Optimal Terms of Foreign Assistance

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As debt service payments of the less-developed world increase very rapidly, an increasing number of writers warn that the terms of foreign assistance must be softened or a large number of less-developed countries will find the burden of their debt service intolerable (see, for example, the Organization for Economic Cooperation and Development [1968]). On the other hand, the writings of a number of economists have provided arguments for the use of relatively high rates of interest, at least as high as the government borrowing rate in the donor country (see Schmidt [1964] and Cooper [1965]). The purpose of this paper is to analyze optimal terms of foreign assistance under three different types of assumptions: (1) perfect certainty and a constant return to capital, (2) limitations on absorptive capacity of the borrowing country, and (3) uncertainty of returns in the borrowing country. These three cases lead to quite different lending criteria.

1. Perfect Certainty and Fixed Returns

A measure of the hardness or softness of a loan is the *grant value* of a loan. The grant value of a loan is the face value of the loan less the discounted stream of payments of interest and principal. The grant value expressed as a percentage of the face value of a loan is the *grant element* of a loan (see Pincus [1963]). If the donor country’s opportunity cost of capital
is used as a rate of discount, the grant value of a loan measures the present cost to the donor of making the loan. If the recipient's rate of return on capital is used as a rate of discount, the grant value measures the present benefit of the loan to the recipient. Note that if the rate of discount exceeds the rate of interest on the loan, the grant value is negative. Thus if the opportunity cost of capital \((p)\) in the donor country is less than the rate of return on capital \((q)\) in the recipient country, and the rate of interest on the loan \((r)\) is higher than \(p\) but less than \(q\), the grant value of the loan is negative (a benefit) for the donor and positive (also a benefit) for the recipient. Thus a rate of interest between \(p\) and \(q\) benefits both the donor and the recipient.

This point is illustrated in figure 1. The face value of a loan is represented by the vertical axis, and the rate of interest on the loan, by the horizontal axis. The curve \(AB\) is an iso-benefit curve to the donor. All points along \(AB\) represent combinations of face value and interest rates having the same grant value, in this case a negative grant value or benefit to the donor. The curves \(C_1D_1\), \(C_2D_2\), and \(C_3D_3\) are iso-benefit curves for the recipient; that is, each represents those combinations of interest rate and face value of a loan with the same positive grant value. Note, however, that the benefit to the recipient increases from \(C_1D_1\) to \(C_2D_2\).
Thus if the face value of the loan is $L^*_1$ and the rate of interest is $r^*_1$, the recipient's benefit will be increased if one lowers the interest rate to $r^*_2$ and increases the face value of the loan to $L^*_2$, while the benefit to the donor is no less than before. Thus if we maximize the benefits to the recipient while holding the benefit to the donor constant, the optimal terms are an interest rate approaching $p$, donor's opportunity cost of capital, and an amount of lending approaching infinity.

If we fix the benefit to the recipient and attempt to maximize the benefit to the donor, the donor is always better off by increasing the interest and the face value of the loan, moving toward the opportunity cost of capital $q$ to the recipient as a limit. This is illustrated in figure 2, where the curve $CD$ represents a fixed benefit to the recipient and $A_1B_1$, $A_2B_2$, and $A_3B_3$ represent increasing iso-benefit curves to the donor. The donor becomes increasingly better off by moving from $P_1$ to $P_2$ to $P_3$.

Alternatively, we could maximize some joint welfare function of the donor's benefit and recipient's benefit subject to a constraint on the total face value of the loan. The optimal plan would always be to lend the full amount possible. The terms, however, depend on the relative values placed on recipient and donor country benefits. For example, if no weight is placed on donor's benefits and the goal is simply to maximize recipient benefits, the optimal terms of aid would be a zero rate of interest.

\[ \text{Fig. 2} \]
All of the above analysis is based on the assumption that the opportunity cost of capital is higher in the recipient than in the donor country. If the reverse is true, it is nonoptimal to charge any rate of interest. Flows from the donor to the recipient should take the form of grants. A simple extension of the opportunity set available to the recipient, however, removes this kind of case from consideration. If the recipient always has the option of reinvesting any grant from the donor in the donor country, he may obtain a rate of return equal at least to the opportunity cost of capital in the donor country. This places a floor on the rate of return which the recipient can obtain from any capital transfer.

