Housing, land prices, and the link between growth and saving

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1. Introduction

This paper is concerned with the relationship between saving and growth. Within an overlapping-generations model of economic growth, we ask how the existence of a fixed supply of land for housing, for which consumers must save prior to ownership, and whose price is bid up by economic growth, affects the steady state growth path of the economy.

Our motivation comes from some dissatisfaction with the available explanations for the empirical relationship between saving and growth, particularly those where the causation runs from growth to saving. Among these, the most famous is the life-cycle hypothesis of saving, through which higher growth rates of household real income redistribute lifetime resources from old dissavers to young savers, thus raising the aggregate household saving rate. Empirical research using household survey data has provided little support for this mechanism because the life-cycle age profile of saving does not conform very well to the predictions of the hypothesis, see Paxson (1996), Poterba (1997) and Deaton and Paxson (1997). While there are some grounds for supposing that these studies understate the size of the life-cycle effect of growth on saving, making the appropriate corrections still predicts small effects, Deaton and Paxson (1998).

An alternative mechanism through which higher rates of economic growth might drive up the saving rate is the role played by consumer habits or more generally, by a slow rate of adjustment of consumption to rising incomes. While some researchers have found evidence in favor of such an account, Carroll and Weil (1994), the household survey evidence, although consistent with the existence of such an effect, suggests that it is small. Paxson (1996), uses time-series of cross-sectional household surveys from Britain, the United States, Taiwan, Thailand and Indonesia to construct data on saving rates and earnings by birth cohorts that can be followed through successive surveys. In Britain and Taiwan, cohorts saved (modestly) more when their earnings growth had been particularly rapid relative to the standard age profile; in the other countries, the effects were small, and either insignificantly different from zero or perverse.

The analysis of this paper was originally inspired by anecdotal evidence on the central role of housing in personal saving decisions in countries such as Korea and Taiwan. When people in their twenties and thirties were asked why they were saving a third of their incomes, a common response was that they were saving for a house or an apartment, that even a "starter" apartment cost many multiples of income, and that mortgage finance was either unavailable or severely restricted. Household surveys in Taiwan (as elsewhere) often show substantial saving rates among the elderly, contrary to simple life-cycle accounts. When asked, the elderly often give the same response as their children, that they are saving for housing, not for themselves, but for their children. Of course, such accounts raise as many questions as they answer, and are not even obviously internally consistent, at least in equilibrium. Nevertheless, consumers in the Asian tigers saved large fractions of their incomes, they faced high prices for housing relative to their incomes, and there were poor or non-existent mortgage markets. It is therefore worth investigating whether the presence of a limited amount of housing land, made ever scarcer by high saving rates and rapid economic growth, can act as a catalyst in a virtuous circle of saving, growth, and rising prices.

There is earlier related work by Jappelli and Pagano (1994) on the relationship between housing, credit restrictions, and saving. Jappelli and Pagano point out that if it is necessary to accumulate a substantial fraction of the price of a house prior to purchase, there will be additional saving early in the life-cycle that will act so as to reinforce the effect of growth on saving; redistributing lifetime resources to the young has a greater effect on aggregate saving if the young are saving. Jappelli and Pagano also find that maximal loan to value ratios for housing enter negatively in a cross-country regression of growth rates; countries that restrict lending grow faster. These are suggestive results, but they leave a number of questions unanswered. In particular, as is true for the life-cycle explanation of growth and saving, Jappelli and Pagano's results require that the life cycle age profile of saving be negatively correlated with age, which once again runs counter to the evidence in the survey data.

In this paper, we develop an overlapping generations growth model which can be used to compare a world with and without housing land. We focus on land, rather than on housing itself, because housing is a produced good, in contrast to land, which we assume is in fixed supply. Land combines with housing goods to generate housing services, so that in a model where housing is not distinguished from other goods, land and consumption of goods are the arguments of the utility function. In the baseline model, without land, developed in Section 2, the younger generation works for wages using the capital owned by the older generation, and saves or consumes the proceeds. Their saving becomes the capital stock in next period, during which they live on the income from capital. There is a fixed rate of population growth n. With these (and other) assumptions, the model will always have as a stationary solution the Golden rule with the rate of interest equal to the rate of population growth, a result that is destroyed by the introduction of land. Instead, all equilibria with land have associated capital stocks smaller than the Golden Rule capital stock, and a real rate of interest larger than the rate of population growth. The market for the unproductive asset, land, lowers consumption and welfare in the long run stationary equilibrium. One way to attain the long-run social optimum would be to nationalize land and "rent" it at no charge. These results are very different from those for which we set out to look, and clearly provide no support for the "virtuous circle" of growth, saving, and land prices.

