Name:______________________________

Princeton University

ORF 363/COS 323
Midterm Exam, Fall 2018

October 25, 2018, from 1:30 pm to 2:50 pm

Instructor:
A.A. Ahmadi

AIs:
Duan, Gong, Mirabelli,
Pumir, Tang, Yu

PLEASE DO NOT TURN THIS PAGE. START THE EXAM ONLY WHEN YOU ARE INSTRUCTED TO DO SO.

1. You are allowed a single sheet of paper, double-sided, hand-written or typed.
2. Cell phones should be off or in airplane mode. No other electronic devices are allowed (e.g., calculators, laptops, etc.).
3. All questions should be answered in the booklet. Please write out and sign the following pledge on the booklet "I pledge my honor that I have not violated the honor code during this examination."
4. Make sure you write your name on the first page of these questions and return the questions to us at the end of the exam. Please don't forget to write your name on the booklet as well.
5. You are allowed to cite results proved in lecture or lecture notes without proof.

Each question has 20 points. You need to justify your answers to receive full credit.

Problem 1: Find all the local minimizers, local maximizers, global minimizers, and global maximizers of the following function over $\mathbb{R}^2$ (or argue if some do not exist):

$$f(x_1, x_2) = \frac{1}{2}x_1^2 + 4x_1x_2 + \frac{1}{2}x_2^2 - x_2^3.$$

Problem 2: Show that the gradient of a differentiable convex function $f : \mathbb{R}^n \to \mathbb{R}$ must satisfy

$$\big(\nabla f(x) - \nabla f(y)\big)^T (x - y) \geq 0$$

for all $x, y \in \mathbb{R}^n$. Is the same claim true for differentiable quasiconvex functions? Justify.

Problem 3: Suppose $f : \mathbb{R}^n \to \mathbb{R}$ is a norm.

(a) Show that f^2 is convex.

(b) Show by a counterexample that f^2 is not necessarily strictly convex. Justify.

Problem 4: True or False? (Provide a proof or a counterexample.)

(a) The sum of a convex and a strictly convex function is strictly convex.

(b) The sum of a convex and a quasiconvex function is quasiconvex.

Problem 5: Consider a univariate quadratic function $p(x) = \alpha x^2 + bx + c$ with $\alpha > 0$. Denote its global minimum by x^*.

(a) What is the maximum number of iterations (over all possible α, b, c with $\alpha > 0$, and all starting points $x_0 \in \mathbb{R}$) that the Newton method can take to land exactly on x^*? Justify.

(b) What is the maximum number of iterations (over all possible α, b, c with $\alpha > 0$, and all starting intervals $[a_0, b_0]$ with $a_0 < x^* < b_0$) that the bisection method can take for the midpoint of its interval to land exactly on the global minimum of p? Justify.