2. Optimal Terms—Declining Returns to Capital

Let us assume that the recipient country is small in relation to the donor country so that the opportunity cost of capital in the donor country is constant for any amount supplied to the recipient. Suppose, however, that the rate of return to capital in the recipient country declines as more
is lent in any short period of time because of limited capacity to implement high-return investment projects.

This is illustrated in figure 3, which shows a declining opportunity cost of capital \( q(L) \) in the recipient country. The opportunity cost of capital in the donor is \( p \). The opportunity cost of capital in the recipient country declines as more is lent but never falls below \( p \). The curves \( C_1D_1 \) and \( C_2D_2 \) represent iso-benefit curves to the recipient. \( A_1B_1 \) is an iso-benefit curve for the donor (negative grant value), and \( A_2B_2 \) is an iso-cost curve for the donor (positive grant value). The curves \( A_1B_1 \) and \( C_1D_1 \) have a point of tangency at \( P_2 \). Similarly, \( A_2B_2 \) and \( C_2D_2 \) have a point of tangency at \( P_1 \). The set of all such points of tangency forms a contract curve \( EF \). The curve \( EF \) represents combinations of terms of lending and amounts of lending which are Pareto optimal. That is, both donor and recipient are better off at some point on the contract curve than at a point off the curve.

If some joint welfare function, including both benefits to the donor and benefits to the recipient, is maximized, any point on the contract curve may prove optimal. Unlike the case of constant opportunity costs, rates of interest less than the opportunity rate in the donor may be Pareto optimal. That is, grant assistance or very soft terms may be perfectly rational from both the donor’s and the recipient’s points of view.

### 3. Risk

So far we have abstracted from the problem of risk. A decision to borrow funds from abroad to implement an investment program involves many kinds of risks regarding not only the chances of success or failure of particular projects but also whether sufficient savings and foreign exchange earnings will be available to meet the service payments on the debt.

Suppose the expected return on an investment program is a proportion \( k \) of the total investment \( (L + C^*) \), where \( L \) is the amount of the investment financed through international borrowing and \( C^* \) is the domestic capital invested. Given that the return is partially random from year to year, the actual gross return from the project in year \( t \) is \( e_t \), where \( e_t \) is a random variable with mean 0. The net or national return, assuming a rate of interest \( r \) on international borrowing and no amortization is

\[
(k + e_t)(L + C^*) - rL, \tag{1}
\]

while the expected net return is

\[
k(L + C^*) - rL \tag{2}
\]

with variance equal to

\[
\text{var} [e_t \cdot (L + C)] = (L + C^*)^2 \cdot \text{var} (e_t). \tag{3}
\]
Let us assume that the recipient is a risk averter so that the level of variance affects the benefits to be derived by the recipient. The benefits derived in a given year \( (B_t) \) are a function of the expected return and the variance

\[
B_t = k(L + C^*) - rL - \alpha(L + C^*)^2 \cdot \text{var}(e), \quad (4)
\]

where \( \alpha \) is positive and indicates the marginal "disutility" of variance in the net return. The discounted value of the net return, assuming an infinite horizon, a discount rate of \( q \), and a constant variance \( \text{var}(e_t) = \text{var}(e) \) for all \( t \), is

\[
B = \frac{k}{q} (L + C^*) - \frac{r}{q} (L) - \frac{\alpha(L + C^*)^2}{q} \cdot \text{var}(e). \quad (5)
\]

We can maximize \( B \) with respect to \( L \) by differentiating and setting the derivative equal to zero. The result is

\[
L = \frac{(k - r)}{2\alpha \text{var}(e)} - C^*. \quad (6)
\]

Assuming that the rate of return \( k \) on the project is greater than the interest rate \( r \) and that \( \alpha \) and \( \text{var}(e) \) are not too large, a positive amount will be borrowed. The amount borrowed will decrease, however, as the rate of interest \( r \) increases. At some point, the rate of interest is so high that no amount will be borrowed because the variance of the return will be too large and will not be compensated by even a positive expected return. This is illustrated in figure 4. The straight line \( L^*QPr^* \) shows the optimal amounts of borrowing for the recipient as a function of the interest rate.

