enhancing $p_T$. For $p_B > 0$, we define three regions, separated by dash-dotted lines. As $p_{TOT}$ is increased, $f_{pk}$ decreases in regions (1) and (3), but increases in region (2). In regions (0) and (1), $f_{pk}$ vs. $p_{TOT}$ fits $f_{pk} \propto p_{TOT}^\gamma$, with $\gamma$ in a typical range of 0.5 ± 10%. $\Delta f$ shows a minimum around the balanced state, and a local maximum around the middle of region (2).
Figure 5.7: $f_{pk}$ and $\Delta f$ vs. $p_{TOT}$, for M433 ($d = 1.7$), at 14 T. In region (0), $p_B = 0$, $p_T = p_{TOT}$. On the right side of the dashed line, $p_T = 2.52 \times 10^{10} cm^{-2}$, $p_B = p_{TOT} - p_T$. The balanced state is marked by a vertical line “BAL”. In region (0), both $f_{pk}$ and $\Delta f$ decreases on enhancing $p_T$. For $p_B > 0$, we define three regions, separated by dash-dotted lines. As $p_{TOT}$ is increased, $f_{pk}$ decreases in regions (1) and (3), but increases in region (2). Compared with M465, $f_{pk}$ feature in region (2) is less prominent. $\Delta f$ decreases monotonically on increasing $p_{TOT}$. In region (2), $\Delta f$ shows a step, with a hint of the local maximum.
Figure 5.8: $f_{pk}$ and $\Delta f$ vs. $p_{TOT}$, for M433 (in a different cool down, with $\bar{d} = 1.8$), at 14 T. In region (0), $p_B = 0$, $p_T = p_{TOT}$. On the right side of the dashed line, $p_T = 2.85 \times 10^{10} \text{cm}^{-2}$, $p_B = p_{TOT} - p_T$. The balanced state is marked by a vertical line “BAL”. Both $f_{pk}$ and $\Delta f$ decreases on increasing $p_{TOT}$.
magnetism. From $f_{pk}$ vs. $p_{TOT}$, we see that the enhanced pinning persists, under an imbalance $|p_T - p_B|/(p_T + p_B)$ as large as 20%. For these imbalanced states, the enhanced pinning is consistent with an extension of the balanced state picture, in which at least part of the carriers stay coherently between the two layers, so the BWC experiences the enhanced disorder. Generally, the carrier wavefunction may not spread equally in the two layers, so the weights may change with density imbalance as well. In the pseudospin language, this would also mean that at least some pseudospins have in-plane component, the orientation of the pseudospins can generally change with density imbalance.

Although $f_{pk}$ shows no sharp feature at balance, the $\Delta f$ minimum is just at balance. This suggests an imbalance induced effect, which increases damping of the resonance without affecting pinning. Based on the classification of BWC at balance, this damping is present only when the BWC at balance is an easy-plane ferromagnet. We note that, in the imbalanced state, the perfect easy-plane ferromagnetic ordering cannot be maintained. The damping appears to result from excess carriers of the majority layer, which we speculate may act as defects, or themselves form an condensed phase.

In region (1), as shown in Figure 5-6, $f_{pk}$ vs. $p_{TOT}$ roughly follows the power law, extrapolated from the region (0). This behavior is explainable as due to the carrier-carrier interaction effect only, hence, indicates that the pinning does not change dramatically from the single layer condition \(^2\). This suggests that the carriers do not spread between the two layers, hence the pseudospin does not have an in-plane component.

Region (0) (with $p_B = 0$), as presented in this chapter, is derived in the capacitive model. The carrier transfer effect, as introduced in Section 3-3, could extend the single layer region into the region where $p_B$ is not zero. Based on a rough estimation\(^2\), Region (0) is single layer region in the capacitive model. This is still the case, even considering the carrier transfer effect, as introduced in Section 3-3.
that will be presented in Appendix A, even all of region (1) could be a single layer. For M440 and M465, $f_{pk}$ vs. $p_{TOT}$ follows the same power law in both region (0) and region (1), consistent with the above picture. However, for M417, the power law is not well followed in region (1). For M465 and M417, within region (1), at some magnetic fields $^{3}$, increasing of $\Delta f$ on increasing $p_{TOT}$ is observed, and this behavior is not typical of a single layer Wigner crystal [Li et al., 1997].

Region (2) can be interpreted as a transition, between different types of BWC phases, i.e. between region (1) with pinning of individual layers (smaller $f_{pk}$) and region (3) with enhanced pinning (larger $f_{pk}$). The local $\Delta f$ maximum, which is observed in the middle of region (2), could then be interpreted as due to this transition. At the transition region, multiple BWC phases can in principle coexist. The broadening effect possibly originates from dissipative excitations that are associated with the phase boundaries. In the absence of the transition, for M433 with $\tilde{d} = 1.8$, the local $\Delta f$ maximum disappears. Instead, $\Delta f$ decreases on increasing $p_{TOT}$, which can be interpreted as due to the carrier-carrier interaction effect.