Our results are nevertheless closely related to earlier results in the literature, particularly the important paper by Drazen and Eckstein (1988). Drazen and Eckstein develop a dual economy model of economic development, in which land is used by the “backward” agricultural sector as a factor of production. They show that, in a fully competitive equilibrium with land markets, the stock of capital can be lower than when land is not traded but passed on from one generation to the next. The mechanism is the same as in the current paper; capital accumulation is a by-product of life-cycle retirement saving, and saving in the form of land crowds out capital. The same mechanism was earlier invoked by Feldstein (1977) to argue that taxation of land need not result in a decrease in its price by the capitalized value of the tax. Our own model is set up differently from that of Drazen and Eckstein. Because we are concerned with housing, land appears in the utility function, not as a factor of production. Moreover, we have a single production sector, not two. Nevertheless, the differences are more in the labeling than in the substance.

2. An overlapping generations model of saving and growth

We start with the specification of a simple OLG economy without land. People live for two periods, so that there are two generations alive at any given time, and population grows at rate n. The young work for wages with capital owned by the old, which they accumulated when they were young. Capital lasts for one period and generates profits for the older generation who do not work. There is an “outside” or “extrinsic” asset, in constant quantity M, which is passed on from generation to generation at a price determined within the system. When it is positive, this asset can be thought of as a claim on another country, or more exotically, as some object that is passed from one generation to another in exchange for goods. When it is negative, it is best thought of as a debt to another country. It is important that this asset have no “intrinsic” worth of its own, and that it not generate utility; this separates it from land which is introduced in the next section. The precise role played by the extrinsic asset will be discussed in some detail as we work through the solutions, where we give particular attention to the case where $M = 0$.

We proceed informally, attempting to convey the essentials of the argument. Assumptions and proofs are gathered together in the Appendix.

For a person who is young in period t, the utility function to be maximized is written as

$$u(c_{1t}) + \upsilon(c_{2t+1}), \tag{1}$$

where c is consumption, $u(.)$ and $\upsilon(.)$ are the youth and old utility functions respectively, and the first suffix indicates age, and the second period, so that, for example, c_{2t+1} refers to the consumption of the old (2) in period $t+1$, i.e. of those who are currently young. Each consumer inelastically supplies one unit of labor when young and none when old. The population grows exogenously at rate n. We work in per capita terms, so that the budget constraint of each young person in period t is written as

$$c_{1t} + x_t + p_t m_t = y_{1t} + w_t, \tag{2}$$

where x_t is saving, which in aggregate is the capital stock of period $t+1$, m_t is the quantity of the extrinsic asset that each of the young buys from the old at price p_t, w_t is the wage, and y_{1t} is exogenous income ("manna") received by the young independently of work. In period 2, the same individuals will face the new constraint

$$c_{2t+1} = p_{t+1} m_t + (1 + r_{t+1}) x_t + y_{2t+1} \tag{3}$$

where y_{2t+1} is the "manna" received by the old, and r_{t+1} is the interest rate on savings invested at t but paid at t+1.

The production technology is represented by a constant returns to scale production function

$$O_t = f(K_{t-1}, L_t) \tag{4}$$

for capital K and labor L, and the dating convention emphasizes the fact that the capital stock is last period's saving. The arguments of (4) are given by

$$L_t = (1+n)^t \tag{5}$$

where the initial labor force has been normalized to unity, and

$$K_{t-1} = (1+n)^{t-1} x_{t-1} \tag{6}$$

which is saving per head of last generations young workers multiplied by their number. There is also a feasibility constraint for the economy as a whole, that total consumption plus investment should sum to production plus manna. Writing this constraint per head of the younger generation alive at t, we have

$$c_{1t} + \frac{c_{2t}}{1+n} + x_t = y_{1t} + \frac{y_{2t}}{1+n} + F\left(\frac{x_{t-1}}{1+n}\right). \tag{7}$$

The last term on the right hand side is output per head of the young generation,

$$\frac{O_t}{(1+n)^t} = \frac{f\left[(1+n)^{t-1}x_{t-1}, (1+n)^t\right]}{(1+n)^t} = f\left(\frac{x_{t-1}}{1+n}, 1\right) \equiv F\left(\frac{x_{t-1}}{1+n}\right) \tag{8}$$

where the last equivalence defines the "intensive" production function for output per head in terms of capital per head, and rests on the constant returns to scale assumption. Because the total quantity of the extrinsic asset M is fixed, the amount per head m_t satisfies

$$m_t = M(1+n)^{-t}. \qquad (9)$$

Factors are paid their marginal products, so that the capitalists, who are the retirees, each get the marginal product of capital, so that

$$1 + r_t = F'\left(\frac{x_{t-1}}{1+n}\right) \qquad (10)$$

while the younger generation, who are workers, receive the wage

$$w_t = F\left(\frac{x_{t-1}}{1+n}\right) - \frac{x_{t-1}}{1+n} F'\left(\frac{x_{t-1}}{1+n}\right). \qquad (11)$$

There are two assets in the model, capital and the "extrinsic" asset *M*. In equilibrium, both are present; under the assumption that its marginal productivity is infinity at zero, the capital stock x_t will be positive, and the restriction $x_t \geq 0$ will not be binding. In consequence, each must yield the same rate of return, so that we must have

$$\frac{p_{t+1}}{p_t} = 1 + r_{t+1} = F'\left(\frac{x_t}{1+n}\right). \qquad (12)$$

