HYPOTHESIS TESTS ON MEANS AND PROPORTIONS

(1) population mean - 1 sample - known population variance:

\[ Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \]

(2) population mean - 1 sample - unknown population variance:

\[ t = \frac{\bar{X} - \mu}{s / \sqrt{n}} \]

d.o.f. = n - 1

(3) difference in population means - 2 independent samples - known population variances:

\[ Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} \]

(4) difference in population means - 2 independent samples - unknown population variances:

\[ t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} \]

d.o.f. = min(n_1, n_2) - 1

(5) difference in population means - 2 independent samples - unknown population variances that are believed to be equal:

\[ t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{sp^2 (1/n_1 + 1/n_2)}} \]

\[ sp^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2} \]

d.o.f. = n_1 + n_2 - 2
(6) difference in population means - 2 dependent samples:

\[ t = \frac{\bar{D} - \Delta}{s_D / \sqrt{n}} \]

d.o.f. = n - 1

where \( n \) is the number of pairs of observations

(7) population proportion - 1 sample - (at least 100 observations):

\[ Z = \frac{\bar{p} - p}{\sqrt{pq/n}} \]

(8) difference in population proportions - 2 independent samples - (at least 100 observations in each sample):

\[ Z = \frac{\bar{p}_1 - \bar{p}_2 - (p_1 - p_2)}{\sqrt{pq(1/n_1 + 1/n_2)}} \]

\[ p = (X_1 + X_2)/(n_1 + n_2) \]

\[ q = 1 - p \]
CONFIDENCE INTERVALS FOR MEANS AND PROPORTIONS

(1) population mean - known population variance:

\[ \bar{X} - Z \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + Z \frac{\sigma}{\sqrt{n}} \]

(2) population mean - unknown population variance:

\[ \bar{X} - t \frac{s}{\sqrt{n}} < \mu < \bar{X} + t \frac{s}{\sqrt{n}} \]

d.o.f. = n - 1

(3) difference in population means - known population variances:

\[ (\bar{X}_1 - \bar{X}_2) - Z \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + Z \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \]

(4) difference in population means - unknown population variances:

\[ (\bar{X}_1 - \bar{X}_2) - t \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + t \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \]

d.o.f. = \text{min}(n_1, n_2) - 1

(5) difference in population means - unknown population variances that are believed to be equal:

\[ (\bar{X}_1 - \bar{X}_2) - t \sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + t \sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)} \]

\[ s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \]

d.o.f. = n_1 + n_2 - 2
(6) population proportion (at least 100 observations):

\[
\hat{p} - Z\sqrt{\frac{pq}{n}} < p < \hat{p} + Z\sqrt{\frac{pq}{n}}
\]

\[
q = 1 - \hat{p}
\]

(7) difference in population proportions (at least 100 observations in each sample):

\[
(\hat{p}_1 - \hat{p}_2) - Z\sqrt{\frac{\hat{p}_1q_1}{n_1} + \frac{\hat{p}_2q_2}{n_2}} < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + Z\sqrt{\frac{\hat{p}_1q_1}{n_1} + \frac{\hat{p}_2q_2}{n_2}}
\]

\[
q = 1 - \hat{p}
\]