In summary, we interpreted the data in the picture of enhanced pinning due to the carriers staying coherently between the two layers, or to in-plane ferromagnetism in the pseudospin language. By studying the peak frequency in the imbalanced states, we found that the enhanced pinning exists not only at balance but also over a considerable range of imbalanced states (around the balance). In these imbalanced states, the presence of this enhanced pinning suggests that carriers still spread between the two layers, so the BWC still possesses some in-plane ferromagnetism although the total pseudospin magnetization of the BWC can not stay in-plane. At sufficiently large imbalance, the enhanced pinning disappears, indicating that a carrier is completely in one layer or the other, so the in-plane ferromagnetism does not exist. Our data suggests transition corresponding to the loss of the in-plane ferromagnetism. Moreover,

$^{3}$For example, at $B = 8$ T, for M465, as shown in Figure 5-6
the results of resonance line-width can be interpreted consistently with this picture.
Chapter 6

Conclusions

The main results of this thesis are summarized as follows.

1. In the balanced state, only for samples showing the $\nu = 1$ QHE, i.e. for small enough $d$, development of the resonance exhibits features around $\nu = 1/2$ or $2/3$. This suggests that the FQH correlations play a role in the low $\nu$ BWC insulating phase as well.

2. We compared pinning mode of the balanced bilayer $(p, p)$ with that of a single layer $(p, 0)$. Our study suggested that BWC experiences an enhanced pinning (compared with the pinning of WC of the individual layers), only for $\tilde{d} \lesssim 1.7$. The enhancement of pinning has been interpreted as an evidence of the easy-plane ferromagnetism within the BWC phase.

3. By measuring the resonance of imbalanced states, we found the enhanced pinning exists not only at the balanced state, but also at imbalanced states with $|p_T - p_B|/(p_T + p_B)$ as large as 20%.

In this thesis, we focused on the resonance at small $\nu$, in both the balanced and imbalanced states. We can, in the future, expand our studies into the higher $\nu$ (lower $B$) region. For samples M440, M465, and M417, with the smallest $d$, in their
balanced states, there is a reentrant insulating phase (RIP) at \( \nu \) slightly larger than 1 [Tutuc et al., 2003a]. This RIP is interpreted as pinned BWC, and present only around the balanced state. An interesting experiment would be measuring \( \text{Re}(\sigma_{xx}) \) at \( \nu \) larger than 1, at both balanced and imbalanced states.

Besides perpendicular \( B, d \) and bilayer densities, there are more parameters that can be changed. For example: (1) We can study effects due to temperature change. In single layer WC, the melting temperature has been found to depend on the inter-carrier quantum correlation [Chen, 2005]. A bilayer WC, as suggested in this thesis, would also have the interlayer quantum correlation, in addition to the quantum correlation in each layer. Temperature dependence measurement, in both the balanced state and imbalanced states, could give us more information about these correlations. (2) Another physical parameter is the in-plane magnetic field. It has been understood that introducing the in-plane field can modify the interlayer tunneling matrix [Hu and MacDonald, 1992], and affect the interlayer quantum correlation [Yang et al., 1994, Zheng and Fertig, 1995]. In DC transport measurement, the in-plane field has no effect on the QH liquid states, possibly because all these samples have negligible tunneling [Tutuc et al., 2003a,b]. However, it is still worthwhile to study the BWC, under the in-plane field [Chen, 2006].

It would be desirable to have independent control of density in each layer. We can immediately see two advantages. (1) We can change the density imbalance, while keep a constant \( p_{TOT} \), so that the imbalance effects can be isolated. (2) \( f_{pp}/f_{p0} \) vs. \( \tilde{d} \) can be measured using one sample. Hence, a more detailed and more reliable phase diagram could be plotted.
Appendix A

Carrier Transfer between Layers

In Chapter 3, the bilayer densities $p_T$ and $p_B$ are derived, based on the simple capacitive model. Within the capacitive model, the kinetic energy is neglected, and the carriers are considered as independent particles. Outside the capacitive model, estimating actual bilayer densities requires consideration of a carrier transfer on the basis of $p_T$ and $p_B$. This is because as the kinetic energy and carrier-carrier interaction are “turned on” to realize the real 2DS, the bilayer with layer densities $p_T + \delta p$ and $p_B - \delta p$ minimizes the total energy.

This carrier transfer may play a role in interpretation of data in Chapter 5, especially for the small $d$ samples. In this section, we will discuss and estimate this carrier transfer at zero $B$, and high $B$ limit, for M440, M465, and M417. The interlayer tunneling is reasonably taken to be zero.

