CURRENCY INCONVERTIBILITY, PORTFOLIO BALANCE
AND RELATIVE PRICES

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Introduction

The development of private international financial intermediation has led to an unprecedented growth of trade in assets taking place not only among industrialized countries but also spreading to semi-industrialized and even to some less developed countries. This has provided channels for individuals and organizations to build up foreign exchange balances, so that they can diversify their portfolios across assets denominated in different currencies. In response, many governments have attempted to recapture foreign exchange by offering special advantages to certain types of transactions, namely the ones where evasion of exchange controls would be easier, like tourist services and migrants' remittances. Nevertheless, in many countries with inconvertible currencies, "black" markets for foreign exchange have developed to the point that the relative price between domestic and foreign money that is determined in these markets may have a greater effect on private capital flows than the "official" rate, with obvious repercussions on the balance of payments problem of the country in question.

Awareness of the importance of illegal transactions in international trade and payments is of course not new. The Italian criminologist Baccaria wrote a condition for smuggling to break even at about the same time as Hume wrote about the specie flow mechanism. From Baccaria's analysis, an upper bound for the tariff can be derived, which depends on the comparison between a "normal" profit markup and the probability of success in smuggling. Larger tariff revenues can then be compared with the cost of increased supervision.¹

Restricting the ability to convert national currency into foreign exchange may involve a restriction on international trade, which would therefore tend to promote factor mobility and trade in assets. Conversely, restricting capital mobility would tend to promote trade in goods.² In most IMF member
countries, inconvertibility applies to current account as well as capital account transactions. Even though the definition of convertibility in the Articles of Agreement only refers to the former, in a world of capital mobility and flexible exchange rates among major currencies, black markets for "convertible" currencies will develop when policies toward current account transactions are divorced from policies with respect to capital account transactions.

In this paper, the fruitfulness of the portfolio approach to the determination of the black market rate in situations of currency inconvertibility is suggested in Section I by a brief survey of the traditional and monetarist approaches and a simple dynamic partial equilibrium portfolio model. Section II develops a portfolio balance model of a small open economy with three goods and two assets. One good is traded in the official foreign exchange market, another is smuggled through the black market, and the third is non-traded. On the other hand, private financial wealth is composed of domestic and foreign money. The temporary equilibrium of the model determines the two relative prices, where the relative price of the two traded goods is also the black market premium. Steady-state equilibrium, in turn, determines the long-run equilibrium values of the asset stocks. After analyzing the perfect foresight path of a simplified version of the model, the paper ends with a brief conclusion, summarizing the results.
I. Black markets for foreign exchange: two polar views and the portfolio approach

The formal literature on black markets emerged in the late 40's in the context of the effects of price controls, using the conventional partial equilibrium graphics of demand and supply, but it was not until Sheikh (1976) that an analysis of the black market for foreign exchange linked the early contributions to the illegal transactions literature. Adapting the conventional monetary approach to the balance of payments, on the other hand, black market exchange rates have been used by Blejer (1976a and b) in monetarist models of exchange rate determination and by Giddy (1978) in Box-Jenkins "efficiency" tests. These two extreme views can usefully be contrasted with the portfolio approach to exchange rate determination defended in this paper. There we have a stock demand for foreign exchange like in the conventional money demand function but the foreign exchange market is not irrelevant and indeed, in the long run, the exchange rate is determined by current account balance, given the "elasticities condition." In this section we provide an overview of the traditional and monetarist approaches and then develop a simple dynamic partial equilibrium model of the black market using the portfolio approach.

1. The traditional graphics of black markets

The graphic analysis of the black market for foreign exchange essentially applies the "textbook" or BM view of the foreign exchange market (where the exchange rate equates flow supply and flow demand for foreign exchange and such schedules are derived from the demand and supply of exports and imports respectively) to the Boulding (1947) and Michaels (1954) analysis of black markets.
Figure 1

Graphics of the Black Market for Foreign Exchange
In Figure 1, S and D represent respectively unrestricted flow supply and demand of foreign exchange and their intersection \( E_f \) determines the unrestricted equilibrium price of foreign currency in terms of domestic currency and the unrestricted quantity transacted per unit of time, \( OX_E \). If the official price is set at \( E_0 < E_f \), the authorities would have to sell foreign exchange at the rate \( X_D - X_S \) per unit of time. Otherwise, exporters would resell \( OX_S \) in the black market at a price necessarily higher than \( E_f \) if the D schedule does not shift down as a consequence of penalties on black market trading. However, penalties on sellers are assumed to shift down \( S_o S \) (to \( S_o S' \)) and penalties on buyers to shift down D (to \( D' \)), establishing a rate \( E_b^B \) (for Boulding) such that \( E_0 < E_b^B < E_f \). The quantity bought and sold in the black market is \( X_E^B - X_S \), so that the black market supply is lower than in the no penalties case. Total supply, while higher than with no penalties, is less than in the unrestricted case: \( X_S < X_E^B < X_E \). It is of course unlikely that the shift from official supply to black market supply would occur only for the foreign exchange supplied at a price higher than \( E_0 \). Boulding discussed briefly the "encroachment" of the black market on the official market which would lead to a supply schedule like \( S_o S' \) rather than \( S_o S \), intersecting at \( B' > B \).
Further analysis of the "encroachment" led Michael to challenge the conclusion that \( E_b^S < E_f \). He based the supply and demand of black market foreign exchange on the decreasing availability of official supply as the black market premium increases. If everyone receives the same foreign exchange allocation and the number of buyers is proportional to the "inhibited quantities" demanded, the demand for foreign exchange will be positive in a range and therefore the black market price will be higher than the equilibrium price. To take the "encroachment" into account, draw the supply of foreign exchange to the black market, the broken line, \( E_oS'' \), as the horizontal difference between a downward sloping availability curve \( AS_o \), and the restricted supply \( S_eS' \). Under this rationing scheme, the demand curve, the broken line \( ED'' \), is the difference between the "rationing curve" \( AD''S_o \) and the restricted demand \( DD' \) and it is upward sloping initially because of the dominance of the effect of the decreased supply. Equilibrium now obtains at \( E_b^M \) (for Michael) such that \( E_b^M > E_f \). Using the rationing assumption of Brofenbrenner (1947), Sheikh (1976) has shown that this result hinges on the impossibility of resale of official foreign exchange in the black market. Indeed, with perfect resale \( E_b^M = E_f \), a limiting case in the previous analysis. Even though Sheikh's analysis is carried out for the foreign exchange market, by incorporating a demand for capital flows independent of the exchange rate, it remains within the bounds of the flow approach to exchange rate determination implicit in most of the illegal transaction literature.\(^6\)