The use of this arbitrage condition allows a considerable simplification when we combine the two budget constraints (2) and (3), leading to the familiar

$$c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} = y_{1t} + \frac{y_{2t+1}}{1+r_{t+1}} + w_t \qquad (13)$$

which is the standard two period intertemporal budget constraint that constrains the present value of outgoing to the present value of incomings. For the generation born at *t*, (13) is the budget constraint against which they maximize utility (1); the solutions are the levels of consumption, which we write in the standard way:

$$c_{1t} = g_1\left(y_{1t} + \frac{y_{2t+1}}{1+r_{t+1}} + w_t, (1+r_{t+1}) \right) \tag{14}$$

$$c_{2t+1} = g_2\left(y_{1t} + \frac{y_{2t+1}}{1+r_{t+1}} + w_t, (1+r_{t+1}) \right). \tag{15}$$

Lemma 1 in the Appendix demonstrates that, as capital per head goes to infinity, so that the productivity of capital goes to zero, the consumption of the young goes to infinity, driven by infinite borrowing. Similarly, as capital per head tends to zero, and its marginal productivity goes to infinity, consumption of the old goes to infinity. We will use these patterns of intertemporal substitution in the argument below.

The model can be solved (or simulated) using equations (14) and (15) together with the feasibility constraint. Suppose that we are at time *t* so that the previous capital per head x_{t-1} is given, which gives the current wage rate from (11), and we are trying to find the current value of x_t. Given a trial value, the future rate of profit is given by (10), so that (14) determines c_{1t}. Along a perfect foresight equilibrium, the value of c_{2t} comes from the lagged value of (15), which is known from x_{t-1}. We therefore have both c_{1t} and c_{2t} so that if we apply the feasibility constraint (7), we have a single equation in x_t, which can be analyzed to derive possible solutions.

In this paper, we concern ourselves only with *stationary* solutions, defined as those for which capital per head is constant, and in which the environment is constant, so that $y_{1t} = y_1$

and $y_{2t} = y_2$. The aggregate stock of the extrinsic asset M is constant so that, with population growth at rate n, the amount per head m_t falls at rate n, so that for $p_t m_t$ to be constant, the price p_t must rise at the same rate. In an equilibrium in which the two assets coexist, we must therefore have $n = r$; otherwise $M = 0$. If the stationary solution for capital per head is x, then we can write the choice of the intertemporal profile (1) subject to (13) in a way that emphasizes the dependence on x, as

$$\max \quad u(c_1) + \upsilon(c_2) \tag{16}$$

subject to

$$c_1 + \frac{c_2}{F'[x/(1+n)]} = y_1 + \frac{y_2}{F'[x/(1+n)]} + F\left(\frac{x}{1+n}\right) - \frac{x}{1+n}F'\left(\frac{x}{1+n}\right). \tag{17}$$

So that, again emphasizing the dependency on x, the solutions (14) and (15) can be written

$$c_1 = \xi_1(x,y_1); \quad c_2 = \xi_2(x,y_1) \tag{18}$$

where the dependence on y_1 is carried forward for use in the next section. From (18), the feasibility restriction will hold provided that the function $G(x,y_1)$ is zero, where

$$G(x,y_1) \equiv \xi_1(x,y_1) + \frac{\xi_2(x,y_1)}{1+n} + x - y_1 - \frac{y_2}{1+n} - F\left(\frac{x}{1+n}\right) \tag{19}$$

Figure 1 sketches one particular case for the function $G(x, y_1)$. The general shape of $G(x, y_1)$, that it goes to infinity as *x* tends to zero, or as *x* tends to infinity, comes from the effect of *x* on the marginal productivity of capital and the associated patterns of intertemporal substitution. Because the functions $\xi_1(x, y_1)$ and $\xi_2(x, y_1)$ inherit the properties of the functions $g_1(.)$ and $g_2(.)$ in (14) and (15), as proved in Lemma 1 in the Appendix, second period consumption goes to infinity as *x* tends to 0, and first period consumption goes to infinity as *x* tends to infinity, and in both cases, $G(x, y_1)$ goes to infinity. The function must therefore have the general shape as shown, with either no or an even number of solutions (unless there is a tangency with an associated single solution.)

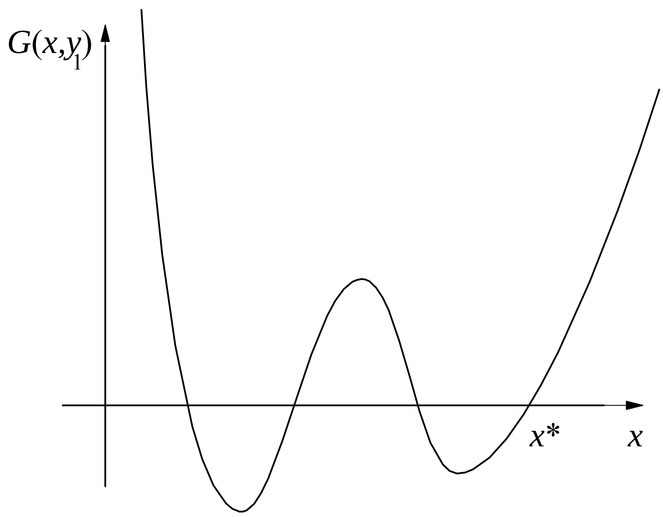

Figure 1: Stationary solutions for capital per head

The Golden Rule capital stock x^* is the stationary value of capital that maximizes the resources available for consumption which, from (7), is the value of *x* that maximizes the difference $F[x/(1+n)] - x$. It is therefore the capital stock that sets the marginal productivity of capital, or the rate of profit, equal to the rate of population growth. It turns out the model has a stationary equilibrium with this capital stock. Start from the definition; x^* is defined as the solution to

