Name:_______________________________

Princeton University

ORF 363/COS 323
Midterm Exam, Fall 2023

October 12, 2023, from 1:30 pm to 2:50 pm

Instructor:
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AIs:
Beneventano, Chaudhry, Hua, Li, Lok

PLEASE DO NOT TURN THIS PAGE. START THE EXAM ONLY WHEN YOU ARE INSTRUCTED TO DO SO.

1. You are allowed a single sheet of paper, double-sided, hand-written or typed.
2. Cell phones should be off or in airplane mode. No other electronic devices are allowed.
3. All questions should be answered in the booklet. Please write out and sign the following pledge on the booklet "I pledge my honor that I have not violated the honor code during this examination."
4. Make sure you write your name on the first page of these questions and return the questions to us at the end of the exam. Please don't forget to write your name on the booklet as well.
5. You are allowed to cite results proved in lecture or lecture notes without proof.

You need to justify your answers to receive full credit.

Problem 1 (30 pts): Find all the local minimizers, local maximizers, global minimizers, and global maximizers of the following function over $\mathbb{R}^2$ (or argue if some do not exist):

$$f(x_1, x_2) = \frac{1}{4}x_1^4 - x_1 x_2 + \frac{1}{4}x_2^4$$

You can use the fact that this function has at least one globlal minimizer without proof.

Problem 2 (20 pts): Let the function $f : \mathbb{R}^2 \to \mathbb{R}$ be as in the previous problem.

(a) Is f quasiconvex? Why or why not?

(b) What is the smallest value of α for which the function $f(x_1, x_2) + \alpha(x_1^2 + x_2^2)$ is convex?

Problem 3 (50 pts): For two convex functions $f, g : \mathbb{R}^n \to \mathbb{R}$, consider the optimization problem

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & f(x) \\ \text{s.t.} \quad & g(x) \leq 0, \end{aligned} \tag{1}$$

whose optimal value is known to be equal to zero.

(a) Given an example of f and g where problem (1) has no optimal solutions.

(b) Given an example of f and g where problem (1) has a unique optimal solution even though f is not strictly convex.

(c) Argue why it is impossible for problem (1) to have exactly two optimal solutions.

(d) Write down a single optimization problem whose optimal value allows you to decide if problem (1) has a unique optimal solution. Is your optimization problem convex? Why or why not?

(e) Describe an algorithm that involves examining the optimal values of at most $2n$ *convex optimization problems* and that allows you to decide if problem (1) has a unique optimal solution.