2. The monetarist approach

Another strand in the black market literature is basically empirical and of monetarist persuasion. The basic reference is the model of Blejer (1978a), who has adapted his work on the monetary approach to the Mexican balance of
payments (1977) to derive a monetarist model of the black market rate which collapses into the usual monetarist-PPP model of exchange rate determination when the real official exchange rate is fixed by a purchasing power parity reaction function.\textsuperscript{7}

Since real money demand, the money multipliers and world inflation play no essential role in Blejer's model, they will be taken as constant and set to one by choice of units. The deflator of nominal money balances, $P$, is a weighted average of traded and non-traded goods. Denoting proportional changes by hats,

\begin{equation}
\hat{P} = (1 - \beta) \hat{\varepsilon} + \beta \hat{P}_{NT}
\end{equation}

where $\hat{\varepsilon}$ is the official exchange rate, representing the domestic currency price of the traded good;

$P_{NT}$ is the price of non-traded goods.

The ex-ante excess supply of money is given by the real increase in domestic credit as a proportion of the money stock. Denoting time derivatives by dots,

\begin{equation}
\delta = \frac{\dot{C}}{\dot{M}} - \hat{P}
\end{equation}

where $C$ is domestic credit;

$M = C + \kappa$ is the money stock, $\kappa$ being the domestic currency value of foreign exchange reserves.

Arguing that excess demand for non-traded goods is a given fraction of excess demand in the goods market, and therefore of the excess supply of money, Blejer posits that changes in the relative price of non-traded goods are proportional to $\delta$.\textsuperscript{8}
(3) \[ \hat{p}_{NT} - \hat{\delta} = \lambda \hat{\delta} \]

With this specification domestic inflation is a weighted average of the rate of crawl and domestic credit creation

(4) \[ \hat{p} = \tau \hat{\delta} + (1 - \tau) \hat{C}/M \]

where \[ 1/\tau = 1 + 2\lambda \]

If all goods were traded and \( \delta = 0 \), we would have inflation given by the law of one price. The same would happen if the relative price of non-traded goods were not responsive to real domestic credit creation \( (\lambda = 0) \). Note that monetary equilibrium implies that if no good were traded, \( \delta = 0 \) at all times.

The black market for foreign exchange is also modelled in a peculiar way. Flow supply, \( S_B \), is a log linear function of the black market premium and the log of flow demand is a linear function of the expected rate of change in the real black market rate

(5) \[ \log S_B = C_{11} + C_{12} \log p \]

(6) \[ \log D_B = C_{21} + C_{22} E(\hat{\delta}) \]

where \( p = e/\bar{e} \) is the black market premium;

\( r = e/\bar{p} \) is the real black market exchange rate;

\( E \) denotes expectation.

The expected rate of change in the real exchange rate is in turn given by its deviation from unity:

(6') \[ E(\hat{r}) = -\log r \]
Substituting \((6')\) into \((6)\), log differentiating \((5)\) and \((6)\) and equating yields the rate of change of the black market rate as a weighted average of the rate of crawl and domestic inflation:

\[
\hat{e} = c\hat{e} + (1-c)\hat{p}
\]

where \(c = \frac{c_{12}}{c_{12} + c_{22}}\).

Substituting \((4)\) into \((7)\) we obtain

\[
\hat{e} = [c(1-\tau) + \tau]\hat{e} + (1-c)(1-\tau)\hat{c}/M .
\]

Clearly, if all goods are traded and \(\tau = 1\), the black market premium is constant. The same happens when the flow supply of foreign exchange does not respond to the premium and \(c = 1\).

The model is completed by a reaction function according to which the official rate of crawl is a fraction of domestic inflation:

\[
\hat{p} = \psi\hat{p} .
\]

This makes domestic inflation a function of credit creation alone

\[
\hat{p} = \frac{1 - \tau}{1 - \tau\gamma} \frac{\hat{c}}{M} .
\]

Similarly the black market rate only depends on credit creation

\[
\hat{e} = [1 - c(1 - \gamma)] \frac{1 - \tau}{1 - \tau\gamma} \frac{\hat{c}}{M} .
\]

It is clear from \((9)\) that if \(\gamma = 1\) and the official real exchange rate is fixed, the change in the black market rate equals the ex-ante excess supply of money, while if \(\beta = 0\) or \(\lambda = 0\) (so that \(\tau = 1\)), it is independent of
monetary disequilibrium and given by the rate of crawl from (7').

Other implications of (9) can be seen by using the balance sheet of
the central bank and monetary equilibrium to substitute for $\hat{C}/M$. In fact,
we can rewrite (2) as:

\[(2') \quad \delta = -\hat{R}/M\]

and thus rearrange (7') as:

\[(10) \quad \hat{r} = [c(1 - \tau) + \tau]\hat{r} - (1 - c)(1 - \tau)\hat{R}/M\]

where $\hat{r} = \hat{e}/\hat{p}$ is the real official exchange rate.