$$F'\left(\frac{x^*}{1+n}\right) = 1 + r = 1 + n. \tag{20}$$

At the Golden Rule value of x^*, the wage is given by

$$w = F\left(\frac{x^*}{1+n}\right) - \frac{x^*}{1+n}F'\left(\frac{x^*}{1+n}\right) = F\left(\frac{x^*}{1+n}\right) - x^*. \tag{21}$$

The intertemporal budget constraint is

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} + w, \tag{22}$$

so that, substituting *n* for *r*, and using (21) for the wage, (22) becomes

$$c_1 + \frac{c_2}{1+n} + x^* = y_1 + \frac{y_2}{1+n} + F\left(\frac{x^*}{1+n}\right) \tag{23}$$

which is the feasibility constraint (7). In consequence, if we start from x^*, the demand functions (18), which by construction satisfy the budget constraint (22), will automatically satisfy the feasibility constraint, so that the function $G(x, y_1)$ is zero at

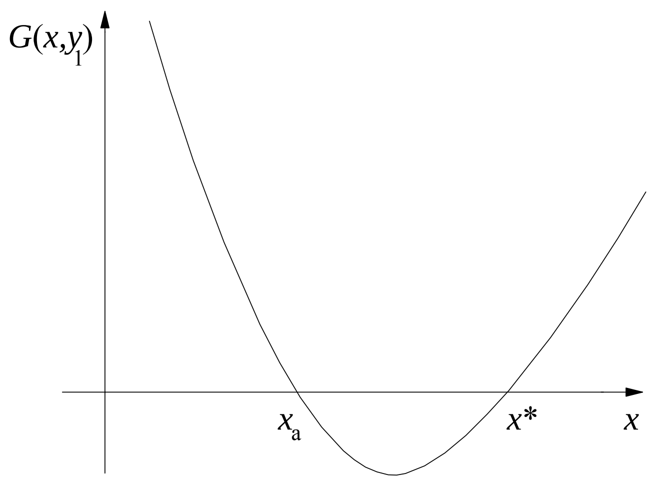

Figure 2: Golden Rule and one other solution

x^*. The Golden rule solution is illustrated in Figure 1 as the largest solution, though this need not be the case in general.

One case to which we shall devote some attention is illustrated in Figure 2. Here there are two solutions, the greater of which is the Golden Rule solution. At the other, x_a, where capital per head is less than at the Golden Rule, the interest rate is larger than the rate of population growth.

Just as the Golden Rule is one of the stationary solutions, the shape of *G*(.) shows that (in the absence of a tangency) there is another solution, the "autarkic" equilibrium, in which the price of the "exogenous" asset p_t is zero. In Figure 2, where there are only two solutions, this is marked as x_a.

To see what the autarkic equilibrium looks like, write the two consumption levels in the form

$$c_1 = y_1 + w - x; \quad c_2 = x(1 + r) + y_2, \tag{24}$$

without the exogenous asset, or equivalently, with its price set equal to zero. Given (24), and the wage and interest rate conditions,

$$1 + r = F'\left(\frac{x}{1+n}\right); \quad w = F\left(\frac{x}{1+n}\right) - \frac{x}{1+n}F'\left(\frac{x}{1+n}\right) \tag{25}$$

it is straightforward to check that the feasibility constraint is satisfied. The equilibrium is a value x_a of the capital stock, such that when *w* and *r* are given by (25) with $x = x_a$, the consumer's problem

$$u(y_1 + w - x) + \upsilon[y_2 + x(1 + r)]. \tag{26}$$

has its maximum at x_a. As a result, an autarkic equilibrium (x_a, c_1, c_2) is a solution of the system of equations

$$u'(c_1) = F'\left(\frac{x_a}{1+n}\right)\upsilon'(c_2) \tag{27}$$

$$c_2 = y_2 + F'\left(\frac{x_a}{1+n}\right)x_a. \tag{29}$$

$$c_1 = y_1 + F\left(\frac{x_a}{1+n}\right) - \frac{x_a}{1+n}F'\left(\frac{x_a}{1+n}\right) - x_a \tag{28}$$

The existence of such an equilibrium is guaranteed by the shape of $G(x, y_1)$ and the fact that any solution of $G(x, y_1) = 0$ such that $x \neq x^*$ is an autarkic equilibrium.

