Princeton University

ORF 523

Midterm Exam, Spring 2024

March 7, 2024, 1:30PM - 2:50PM EST.

Instructor:
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AIs:
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Please read the exam rules below before you start.

1. Please write your names on the exam booklet and on the exam sheet. Please return both items to us once the exam is over.
2. All questions should be answered in the booklet. Please write out and sign the following pledge on the booklet: "I pledge my honor that I have not violated the honor code during this examination."
3. You are allowed a single sheet of A4 paper, double sided, hand-written or typed.
4. No electronic devices are allowed (e.g., cell phones, calculators, laptops, etc.), except for checking the time.
5. You can cite results proven in lecture or on problem sets without proof.
6. Good luck!

You need to justify your arguments (correct proofs or counterexamples) to receive full credit.

Problem 1: Local minima of quasiconvex functions
Let $f : \mathbb{R}^n \mapsto \mathbb{R}$ be a quasiconvex function.

(a) Must every local minimum of f be a global minimum?

(b) Must every strict local minimum of f be a global minimum?

Problem 2: Distance between sublevel sets
Let $f, g : \mathbb{R}^n \mapsto \mathbb{R}$ be two continuous functions with non-empty and non-intersecting zero sublevel sets. Consider the problem of finding the distance between their zero sublevel sets:

$$\begin{aligned} \min_{x,y \in \mathbb{R}^n} \quad & \|x - y\| \\ \text{s.t.} \quad & f(x) \leq 0 \\ & g(y) \leq 0. \end{aligned}$$

In the following situations, does the problem above necessarily have an optimal solution? If so, is the optimal solution necessarily unique?

(a) f and g are strictly convex.

(b) f is convex and coercive and g is convex.

(c) f is strictly convex and coercive and g is quasiconvex.

Problem 3: Optimality condition over a polyhedron
Let $f : \mathbb{R}^n \mapsto \mathbb{R}$ be a convex function and consider the optimization problem

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & f(x) \\ \text{s.t.} \quad & Ax \leq b, \end{aligned}$$

where $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$. Show that a feasible point $\bar{x}$ is optimal to this problem if and only if there exists a vector $\mu \in \mathbb{R}^m$ such that

$$\nabla f(\bar{x}) = -A^T \mu, \quad (A\bar{x} - b)^T \mu = 0, \quad \mu \geq 0.$$