The role of the given official real rate in (10) even when $c = 0$
raises further doubts on the appropriateness of the specification of flow
demand for foreign exchange and expectations in (6) and (6'). More impor-
tantly, there is no reason for the black market rate to vary inversely with
central bank reserves when there are no capital movements. When there are
capital movements, on the other hand, the rate of change of the black market
rate could only be a function of the current account (in foreign currency)
if there was a stock rather than a flow demand for black market foreign
exchange. This inconsistency is apparent when $\gamma = 1$, so that $\hat{r} = 0$ and
$\hat{r}$, which becomes the black market premium, only depends on reserves. Yet
that $\hat{r} = 0$ is precisely the implication of Blejer's empirical results.

Equation (9) is tested by first constructing a measure of excess supply
of money at world prices $\delta^* = \hat{C}/M - \hat{p}^* - m_d$ where $p^*$ is the foreign
currency price of the traded good and $m_d$ is the demand for real balances.
Fitting $m_d$ by a regression on permanent income and an adaptive domestic
expected inflation, Blejer constructs an annual time series for $\delta^*$ and
regresses $e$ on $\delta^*$ and $\delta^*$ lagged once, to find that the two coefficients
sum to one. This result leads the author to conclude that "almost all ex-ante money-market flow disequilibria are transmitted to the black market rate within a two-year period" (p. 127). Leaving aside the econometrics, this finding suggests that, integrating this continuous time model over the interval in question, one would find that the coefficient in (9) is unity. This, however, can only happen if \( \gamma = 1 \), so that the official exchange rate is determined by relative prices, as if it were a unified float. Then \( e = 0 \) and, from (10), \( \hat{s} = \hat{e} \) so that the money supply is exogenous, we have the usual monetarist equation for the official exchange rate and the black market is irrelevant:

\[
(11) \quad \hat{e} = \hat{M} - \hat{\pi} - \hat{\pi}^* - \delta^* + \hat{r}/\hat{M}.
\]

3. A simple portfolio model

It is easy to suggest how a stock demand for black market foreign exchange explains the determination of the black market rate, without having to introduce non-traded goods or reaction functions. If the black market rate is determined so that the private stock of foreign assets, \( F \), is willingly held, we can write:

\[
(12) \quad eF = A
\]

where \( A \) is the given demand for foreign assets.

The change in \( F \) is given by smuggling and other unreported current account transactions. Applying the BRM-Michael framework developed above, an increase in the black market premium will increase the supply of exports as well as the under invoicing of reported exports, so that the share of black market trade increases, and it will decrease the demand for imports as well on the share of black market trade. If the BRM condition holds, and the share elasticities are non-zero, then the unreported current account \( B \) is given by:

\[
(13) \quad \hat{F} = \delta^* (e, \hat{e}).
\]
Differentiating (12) and substituting from (13), we have Kouri's "acceleration" result applied to the black market for foreign exchange:\textsuperscript{10}

\begin{equation}
\hat{c} = \frac{\delta (e, \delta)}{p}.
\end{equation}

Assuming that the opposite holds for the reported current account, \( \delta (e, \delta) \), we show temporary equilibrium in Figure 2. On the right panel we have portfolio balance, while in the left panel we represent the two current accounts. Suppose that \( A \) increases from a situation of initial balance at \( e_0 \), so that \( e \) jumps to \( e_1 \), where the unreported current account is in surplus and the reported current account is in deficit. As the unreported surplus increases \( F \), \( e \) returns to its original value. Suppose, however, that central bank reserves are not sufficient for transition and therefore the official rate is devalued. This shifts the two loci upward, say to \( e_1 \) so that the stock of foreign assets remains the same and \( e_1 \) becomes the new equilibrium black market rate. It is easy to show that the model will be stable under stationary expectations if the "own" effect of the black market rate on the unreported current account is stronger than the "cross" effect of the official rate and conversely.\textsuperscript{11}
Temporary Equilibrium Under Currency Inconvertibility

Figure 2
II Portfolio diversification under currency inconvertibility

1. A three-good two-asset model

Consider an economy producing and consuming two traded goods and one non-traded good. Receipts and payments for the "official traded good" go through the official foreign exchange market, while for the "smuggled good" they go through the black market. We are thus aggregating imports and exports channeled through each one of the two foreign exchange markets. Assuming full employment of the factors of production, we can express the domestic supply of these goods as a function of relative prices \( p \) and \( q \):

\[
X^B = X^B(p^B/p^0, p^{NT}/p^0) = X^B(p, q)
\]

\[
X^O = X^O(p^B/p^0, p^{NT}/p^0) = X^O(p, q)
\]

\[
X^{NT} = X^{NT}(p^B/p^0, p^{NT}/p^0) = X^{NT}(p, q)
\]

where \( X^B(X^O, X^{NT}) \) is the supply of the smuggled (official traded, non-traded) good;

\( p^B(p^0, p^{NT}) \) is the respective domestic currency price.

Domestic consumption, in turn, depends on prices, income and wealth:

\[
D^B = D^B(p^B, p^0, p^{NT}, Y, W)
\]

\[
D^O = D^O(p^B, p^0, p^{NT}, Y, W)
\]

\[
D^{NT} = D^{NT}(p^B, p^0, p^{NT}, Y, W)
\]

where \( Y = X^Bp^B + X^Op^0 + X^{NT}p^{NT} \) is nominal income;

\( W \) is nominal wealth.
Consumption is homogeneous of degree zero in all prices and income. Define the price of the official traded good as the numeraire and rewrite consumption in terms of the numeraire as

\[(18') \quad D^B = D^B(p, q, \bar{w})\]

\[(19') \quad D^O = D^O(p, q, \bar{w})\]

\[(20') \quad D^{NT} = D^{NT}(p, q, \bar{w})\]

where \(\bar{w} = w/f^O\).