Finally, we look at the sign of the extrinsic financial asset *M* in the case of Figure 2, where there is one autarkic equilibrium with lower capital per head than the Golden Rule equilibrium. By the definition of the autarkic equilibrium, there are no exogenous assets, and $M = 0$. Consider the function

$$\theta(x) \equiv \xi_2(x) - F'\left(\frac{x}{1+n}\right)x - y_2, \tag{30}$$

which can be interpreted as the amount of second period consumption, c_2, that is not covered by either second period's "manna" or the return from capital saved in the first period. It is therefore the amount of second period's consumption financed by the transfer of the asset from the young to the old. An autarkic equilibrium with no extrinsic assets is characterized by $\theta(x_a) = 0$, a property that is easily checked from the definition of the demand functions $\xi_1(x)$ and $\xi_2(x)$. When x approaches zero, as we have already seen, $\theta(x)$ tends to infinity. In the situation shown in Figure 2, since $\theta(x)$ is continuous, and since there is only one autarkic equilibrium, $\theta(x)$ must be negative for x greater than x_a, and in particular is negative at the Golden Rule value of x, x^*. At the Golden Rule equilibrium, there must be *negative* external assets, for example net foreign debt. If this is not feasible, so that the problem must be solved with $M \geq 0$, the Golden Rule equilibrium is not feasible either.

3. Growth, saving, and land

Given the baseline model of Section 2, we can examine what happens when we replace the exogenous outside asset with land. We think of the land as being used for housing; the single good in the model is combined with land according to some technology that generates shelter. We represent this by extending the intertemporal utility function (1) to read

$$u(c_{1t}) + \upsilon(c_{2t+1}, h_t), \tag{31}$$

where h_t is the amount of land that is combined with goods to generate second period utility. Individuals do not own land in the first period; we think of them as living with their parents, or in infinitely high tower blocks, which provide accommodation without land.

Young consumers are required to purchase their second period land in the first period, although they do not get to use it until later; the treatment of land therefore parallels the treatment of capital, which must be saved in the first period, but yields no return until the second period. However land, unlike capital, never deteriorates, and so is purchased in the first period from the older generation, who, in spite of living on the land, can thus use its sales to finance second period consumption. (Richer structures than this are clearly possible, but would require more than a two-period model. But a critical element in any story of land and housing is to ensure that, if someone is saving to buy land, or is being forced to save because of rising land prices, someone else is receiving the purchase, and can consume the capital gains.) The budget constraints in the two periods are then written

$$c_{1t} + p_t h_t + x_t = y_{1t} + w_t \tag{32}$$

for the first period, and

$$c_{2t+1} = p_{t+1} h_t + (1 + r_{t+1}) x_t + y_{2t+1} \tag{33}$$

in the second period.

Feasibility requires the same condition on output as before, reproduced here for convenience

$$c_{1t} + \frac{c_{2t}}{1+n} + x_t = y_{1t} + \frac{y_{2t}}{1+n} + F\left(\frac{x_{t-1}}{1+n}\right). \tag{34}$$

In addition, total land is fixed, so that the land purchased by each person of the younger generation must be falling at rate *n*,

$$h_t = \frac{h_0}{(1+n)^t}. \tag{35}$$

If, as before, and ignoring irrelevant corner solutions, we eliminate *x* between the first and second period budget constraints, (32) and (33), we get a new version of the intertemporal budget constraint

$$c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} + \left(p_t - \frac{p_{t+1}}{1+r_{t+1}}\right)h_t = y_{1t} + \frac{y_{2t+1}}{1+r_{t+1}} + w_t. \tag{36}$$

The term multiplying h_t in (36) is, of course, the user cost of land, which we denote π_t, so that

$$\pi_t = \left(p_t - \frac{p_{t+1}}{1+r_{t+1}}\right). \tag{37}$$

The first-order conditions for the maximization of (31) subject to (37) are

$$u'(c_{1t}) = (1+r_{t+1})\upsilon_1'(c_{2t+1}, h_t) \tag{38}$$

$$\pi_t u'(c_{1t}) = \upsilon_2'(c_{2t+1}, h_t). \tag{39}$$

In any stationary equilibrium of this model, we require that the value of land per head, $p_t h_t$, be constant, which, in view of (35), requires that p_t increase at rate *n*. In equilibrium, the user cost π_t has to be strictly positive, otherwise the demand for land would be unbounded. It then follows from (37) that the rate of interest must be strictly larger than the rate of growth of the population. In consequence, the equilibrium capital stock will always be less than its Golden Rule value. Furthermore, there is no scope for the introduction of a fixed quantity of an extrinsic financial asset as in the previous section.