1. The case of $B = 0$ has been studied [Eisenstein et al., 1994, Katayama et al., 1995, Papadakis et al., 1997, Ruden and Wu, 1991, Ying et al., 1995]. In the Hartree-Fock approximation (HFA), the total relevant energy of a bilayer has been written as $^{1}$

$^{1}$The exchange energy term (the last term on the right side) in this equation is smaller than that used in some earlier studies [Eisenstein et al., 1994, Katayama et al., 1995, Ruden and Wu, 1991] by a factor of $\sqrt{2}$. The reason is, at zero $B$, the 2D gas is unpolarized for the real spin degree of freedom, as pointed out later by Zheng [Zheng et al., 1997].

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\[ E_{\text{total}} = \frac{1}{2} D(E)^{-1}[(p_T + \delta p)^2 + (p_B - \delta p)^2] - D(E)^{-1}(p_T - p_B)\delta p \]

\[ + \frac{1}{2} \frac{e^2}{\epsilon_0 \epsilon_r} \delta p^2 \]

\[ - \frac{4}{3} \left( \frac{1/\pi}{4\pi \epsilon_0 \epsilon_r} \right) \frac{e^2}{4\pi \epsilon_0 \epsilon_r} [(p_T + \delta p)^{3/2} + (p_B - \delta p)^{3/2}] \]  

(A.1)

where \( D(E) = \frac{m^*}{\pi \hbar^2} \) is the density of states of 2DS, \( m^* \) is the effective mass for a hole carrier. \( p_T \) and \( p_B \) are bilayer densities within the capacitive model, as plotted in Figure A.1. \( \delta p \) is estimated by looking for the minimum of \( E_{\text{total}} \). After consideration of this carrier transfer \( \delta p \), the top and bottom layer densities are plotted in Figure A.1 as well.

2. In the high \( B \) limit, all carriers stay in LLL. As the carrier-carrier interaction is turned on, carriers would form WC. The energy (per unit density) for a classical WC in a single layer has been shown as \( \varepsilon_{WC} \approx -0.78(\epsilon^2/4\pi \epsilon_0 \epsilon_r)(2\pi)^{1/2}p^{3/2} \) [Bonsall and Maradudin, 1977]. Hence, the total relevant energy can be written as

\[ E_{\text{total}} = \frac{1}{2} \frac{e^2}{\epsilon_0 \epsilon_r} \delta p^2 - 0.78 \frac{e^2}{4\pi \epsilon_0 \epsilon_r} (2\pi)^{1/2}[(p_T + \delta p)^{3/2} + (p_B - \delta p)^{3/2}] \]  

(A.2)

The estimated bilayer densities in this high \( B \) limit are also plotted vs. \( V_g \) in Figure A.1.
Figure A.1: For (a) M440, (b) M465, and (c) M417, we estimate top and bottom layer densities vs. $V_g$ under several conditions. Colored dash line and solid line indicate $p_T$ and $p_B$ in the capacitive model, as derived in Chapter 3. The dash-dotted line and dotted line indicate the top and bottom densities at $B = 0$ in HFA. The solid line and dash line indicate those in the high $B$ limit, where carriers form classical WC in each layer.
Appendix B

Supplementary Data of Balanced Bilayers

This section presents supplementary data of several bilayer samples, in their balanced states. Figures B. 1 - 4 show spectra of M440, M417, M433, and M443, at several magnetic fields (or total filling factors $\nu$) and at $T \approx 65\text{mK}$. $f_{pp}$, $Q$ vs. $\nu$ are plotted in Figure B. 5. $\sigma_{pk}$ and $S_{FL}/f_{pp}$ vs. $\nu$ are summarized in Figure B. 6, where $\sigma_{pk}$ is peak conductivity of the resonance, $S_{FL}$ is oscillator strength.
Figure B.1: Spectra of M440, in its balanced state, at several magnetic fields (total filling factors \( \nu \)). The spectra are offset, proportional to \( \nu \). The area between a spectrum and its “zero” is shaded. For each spectrum, total filling factor \( \nu \) and \( B \) are labeled.
Figure B.2: Spectra of M417, in its balanced state, at several magnetic fields (total filling factors $\nu$). The spectra are offset, proportional to $\nu$. The area between a spectrum and its “zero” is shaded. For each spectrum, total filling factor $\nu$ and $B$ are labeled.
Figure B.3: Spectra of M433, in its balanced state, at several magnetic fields (total filling factors \( \nu \)). The spectra are offset, proportional to \( \nu \). The area between a spectrum and its “zero” is shaded. For each spectrum, total filling factor \( \nu \) and \( B \) are labeled.
Figure B.4: Spectra of M443, in its balanced state, at several magnetic fields (total filling factors $\nu$). The spectra are offset, proportional to $\nu$. The area between a spectrum and its “zero” is shaded. For each spectrum, total filling factor $\nu$ and $B$ are labeled.