Note that the income effect cancels because of the full employment assumption, so that the total effect of a change in relative prices is the same as the compensated effect.\(^{13}\) Noting that excess supply functions are also homogeneous of degree zero, we express the reported and unreported current account as, respectively, the excess supply of the official traded good and the smuggled good. We assume that the market for non-traded goods always clears but the unreported current account equals the increase in the stock of foreign assets of the private sector \(F\) and the reported current account equals the increase in the stock of foreign assets of the central bank, \(F^G\):

\[(21) \quad B(p, q, \bar{w}) = X^B - D^B = \bar{F}\]

\[(22) \quad B(p, q, \bar{w}) = X^O - D^O = \bar{F}^G\]

\[(23) \quad N(p, q, \bar{w}) = X^{NT} - D^{NT} = 0\]

Wealth is composed of two assets, domestic money \(M\) and foreign money
F. Asset stocks are always willingly held and the asset demand functions depend on returns, goods prices and wealth:

\begin{align*}
(24) & \quad M = M(X^M, R^F, p^B, p^O, p^N, \bar{w}) \\
(25) & \quad eF = f(M, R^F, p^B, p^O, p^N, \bar{w}) \\
(26) & \quad \bar{w} = M - eF
\end{align*}

where $e$ is the black market rate; $\bar{R}^M (R^F)$ is the return on domestic (foreign) money in domestic currency.

Asset demand functions are homogeneous of degree one in prices and wealth so that expressing them in terms of the price of the official traded good we have

\begin{align*}
(24') & \quad \bar{M} = m(X^M, R^F, p, q, \bar{w}) \\
(25') & \quad eF/p^O = f(M, R^F, p, q, \bar{w}) \\
(26') & \quad \bar{w} = \bar{M} + eF/p^O
\end{align*}

where $\bar{M} = M/p^O$ is the real money stock.

We know that $\partial m/\partial \bar{w} + \partial f/\partial \bar{w} = 1$ and that all other partials sum to zero. If, in addition, $\partial m/\partial R^M = \partial m/\partial R^F$ and we take nominal returns in the own currency as given, relative returns are captured by the expected rate of change of the black market rate, $\psi$. We assume further that, while an increase in the price of smuggled good increases demand for foreign assets, an increase in the other two prices increases the demand for domestic money. Also, since $e/p^O = p/p^B_*$, where $p^B_*$ is the foreign currency price of the smuggled good, if
we set \( p^N = 1 \) by choice of units, and use the wealth constraint in (26') to eliminate (24'), then portfolio balance can be written as:

\[
pF = f(\psi, p, q, \bar{M} + pF).
\]

To close the model we need to specify the money supply process. We will take the money multiplier as given so that \( \bar{M} = C + \delta p^2 \), \( C \) being domestic credit.\(^{15} \)

We can then write the change in the real money stock as:

\[
\dot{M} = \delta(p, q, \bar{w}) - \bar{M}
\]

where \( \delta = \dot{\bar{M}} - \dot{p}^0 \) is the real increase in domestic credit creation.

2. Temporary Equilibrium

Equations (23) and (27) above define the temporary equilibrium values of \( p \) and \( q \), given asset supplies and expectations. From (23), an increase in \( q \) creates an excess supply of the non-traded good, which, given asset supplies, can only be offset by an increase in the black market premium. The combination of \( p \) and \( q \) consistent with equilibrium in the non-traded goods market are thus given by a locus with slope greater than \( p/q \), drawn as the NT locue in Figure 3. From (27), an increase in \( q \) generates a decreased demand for foreign assets. Given asset supplies and expectations, this implies a decrease in the premium, so that the combinations of \( p \) and \( q \) consistent with portfolio balance are given by a downward sloping locus, PB in Figure 3. Note that if the effect of the premium on the demand for foreign assets were larger than the valuation effect, the portfolio balance locus would be upward sloping.\(^{16} \) At the intersection of the NT and PB loci, the system is in temporary equilibrium with relative prices \( p_0 \) and \( q_0 \). To ascertain
Figure 3
Temporary Equilibrium
the effects of changes in asset stocks and expectations on the temporary equilibrium, logarithmic differentiation of (23) and (27) yields:

\[
\begin{bmatrix}
1 - \alpha - \varepsilon_1 & \varepsilon_2 \\
-\omega_1 - \nu_1 & \nu_2
\end{bmatrix}
\begin{bmatrix}
\hat{p} \\
\hat{q}
\end{bmatrix}
= \begin{bmatrix}
(1 - \alpha)\eta & -1 + \alpha \\
(1 - \alpha)\omega_1 & \omega_1
\end{bmatrix}
\begin{bmatrix}
\hat{r} \\
\hat{p}
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_3 \\
0
\end{bmatrix}
\]

where \( \alpha = \frac{pF}{\tilde{W}} \) is the share of foreign assets in wealth;
\( \eta = \frac{\partial f}{\partial \tilde{W}} \) is the elasticity of demand for foreign assets with respect to real wealth;
\( \varepsilon_1, \varepsilon_2, \varepsilon_3 \) is the positive elasticity of demand for foreign assets with respect to the premium (the relative price of non-traded goods, relative returns)

\[ e.g. \quad \varepsilon_2 = -\frac{\partial f}{\partial \tilde{q}} \]  

\( \omega_1 = \frac{\partial \hat{N}}{\partial \tilde{W}} \) is the positive semi-elasticity of the excess supply of non-traded goods with respect to real wealth;

\( \nu_1, \nu_2 \) is the positive semi-elasticity of the excess supply of non-traded goods with respect to the premium (the relative price of non-traded goods), e.g. \( \nu_1 = -\frac{\partial \hat{N}}{\partial \hat{p}} \).