The existence of a stationary equilibrium also requires further restrictions on preferences. If the price of land p_t rises at rate *n*, and the real interest rate is constant, then by (37), the user cost π_t must also rise at rate *n*. Equation (37) then specializes to

$$\pi_t = p_0(1+n)^t\left(\frac{r-n}{1+r}\right). \tag{40}$$

Since consumption per head is constant in a stationary equilibrium, (39) implies that $\upsilon_2^{/}(c_{2t+1},h_t)$ grows at rate n. This typically holds only if, for some function φ(.),

$$\upsilon_2^{/}(c_2,h) = \varphi(c_2)/h, \tag{41}$$

so that, integrating,

$$\upsilon(c_2,h) = \varphi(c_2)\ln h + \mu(c_2). \tag{42}$$

But, by the other first-order condition, (38), the first derivative of $\upsilon(c_2,h)$ is constant in stationary equilibrium, and thus independent of h, so that (42) can be specialized further to

$$\upsilon(c_{2t+1},h_t) = v(c_{2t+1}) + \gamma\ln h_t, \tag{43}$$

and life-cycle preferences take the additively separable form

$$u(c_{1t}) + v(c_{2t+1}) + \gamma\ln h_t. \tag{44}$$

We can look for solutions for the model with land in the same way that we looked for solutions for the model without land. We solve the consumer's intertemporal maximization problem, maximizing (44) subject to the intertemporal budget constraint (36), and then substitute the solutions into the feasibility constraint that consumption and investment should not exceed production, together with the new feasibility constraint that the demand for land should equal its fixed supply. Because utility is additive, the solution to the maximization problem can be straightforwardly related to the corresponding solutions in the case without land. Rewrite the intertemporal budget constraint (36) in equilibrium as

$$c_1 + \frac{c_2}{1+r} = y_1 - \pi_t h_t + \frac{y_2}{1+r} + w. \tag{45}$$

In equilibrium, π_t is given by (40) and grows at rate *n*, and h_t is decreasing at the same rate so that $\pi_t h_t$ is constant at its original value $\pi_0 h_0$; in order to find a solution, we need then only solve for the initial price of land p_0 as well as the capital stock per head *x*. The maximization of (44) subject to (45) gives, see (18),

$$c_1 = \xi_1(x, y_1 - \pi_0 h_0); \quad c_2 = \xi_2(x, y_1 - \pi_0 h_0). \tag{46}$$

The two feasibility conditions are thus given by, cf. (19),

$$G(x, y_1 - \pi_0 h_0) \equiv \xi_1(x, y_1 - \pi_0 h_0) + \frac{\xi_2(x, y_1 - \pi_0 h_0)}{1+n} + x - y_1 - \frac{y_2}{1+n} - F\left(\frac{x}{1+n}\right) = 0. \tag{47}$$

and for land,

$$\gamma = h_0 \pi_0 u'(c_1) \tag{48}$$

which equates the marginal utility of land to its user cost multiplied by the marginal cost of the numeraire. Using (40), (48) can be written in terms of the initial price of land p_0

$$\gamma = h_0 \pi_0 u'(c_1) = h_0 p_0 \left(\frac{r-n}{1+r}\right) u'\big(\xi_1(x, y_1 - \pi_0 h_0)\big). \tag{49}$$

A solution of the model is then a value for *x* and for π_0, or equivalently p_0, which solve (47) and (49) simultaneously.

The Appendix provides a proof that at least one equilibrium exists in the model with land. Figure 3 provides an illustration based on Figure 2 and shows the *G*(.) function in (47). Note that this is only one of the two equations that defines the equilibrium—the other is (49)—but it is nevertheless informative. The higher curve is the original function which cuts the horizontal axis twice, at the autarkic and Golden Rule equilibria. Once land is introduced, the user cost of land is deducted from y_1, and since income is normal for consumption in both periods, the consumption levels in both periods are lower for any given value of *x*, and thus so is the value of the *G*(.) function. In consequence, if the autarkic equilibrium lies below the Golden Rule equilibrium as shown, the new *G*(.) function will cut the horizontal axis once below the original autarkic equilibrium, at x_t^L, and once above the Golden Rule. The latter cannot be an equilibrium, because the real rate of interest is below the rate of population growth, so that user cost would be negative, and the demand for land infinite. In consequence, in the case of Figures 2 and 3, the introduction of land removes the Golden Rule equilibrium, and the new autarkic equilibrium has a lower level of capital per head than did the original autarkic equilibrium. If Figure 2 had been such that the two equilibria were ordered so that the Golden Rule equilibrium had lower capital per head than the (inefficient) autarkic equilibrium, the introduction of land would have eliminated the autarkic equilibrium, because the user cost is

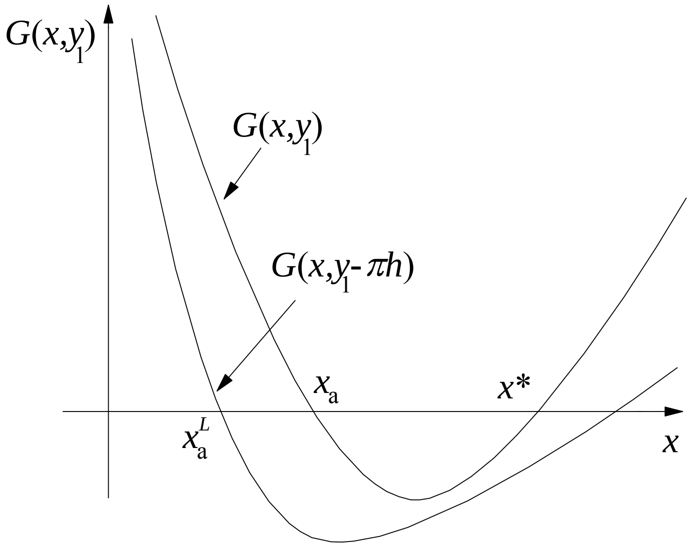

Figure 3: Golden Rule and autarkic solutions with and without land

negative above the Golden Rule. The new, unique equilibrium would lie to the left of the original Golden Rule equilibrium, once again with lower capital per head. In both these cases, the introduction of land decreases the level of capital per head compared with the original equilibrium.