M443, $d = 65\text{nm}$

$2.9 \times 10^{10} \text{cm}^{-2} / \text{layer}$

$d\approx 3.9$

$\nu = 0.17 (14 \text{ T})$

$\nu = 0.2 (12 \text{ T})$

$\nu = 0.24 (10 \text{ T})$

$\nu = 0.3 (8.0 \text{ T})$

$\nu = 0.35 (7.0 \text{ T})$

$\nu = 0.4 (6.0 \text{ T})$

$\nu = 0.44 (5.5 \text{ T})$

$\nu = 0.48 (5.0 \text{ T})$

$\nu = 0.54 (4.5 \text{ T})$

$\nu = 0.6 (4.0 \text{ T})$

$\nu = 0.68 (3.5 \text{ T})$
Figure B.5: $f_{pp}$ and $Q$ vs. total filling factor $\nu$, for (a) M440, (b) M465, (c) M417, (d) M433, (e) M443, and (f) M453.
Figure B.6: $\sigma_{pk}$ and $S_{FL}/f_{pk}$ vs. total filling factor $\nu$, for (a) M440, (b) M465, (c) M417, (d) M433, (e) M443, and (f) M453. $\sigma_{pk}$ is the peak conductivity, $S_{FL}$ is oscillator strength, numerical integral of $\text{Re}(\sigma_{xx})$ over frequency. The dashed lines in the figures indicate $S_{FL}/f_{pp} = (e\pi/2B)2p = (e^2\pi/2h)\nu$, as predicted by F-L, where $p$ is density in each layer.
Appendix C

Supplementary Data of M440 and M417 in Their Imbalanced States

This section presents $f_{pk}$ and $\Delta f$ vs. $p_{TOT}$ for two cool downs of M440 (with $\tilde{d} = 1.57$ and 1.39) and for M417 (with $\tilde{d} = 1.62$), measured at $T \approx 65$mK.
Figure C.1: $f_{pk}$ and $\Delta f$ (full width at half maximum) vs. total bilayer density $p_{TOT}$, for M440 ($d = 1.57$), at 14 T. In region (0), $p_B = 0$, $p_T = p_{TOT}$. On the right side of the dashed line, $p_T = 3.87 \times 10^{10} \text{cm}^{-2}$, $p_B = p_{TOT} - p_T$. The balanced state is marked by a vertical line “BAL”. In region (0), both $f_{pk}$ and $\Delta f$ decreases on increasing $p_T$. For $p_B > 0$, we define three regions, separated by dash-dotted lines. As $p_{TOT}$ is increased, $f_{pk}$ decreases in regions (1) and (3), but increases in region (2). In regions (0) and (1), $f_{pk}$ vs. $p_{TOT}$ fits $f_{pk} \propto p_{TOT}^{-\gamma}$, with $\gamma \approx 0.46$. $\Delta f$ shows a minimum around the balanced state, and a local maximum around the middle of region (2).
Figure C.2: $f_{pk}$ and $\Delta f$ (full width at half maximum) vs. total bilayer density $p_{TOT}$, for M440 ($d = 1.39$), at 8 T. In region (0), $p_B = 0$, $p_T = p_{TOT}$. On the right side of the dashed line, $p_T = 3.0 \times 10^{10} \text{cm}^{-2}$, $p_B = p_{TOT} - p_T$. The balanced state is marked by a vertical line “BAL”. In region (0), both $f_{pk}$ and $\Delta f$ decreases on increasing $p_T$. For $p_B > 0$, $f_{pk}$ vs. $p_{TOT}$ shows a non-monotonic trend. $\Delta f$ shows a minimum around the balanced state.
Figure C.3: $f_{pk}$ and $\Delta f$ (full width at half maximum) vs. total bilayer density $p_{TOT}$, for M417 ($d = 1.62$), at 10 T. In region (0), $p_B = 0$, $p_T = p_{TOT}$. On the right side of the dashed line, $p_T = 3.05 \times 10^{10} \text{cm}^{-2}$, $p_B = p_{TOT} - p_T$. The balanced state is marked by a vertical line “BAL”. In region (0), both $f_{pk}$ and $\Delta f$ decreases on increasing $p_T$. For $p_B > 0$, we define three regions, separated by dash-dotted lines. As $p_{TOT}$ is increased, $f_{pk}$ decreases in regions (1) and (3), but increases in region (2). In regions (0), $f_{pk}$ vs. $p_{TOT}$ fits $f_{pk} \propto p_{TOT}^{-\gamma}$, with $\gamma \approx 0.55$. In region (1), $f_{pk}$ vs. $p_{TOT}$ does not follow this power law. $\Delta f$ shows a minimum around the balanced state, and a local maximum around the middle of region (2).
Bibliography