From (29), the slope of the NT and PB loci can be written as:

\[ \left. \frac{dp/p}{dq/q} \right|_{NT} = \nu_2 / \nu_1 + \omega_1 \quad \left. \frac{dp/p}{dq/q} \right|_{PB} = -\varepsilon_2 / 1 - \alpha \eta - \varepsilon_1 \]
As indicated, the slope of the NT locus will be greater than \( p/q \) and the effect of \( p \) dominates, so that \( \varepsilon_1 + \eta < 1 \). As the premium increases, demand of foreign assets has to increase by less. Since \( \eta = \partial \varepsilon_1 / \partial W \) and \( -\varepsilon_1 \) is the elasticity of money demand with respect to the premium, we can write:

\[
\frac{\partial m}{\partial W} - \varepsilon_1 > 0
\]

According to condition (30), the wealth effect of an increase in the premium on the demand for domestic money has to be stronger than the direct effect of the premium in decreasing money demand. Under condition (30), the premium varies inversely with the currency ratio valued at the official rate, \( F/M \), and with a coefficient less than one, so that if the premium increases, we know that the currency ratio valued at the black market rate, \( pF/M \), will also increase. If marginal and average currency shares are the same (\( \eta = 1 \)), furthermore, condition (30) puts as an upper bound on the premium elasticity of the share of foreign assets in wealth the share of domestic money in wealth: if domestic money is 90% of wealth, the elasticity has to be less than 0.17.

The effects of changes in demand and supply of foreign assets on the temporary equilibrium, can be seen by going back to Figure 3. From \( T_0 \), the effect of an increase in the return to foreign currency will be to shift PB up: given asset stocks, an increase in \( q \) would reduce the demand for foreign assets, while an increase in \( p \) would increase the valuation of the existing stock (since this effect dominates by assumption). The NT locus does not shift, so that the two relative prices increase to \( T_1 \), \( p \) increasing more than 9. Conversely, an increase in \( F \) shifts the PB locus down, while it shifts the NT locus to the right because an excess demand for non-traded goods implies an increase in its relative price, or a decline in the black market premium.
As a consequence p has to decrease to accommodate an increase in F and q may increase or decrease but p/q has to fall. In Figure 3, it is assumed that the PB curve shifts by more so that q is also smaller at T_{2}. To ascertain this, solve (29) for a given ψ, to yield:

\[
\begin{bmatrix}
\hat{p} \\
\hat{q}
\end{bmatrix} = \begin{bmatrix}
-\tilde{\omega}_{1} \varepsilon_{2} & -\omega_{1} \varepsilon_{2} h \\
\omega_{1} (\varepsilon_{2}) & -\omega_{1} \varepsilon_{2} h
\end{bmatrix} \begin{bmatrix}
(1-\alpha)n \\
\Delta
\end{bmatrix} \begin{bmatrix}
\hat{M} \\
\hat{F}
\end{bmatrix}.
\]

(29')

where \( \Delta = \omega_{2}(1-\alpha-\varepsilon_{1}) + (\omega_{1}+\varepsilon_{1}) \varepsilon_{2} > 0 \)

\( h = \omega_{2}/(1-\alpha) \) is the marginal currency ratio

\( \tilde{\omega}_{1} = \omega_{1}/\eta \) is the ratio of the wealth semi-elasticity of excess supply of non-traded goods to the wealth elasticity of demand for foreign assets.

It will be convenient to express (29') compactly as:

\[
\begin{bmatrix}
\hat{p} \\
\hat{q}
\end{bmatrix} = \begin{bmatrix}
P_{M} & P_{F}
\end{bmatrix} \begin{bmatrix}
\hat{M} \\
\hat{F}
\end{bmatrix}.
\]

(29'')

It is clear from (29') that if \( \Delta > 0 \), \( q_{F} < 0 \) and \( p_{F} < 0 \), as depicted in Figure 3. It may seem that even if condition (30) does not hold, the effect of an increase in the stock of foreign assets could still be a decline in p and q, provided that \( \Delta > 0 \) (or \( p_{M} q_{F} > q_{M} p_{F} \)). This result is unlikely, however, because \( \omega_{2} \) captures the "own" relative price effect in the non-traded goods market, which is larger than the "own" relative price and the wealth effects, captured by \( \omega_{1} \) and \( \omega_{1} \) respectively, so that if condition (30) does not hold,
\( \Delta < 0 \). Consider now the effect of monetary expansion. Given condition (30), an increase in \( \tilde{\Phi} \) will unambiguously raise \( q (q_M > 0) \) but it may increase or decrease \( p \). In fact the increase in \( \tilde{\Phi} \) shifts the PB locus up because it generates an increase in the demand for foreign assets and requires an increase in \( p \) or in \( q \) to maintain portfolio balance. The effect on the NT locus is the same as the effect of an increase in \( F \), it shifts to the right and requires an increase in \( q \) or a decrease in \( p \). The condition for \( q \) and \( p \) to increase (or \( p_M > 0 \)) as shown in Figure 3 by \( T_3 \) is the dominance of the "own" effect in the non-traded goods market, or:

\[
(31) \quad \nu_2 > \tilde{\nu}_1 \bar{\nu}_1
\]

Even when \( p \) increases, however, it will only increase by more than 9 if \( p_M > q_M \), which will be true if:

\[
(32) \quad \nu_2 > \nu_1 + \bar{\omega}_1 (1 - \epsilon_1 + \epsilon_2).
\]

The effect of a once-and-for-all devaluation of the official rate is to lower \( p \), \( q \) and \( \tilde{\Phi} \) in proportion. This will generate an excess demand for non-traded goods if \( \nu_2 > \nu_1 + \bar{\omega}_1 \), so that the NT locus will shift to the right to maintain equilibrium and \( q \) will increase. The effect on the demand for foreign assets hinges on whether the demand effect of the premium is stronger than the valuation effect of one premium plus the demand effect of the relative price of non-traded goods. In particular, demand for foreign assets will decrease if \( \eta + \epsilon_1 > 1 + \epsilon_2 \), so that the PB locus will shift down and the system will jump from \( T_0 \) to \( T_2 \), and \( p \), \( q \) and \( p/q \) will fall, just like when \( F \) increases. Note that, if \( \eta + \epsilon_1 < 1 + \epsilon_2 \) the effect of devaluation would be like the effect of monetary expansion, so that \( p \) and \( q \) would increase rather than decrease and the system would jump from \( T_0 \) to \( T_3 \).
3. **Steady-state equilibrium**