The situation is more complex when there are multiple equilibria of the sort shown in Figure 1. In that case, moving the graph down will decrease the capital stock at the first equilibrium, increase it at the second, decrease it at the third, and eliminate the fourth (Golden Rule) equilibrium altogether. At equilibria where the G curve cuts the x-axis from below, and the introduction of land increases the capital stock, we obtain the "virtuous" mechanism from land to capital stock, the search for which originally inspired our work. What separates this sort of equilibrium from the equilibria where the introduction of land reduces the capital stock, and how can we judge which is more likely in practice? The crucial issue here is the effect on consumption in equilibrium of an increase in the capital stock. Higher capital stock is associated with higher production, and therefore income, but also with a lower interest rate, with indeterminate consequences for saving, and thus for consumption. In what we might label the "standard" case, higher capital stock means more consumption, the G function cuts the x-axis from above, and the introduction of land decreases both consumption and the capital stock. In the "non-standard" case, where the lower interest rate decreases consumption by more than the increase in production from the higher capital stock, the introduction of land *increases* the capital stock. Although this outcome seems implausible on empirical grounds, ruling it out requires serious empirical analysis.

A fuller analysis of the effects of land on the equilibrium can conveniently be conducted by considering the effects of varying the parameter γ in (44). When $\gamma = 0$, we have the original model without land, and the preference for land increases with γ. Analyzing the effects of changes in γ requires handling both (47) and (49) simultaneously. In the Appendix, we investigate how changes in γ affect the capital stock. We derive conditions under which it is true that the capital stock in the *smallest* equilibrium is declining in the taste parameter γ. In particular, near the no-land equilibrium, or when the user cost of land is sufficiently low, the derivative of the capital stock with respect to the taste for land, γ, is negative. More generally, we show that whenever the derivative of $\xi_1(x, y_1 - \pi_0 h_0)$ with respect to x is positive, the result carries through. When these conditions are met, the stronger is the taste for land, the lower is the capital stock and the level of (non-land) output per head.

4. Concluding remarks

Our original interest in this paper was to enquire whether the introduction of land into a growth model might help account for a "virtuous" circle in which saving-up for land (or housing) helped generate growth and higher land prices, which would generate further increases in saving, and so on. But our simple overlapping generations model does not support this conclusion, except in what appear implausible circumstances. In the long run, the user cost of land reduces the resources available for consumption of the reproducible goods, all other things equal. Production, together with the associated stock of productive capital, adjusts to demand, which in turn is affected by the new level of production. The interest rate moves with the marginal productivity of capital. The new equilibrium is typically obtained with a lower stock of capital and a higher rate of interest. This is the effect identified by Feldstein (1977) and by Drazen and Eckstein (1988), and which was first explored by Allais (1948). On the asset side, the presence of land causes a portfolio reallocation away from capital towards land. The social optimum here is for land to be nationalized and not to be allocated through markets, but passed down from one generation to the next through customary rules, such as division among the heirs, or by entail, as in Drazen and Eckstein. Land markets, far from generating growth, are inimical to capital formation.

We note that the negative effect of land on capital is not the only theoretical possibility. If consumption increases by enough in response to increases in the interest rate, it is possible for the introduction of land, or for an increase in the taste for land, to increase the capital stock, the virtuous circle effect with which we began. But without empirical support for such an apparently implausible outcome, we do not regard this theoretical outcome as providing much support for the original hypothesis.

The results in this paper are derived under a number of special assumptions, and it is important to consider their robustness. Note first that all of these results concern long-run equilibria, and it would be an interesting exercise to consider the dynamics to see whether transitional paths might be able to give a different account of the stylized facts. Another issue is the question of borrowing constraints, which we have not explicitly imposed. However, in the autarkic equilibria to which we give most attention, and whose level of capital per head is reduced by strengthening the preference for land, agents choose not to borrow, so that explicit borrowing constraints would not affect the equilibrium nor its properties. In autarkic equilibria, the amount of extrinsic asset demanded by each member of the younger generation, m_t, is zero, so that, by (2), and noting that the amount of capital per head is positive, consumption is strictly less than income, defined as the sum of wages and unearned income y_1 (manna). In the model with land, the younger generation must save, not only to purchase the capital stock, but also to purchase land for use in the next period.