To simplify the analysis, suppose that the donor is not a risk averter so that his real cost or benefit is determined by using a discount rate \( p \) which represents the expected rate of return to the donor. Then \( A_1B_1, A_2B_2, \) and \( A_3B_3 \) are iso-cost curves to the donor, and \( A_4B_4 \) is an iso-benefit curve for the donor. Qualitatively, the same conclusions would hold if the donor were also a risk averter but his aversion to risk or variance were in some sense less than that of the recipient. Let us also assume that the expected return \( k \) on capital is greater in the recipient country than in the donor country but that the variance of the return is equal in both cases. Thus the recipient will always invest locally. If we relax this assumption and assume that the variance of return is less in the donor country, optimal behavior by the recipient would require a policy of balancing his portfolio between local investments and investments in the donor country. Again the same qualitative conclusions would hold, but the simpler assumption that the variance of return is equal in both donor and recipient simplifies the analysis considerably and is just as instructive.
Given these assumptions, then, we may investigate the properties of Pareto-optimal solutions to the problem of terms of foreign assistance. From figure 4, the Pareto-optimal curve may be divided into two segments: (a) a downward-sloping segment $QPr^*$ and (b) a horizontal segment $TRQ$.

First, let us consider a point below the downward-sloping segment $QPr^*$, for example, a point such as $U$. At the point $P$ both donor and recipient are better off than at $U$. The donor is better off because he increases his lending at a rate of interest $r^{**}$ which is greater than his opportunity cost of capital. The recipient is better off because $L^*QPr^*$ by definition represents the optimal amount of borrowing at each interest rate. On the other hand, a point above the line segment $QPr^*$ such as $W$ is not sustainable. The recipient is better off if he simply refuses to borrow this amount and reduces his borrowing to the point $P$. Thus the line segment $QPr^*$ is a set of Pareto-optimal points.

For rates of interest $r$ which are less than $p$, a Pareto-optimal point may be determined by maximizing the benefit $B$ in (5) to the recipient subject to the constraint that the cost to the donor $D = (p - r) \cdot L/p$ is held constant. The solution is

$$L = \frac{(k - p)}{2a \var{e}} - C^*.$$
This optimal amount of lending is invariant with respect to the interest rate; that is, the Pareto-optimal curve is horizontal to the left of \( Q \) in figure 4. Comparing (7) and (6), we see that the Pareto-optimal amount of lending is equal to the amount of borrowing that a rational recipient would choose to undertake if he were offered an interest rate \( p \) equal to the expected return on capital to the donor. Thus, combining our analysis for rates of interest below \( p \) and for rates of interest above \( p \), we obtain a Pareto-optimal curve \( TRQPr^* \).

Note that the Pareto-optimal curve to the left of \( R \) implies negative rates of interest. This corresponds to a foreign assistance program which includes both a lump-sum payment and an annuity which promises to pay \(-r\) dollars per year in perpetuity.

If a recipient country borrows on a large scale for some time, its high level of debt service may induce extreme variability in net returns relative to expected net returns. If we postulate increasing risk aversion as variability of return relative to expected return increases, the risk aversion parameter \( \alpha \) in equation (6) will increase and the line segment \( L^*QPr^* \) will shift downward and to the left. At some point the line will have shifted to the left of \( p \), as in figure 5. One can show, then, that the recipient is always better off at no cost to the donor by moving from \( P_1 \) to \( P_2 \) to \( P_3 \). There is no Pareto-optimal solution, but as the rate of interest approaches negative infinity and \( L \) approaches 0, the recipient is increasingly better off at no cost to the donor.

\[
\text{FIG. 5}
\]
4. Conclusions

When the absorptive capacity of the recipient country is limited or when the recipient country is a strong risk averter, optimal terms of lending tend to lie at the softer end of the terms spectrum. Under these conditions, amounts of lending should be strictly limited, and any attempt to increase the flow of real benefits to the recipient country should take the form of a softening of terms of assistance.

References