The dynamics are governed by (21) and (28) above. Substituting for $p$ and $q$, we can express asset accumulation as a function of asset stocks and expectations:

\[
(33) \quad \dot{\tilde{M}} = B(\tilde{\psi}, \tilde{M}, \tilde{F}) + \delta\tilde{M}
\]

\[
(34) \quad \dot{\tilde{F}} = B(\tilde{\psi}, \tilde{M}, \tilde{F})
\]

In long-run equilibrium, when $B = 0$, since the non-traded goods market and the asset markets are always in equilibrium, we have to have $\tilde{B} = 0$ as well. This implies an equilibrium relationship between credit creation and the rate of crawl, such that $\delta = 0$. Taking a linear approximation of the system around the steady-state values $\tilde{M}^*$ and $\tilde{F}^*$ we have:

\[
(35) \quad \begin{pmatrix} \dot{\tilde{M}} \\ \dot{\tilde{F}} \end{pmatrix} = \begin{pmatrix} -\tilde{\pi}_M - \tilde{\mu}_M - \tilde{\omega}[1 + \alpha(p_M - 1)] & -\tilde{\pi}_F - \tilde{\mu}_F - \tilde{\omega}_a(p_F + 1) \\ \tilde{\pi}_M - \tilde{\mu}_M - \omega[1 + \alpha(p_M - 1)] & \tilde{\pi}_F - \mu_F - \omega_a(p_F + 1) \end{pmatrix} \begin{pmatrix} \frac{\tilde{M} - \tilde{M}^*}{\tilde{M}^*} \\ \frac{\tilde{F} - \tilde{F}^*}{\tilde{F}^*} \end{pmatrix}
\]

where $\tilde{\pi}(\pi)$ is the positive semi-elasticity of the reported (unreported) current account with respect to the premium, e.g. $\tilde{\pi} = -p\tilde{\beta}_S / \partial p$;

$\tilde{\mu}(\mu)$ is the positive semi-elasticity of the reported (unreported) current account with respect to the relative price of non-traded goods, e.g. $\mu = -q\tilde{\beta}_B / \partial q$;

$\tilde{\omega}(\omega)$ is the positive semi-elasticity of the reported (unreported) current account with respect to real wealth, e.g. $\omega = -\tilde{\omega}_B / \partial W$;

and $p_M, q_M, p_F, q_F$ are defined in (28).
The determinant of the Jacobian in (35) is given by:

\[(36) \quad \det = \frac{1-\alpha}{\Delta} \left[ \omega_1 \Delta (\pi + \pi_1) + \nu_2 (\pi + \pi_1) + (\nu_1 + \omega_1) (\pi_1 - \mu_1) \right]. \]

It is clear from (36) that if the reported and unreported current account have the same response to wealth and the relative price of the non-traded good (so that \( \pi = \hat{\pi} \) and \( \mu = 0 \)), the determinant will be positive:

\[(36') \quad \det = (1-\alpha)(\pi + \pi_1)[\mu_1 + \nu_2 \omega/\Delta] > 0 \]

Under the same simplifying assumption, and if in addition \( \eta = 1 \), the trace will be given by:

\[(37) \quad -\text{TR} = [\nu_2 (1-\alpha) + \omega_1 \varepsilon_2] (\pi + \pi_1) + \omega_1 \varepsilon_2 (\pi + \omega_1) - \mu (1-\alpha - \varepsilon_1)] . \]

Note that even when \( \varepsilon_1 \) is at the upper bound given by condition (30), and the term in \( \mu \) vanishes, if \( \nu_2 \), \( \pi \) and \( \pi_1 \) are large relative to \( \varepsilon_2 \) and \( \omega_1 \), the expression in (37) will be positive and the system will be stable. The condition for the own effect on the unreported current account to be negative is of course stronger than (37). It requires that the direct premium effect \((\partial B/\partial F < 0)\) dominate the effect of the relative price of non-traded goods and wealth:

\[(38) \quad [\nu_2 (1-\alpha) + \omega_1 \varepsilon_2] \pi_1 > \alpha \omega (\nu_2 \varepsilon_1 - \nu_1 \varepsilon_2) + \mu [\nu_1 (1-\alpha) + \omega_1 \varepsilon_1] . \]

If \( \varepsilon_1 = 0 \), condition (38) will be fulfilled, since the second term in the right hand side becomes positive and since \( \nu_1 \) is likely to be small relative to \( \pi \) and \( \nu_2 \). Even if \( \varepsilon_1 \) is at its upper bound, though, the term multiplying \( \nu_2 \) will be \( \pi - \omega \alpha \), which is surely positive.
On the cross effects in (35), the one on the reported current account 
\( \partial \tilde{B} / \partial \tilde{F} \) is positive because \( q_F \) and \( p_F \) are negative and 
\( |p_F| > 1 \). For \( \partial \tilde{B} / \partial \tilde{M} > 0 \), however we need

\[ \pi (1-\alpha) [\nu_2 - \varepsilon_2 \omega_1] > \omega [\alpha (\nu_2 \varepsilon_1 - \nu_1 \varepsilon_2) + 1] + \mu [(1-\alpha) \nu_1 + \omega_1 (1-\varepsilon_1)] \]

which is stronger than (38), but nevertheless plausible if \( \nu_2 \) and \( \pi \) are
large enough relative to \( \omega \) and \( \varepsilon_2 \).