It should also be noted that the preferences we have used are very special, particularly in the case where there is land. Additive preferences over the two consumption levels and land are analytically convenient, but are not plausible, if only because land requires complementary goods to construct housing, which is what people care about. However, the results in Section 3 are driven by the fact that, when people care about land, they must make room in their budgets to purchase it and in their portfolios to hold it. This mechanism would carry through into more complex models, and would be supplemented by the effects of non-separability, in particular the substitution effects on the consumption profile of the shrinking supply of land per head. One might suppose that, if construction can substitute for land—ever taller buildings, for example— there would be an additional motive to save in the first period, and a higher capital stock. We have not attempted to work through such a model in detail, because it seems unlikely to account for the stylized facts with which we began.

Another issue is whether the results would change in a model where the representative agent lives for more than two periods so that, for example in a three period model, there could be saving for housing in the first period, the purchase of a house in the second period, which is sold in the last period. But the basic mechanism works as before. Consider a long run equilibrium where aggregate consumption plus investment is equal to production net of capital depreciation, and assume an increase in the taste for land at the margin. At a fixed stock of capital and constant interest rate, the only consequence of such a change would be an increase in the user cost of land supported over the life time, and a corresponding reduction in the intertemporal income that can be spent on the reproducible good. Under normal circumstances—the "standard" case—equilibrium would be restored at a lower levels of production and capital stock.

The final topic that needs to be discussed is bequests. There are many different ways of modeling bequests, and we have considered only one, "Barro preferences," by which the utility of each generation depends on the utility of the next. For example, we could modify (43) to read

$$U_t = u(c_{1t}) + v(c_{2t+1}) + \gamma \ln h_t + \beta U_{t+1} \tag{50}$$

where U_t is the utility of the generation born at *t* and β is a discount factor less than to one. The repeated substitution of future utilities into (5) gives an infinite discounted sum of present and future subutilities so that, as usual with this form of bequest motive, we have moved from a model with an infinite number of overlapping generations to a model with essentially one consumer, albeit an infinitely lived dynasty. In the case without land, or $\gamma = 0$, this model has the general structure of a standard infinite horizon optimal growth model, which has a unique efficient equilibrium. In general, the capital stock per head will not tend to the Golden Rule level except when there is no discounting of intergenerational utility, which takes place as β tends to unity. When γ is positive, and land matters, the equilibrium will not change. Land is valued at the discounted value of its service flow, but the dynamics of consumption and the capital stock are the same as in the model without land. In this sense, and provided bequests are modeled in Barro's form, the results in this paper are not robust to the introduction of bequests. This should not be surprising. In the single agent dynasty of Barro's model, there is no room for the divergence between social and private optima that is at the heart of our account of land and growth.

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6. Appendix

6.1 Technical assumptions on utility and production in the model without land.

The subutility functions $u(.)$ and $\upsilon(.)$ in (1) are increasing, concave, and continuously differentiable on $_+$ and satisfy

$$\lim_{c_1 \to 0} u'(c_1) = +\infty \qquad \lim_{c_2 \to 0} \upsilon'(c_2) = +\infty.$$

We require $x_t \geq 0,\ y_{1t} > 0,\ y_{2t+1} > 0,\ p_t \geq 0,\ w_t \geq 0,\ r_t \geq -1.$ For the production function,

$$F(0) = 0, \lim_{x \to \infty} F(x) = \infty, \lim_{x \to 0} F'(x) = +\infty, \lim_{x \to \infty} F'(x) = 0.$$

6.2 Lemma 1.

Let

$$W(x) = F\left(\frac{x}{1+n}\right) - \frac{x}{1+n} F'\left(\frac{x}{1+n}\right), \quad R(x) = F'\left(\frac{x}{1+n}\right)$$

Then

$$\lim_{x \to \infty} g_1\left(y_1 + \frac{y_2}{R(x)} + W(x),\ R(x)\right) = \infty$$

$$\lim_{x \to \infty} g_1\left(y_1 + \frac{y_2}{R(x)} + W(x),\ R(x)\right) = \infty$$

Proof: the consumer demands are determined by the first-order condition associated with the income constraint:

$$u'(c_1) = R(x)\, \upsilon'(c_2)$$

$$c_1 + \frac{c_2}{R(x)} = y_1 + \frac{y_2}{R(x)} + W(x).$$

When x goes to infinity, $R(x)$ goes to zero. If c_1 were to stay finite (implying that $u'(c_1) \geq \underline{u}$

for some $\underline{u} > 0$), the budget constraint would imply that $c_2 \geq y_2 - R(x)c_1$, which tends to y_2. Hence the limit of c_2 is greater than $y_2 > 0$, so that the limit of $\upsilon'(c_2)$ would be smaller than $\upsilon'(y_2)$ which is finite. But this makes the limit of the right hand side of the first-order condition zero, which is a contradiction.

Similarly, when x goes to zero, $R(x)$ goes to infinity. If c_2 were to stay finite (implying that $\upsilon'(c_2) \geq \underline{\upsilon}$ for some $\underline{\upsilon} > 0$), the budget constraint would imply that $c_1 \geq y_1 > 0$, so that $u'(c_1)$ would be smaller than $u'(y_1)$, again contradicting the first-order condition. #