If the own effects are negative and the cross effects are positive,
as assumed, the two loci where \( \dot{M} = 0 \) and \( \dot{F} = 0 \) will be positively sloped.
and, because of condition (36'), the \( \dot{F} = 0 \) locus will be steeper than the 
\( \dot{M} = 0 \) locus in \( \dot{M}, \dot{F} \) space, as depicted in Figure 4. As shown by the
arrows, above the \( \dot{M} = 0 \) locus, the reported current account has a deficit
larger than real domestic credit creation and real money balances are de-
creasing, while below they are increasing. Above the \( \dot{F} = 0 \) locus the
unreported current account is in surplus and the private stocks of foreign
assets is increasing, while below it is decreasing. Steady-state equili-
brum obtains at \( A_0 \).

Consider the long run effects of an exogenous increase in the demand
for foreign assets, because of an increase in the foreign interest rate
from a position of equilibrium at \( A_0 \). Given asset supplies, \( p \) and \( q \) go
up, which moves the reported current account into deficit, so the \( \dot{M} = 0 \)
locus shifts to the right, and the unreported current account into surplus
(the "own" relative price effect dominates) so the \( F = 0 \) locus shifts to
the right as well and the new steady-state is at \( A_1 \), with a larger currency
ratio. Now, from \( A_1 \), an increase in \( \alpha \) shifts the \( \dot{M} = 0 \) locus up because
an increase in \( \dot{M} \) or a decline in \( \dot{F} \) will deteriorate the current account to
compensate for the increase domestic credit. The \( \dot{F} = 0 \) locus does not
Steady-state effects of various disturbances
shift so that if the $\tilde{M} = 0$ locus shifts back to its original position, the new steady-state will be $A_2$. The increase in $F$ is less than the increase in $\tilde{M}$ but, because $p$ also increased, the currency ratio $pF/\tilde{M}$ is larger at $A_2$ than at $A_1$. By the same argument, an increase in the rate of crawl would decrease the currency ratio.

Consider now the effect of a once-and-for-all devaluation of the official rate from $A_1$, which will be assumed to decrease $p$ and $q$ (as well as $p/q$), as discussed above. This brings the reported current account into surplus and the unreported current account into deficit, so that $\tilde{M}$ has to increase and $F$ has to decrease to restore balance. This shifts the $M = 0$ locus up and the $F = 0$ locus to the left, which increases the premium so that the economy will tend to move toward $A_0$. It can, however, be shown that the steady-state stock of foreign assets does not change and that the stock of real balances increases in proportion to devaluation and the premium, so that the new steady-state equilibrium is at $A_3$, where $\tilde{m}$ is the same at $A_1$. Devaluation has therefore no effect on the currency substitution ratio valued at the black market rate, unlike the crawling peg.\(^{18}\) Note that this is true even if the decline in the premium generates an expectation of depreciation of the black market rate and demand for foreign assets goes up, so that expectations of the black market rate are then extrapolative with respect to the official rate.

Consider finally the effects of exchange rate unification. If unification is accompanied by a floating official exchange rate, central bank reserves do not change, portfolio balance is described by (27) above with $p = 1$, domestic money is given and the change in the stock of foreign assets is equal to the aggregation of the reported and unreported current account, so that we have
the conventional portfolio model with non-traded goods. If unification were accompanied by a fixed but fully-convertible official exchange rate, on the other hand, domestic and foreign money would become perfect substitutes and we could apply the simple monetary approach to the balance of payments. It is, however, difficult to believe that such a policy would be credible without full interest liberalization so that the reasons for the development of a black market for foreign exchange would probably remain and the private valuation of foreign currency would still differ from the official exchange rate, with the consequences suggested above.

4. Equilibrium with perfect foresight

To analyze the effect of expectations about the rate of change of the black market rate being continuously realized, rewrite (27) as:

\[(40) \quad \dot{p}F = f(\dot{\epsilon}, p, q, \dot{M} + p\dot{F})\]

Solve out for $q$ in (21) and (22) using (23) assume that $p = 1$ in steady-state and express the system of (21), (22) and (40) around equilibrium as:

\[
\begin{bmatrix}
\dot{p} \\
\dot{\epsilon} \\
\dot{M} \\
\dot{\nu}
\end{bmatrix} =
\begin{bmatrix}
(1 - \alpha \omega_1 + \nu_2)/\epsilon_3 \\
\epsilon_1 - (1 - \alpha) \omega_1/\epsilon_3 \\
(1 - \alpha) \omega_1/\epsilon_3 \\
(1 - \alpha) \omega_1/\epsilon_3
\end{bmatrix}
\begin{bmatrix}
p - 1 \\
\epsilon_2/\epsilon_3 \\
\epsilon_3 \\
\end{bmatrix}
\]

where

\[
\begin{align*}
\tilde{w}_1 &= \gamma - w_1 \epsilon_2/\nu_2 \\
\tilde{\nu}_1 &= \nu_1/\nu_2 \\
\tilde{\Omega} &= \tilde{w} + w_1 \tilde{\nu}/\nu_2 \\
\Omega &= w + w_1 \mu/\nu_2
\end{align*}
\]
It is clear that the determinant of the Jacobian in (41) is positive and given by:

\[ \text{DET} = (1 - \alpha)(\pi \bar{\mu} + \bar{\pi})/\varepsilon_3 \]

Therefore the system has two negative and one positive eigenvalues, so that the steady-state equilibrium is a saddle point. To avoid three dimensional phase diagrams, which are difficult to interpret, assume that the domestic money stock is always at its steady state value and express the loci where \( p = 0 \) and \( \dot{F} = 0 \) as in Figure 5. Steady state equilibrium obtains at \( F_0 \). This differs from Figure 2 above in two ways. First the reported current account is always in balance. Second, the unreported current account responds to wealth. As a consequence, \( \dot{F} = 0 \) is upward sloping rather than horizontal. The effect of an increase in the demand for foreign assets is now a smaller jump in \( \epsilon \), which puts the black market rate in the perfect foresight path to \( F_1 \), as indicated in Figure 6. Conversely, the effect of an exogenous increase in the foreign demand for the smuggled good is a downward shift in the \( \dot{F} = 0 \) locus, which leads to a jump appreciation in \( \epsilon \) and a continuous appreciation along the new perfect foresight path to \( F_2 \).
Figure 5

Steady-state equilibrium
under perfect foresight
Figure 6

Effect of various disturbances
under perfect foresight
Conclusion

This paper has analyzed regimes of currency inconvertibility using the portfolio approach to exchange rate determination according to which the exchange rate depreciates when the current account is in deficit and appreciates when it is in surplus. The traditional and monetarist approaches to the black market for foreign exchange were surveyed and criticized in Section I, where a simple portfolio model without wealth effects was presented. In Section II it was shown that stability required that the effect of relative prices on the demand for traded and non-traded goods dominate their effect on asset demands. In this framework, it was shown that the effect of an increase in the rate of crawl decreases the currency substitution ratio while the once-and-for-all devaluation does not change it. To the extent that the monetary authorities wish to change the currency composition of private financial wealth, a crawling peg rather than a devaluation is therefore the appropriate instrument. A simplified version of the model is analyzed under the assumption of perfect foresight.
NOTES

This chapter revises and extends parts of Essay II of my Ph.D. dissertation. I am grateful to the members of my committee, particularly to Pentti Kouri, for encouragement. Errors are my own.

1. Beccaria (1764-65) is quoted and discussed in Bhagwati and Hansen (1973, footnote 1). Empirically, the positive relation between smuggling and the tariff has been observed by Cooper (1974) in the case of Indonesia.

2. See Mundell (1957) and Girton and Roper (1981). Mikesell (1947) has a discussion of the relation between trade and exchange restrictions.

3. See IMF, Annual Report, Table 1.1 and Annual Report on Exchange Arrangements and Exchange Restrictions, Analytical Appendix. Only 33 out of 140 member countries have convertible currencies, while 50 (Western Europe, Middle-East and Latin America) have accepted convertibility for current account transactions.

4. On the portfolio approach see Tobin (1969), Branson (1977) and Tohin and Macedo (1980), for example. A portfolio model of a dual exchange rate regime is in Macedo (1978). Flood (1978) and Cumby (1979) have also used the portfolio approach to flexible exchange rates in modelling dual foreign exchange markets but ignoring the linkage between the two markets. Taking the current account as given, the linkage was addressed by Fleming (1968) and (1947). Other useful references are Fleming and Mundell (1964), Lanyi (1975) and Swoboda (1974).
5. See Bickerdicka (1920), Robinson (1937), Metzler (1949), Machlup (1939–40), and Dornbusch (1975).


7. Other references are Culbertson (1975) who simply used annual data on black market rates in India, the Philippines and Turkey for PPP calculations, Giddy (1973) who attempted to show the "efficiency" of black markets for foreign exchange in Colombia, Brazil and Israel by testing the randomness of the black market premiums and changes in the premium using weekly data and Blejer (1978b) who used annual data on a black market based on the real exchange rate for Brazil, Chile and Colombia to estimate money demand functions in these countries.

8. In Blejer (1977), relative price levels depended on δ which led to lagged terms in (3) and was replaced by a formulation like (2) in (1978a).

9. Under rational expectations the real rate would be a function of time only given by

\[ r_t = \exp[\log r_0 \exp(-t)]. \]


11. See Macedo (1979a) and (1981c). This assumes that the stock of reserves is "large enough" and disregards the crisis problem of Krugman (1979).
12. The introduction of non-traded goods in monetary models of the small open economy is due to Dornbusch (1973) and Krueger (1974), while Kouri (1975) and Calvo and Rodríguez (1977) introduced non-traded goods in portfolio models of flexible exchange rates. Work with three good models has been done by Jones (1974), Takayama (1974), and Dornbusch (1975) and (1980).

13. To see this consider the effect of a change in $p$ on the demand for the "smuggled good," given $x^N$ and $\beta$:

$$\frac{dD^B}{dp} = \frac{\partial D^B}{\partial \beta} + \frac{\partial D^B}{\partial Y} x^B + \frac{\partial D^B}{\partial Y} (dX^B p + dX^O)$$

By full employment $p = -dX^O/dX^B$ and by the Slutsky decomposition

$$\frac{\partial D^B}{\partial \beta} = \frac{\partial D^B}{\partial \beta} \mid_{\beta} - \frac{\partial D^B}{\partial \beta} \mid_{\beta}$$

so that:

$$\frac{dD^B}{dp} = \frac{\partial D^B}{\partial \beta} \mid_{\beta} < 0$$

See Kouri (1975) and Dornbusch (1980).

14. If the shares of expenditure on the official traded good and the non-traded good are $\alpha$ and $\beta$, a price index for a Cobb-Douglas utility function will be

$$Q = \frac{p^1 - \alpha - \beta}{p^0 \frac{p^\alpha}{p^N \frac{p^\beta}}$$

and the real return on domestic money will be

$$-\alpha \frac{p^0}{Q} - \beta \frac{p^N}{Q} - (1-\alpha-\beta) \frac{p^B}{Q}$$

while the real return on foreign money will be the change in $\frac{p^B}{Q}$.
or \( - (\alpha + \beta) \hat{p} + \delta q \). The difference is therefore \( \hat{p}_B \), which given the foreign currency price \( p_B^* \) is simply the rate of change of the flexible market rate.

15. Domestic Credit creation, in particular to the public sector, is thus exogenous and, in particular, capital gains and losses on foreign exchange reserves are offset by changes in central bank net worth. For the case, discussed in Johnson (1976), that these induce credit creation see Macedo (1979a).

16. Or if an increase in the price of the non-traded good increased demand for foreign assets as assumed in Macedo (1979a).

17. For evidence on these proportions for major currencies see Kouri and Macedo (1978) and Macedo (1981a).

18. Calvo (1979) elaborates on the differences between a once-and-for-all devaluation and an increase in the rate of crawl.

19. See references in footnote 12.
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