

ECO 519. Spring 2006

Homework # 4 (due: May 12th)

1. Let $X \in \mathbb{R}^L$ be continuously distributed random vector. Consider the usual kernel-weighted density estimator

$$\hat{f}(x) = \frac{1}{Nh^L} \sum_{i=1}^N K\left(\frac{X_i - x}{h}\right), \text{ and define } \hat{F}(x) = \int_{-\infty}^x \hat{f}(v) dv.$$

- (a) Provide conditions under which $\hat{F}(x) \xrightarrow{p} F(x) = \Pr(X \leq x)$ —make whatever assumptions you think are necessary—.
- (b) Can you find conditions under which there exists a function $\varphi(N)$ of the sample size such that $\varphi(N)(\hat{F}(x) - F(x)) \xrightarrow{d} \mathcal{N}(0, V)$? If so, what would $\varphi(N)$ (the normalizing scale) and V (the asymptotic variance) be? In the case $L = 1$ (i.e., X one-dimensional) compare your answers to the asymptotic properties of the empirical cdf $\tilde{F}(x) = \frac{1}{N} \sum_{i=1}^N \mathbb{1}\{X_i \leq x\}$.
2. Take $Y \in \mathbb{R}$ and $X \in \mathbb{R}^L$, assume that X can be partitioned as $X = (X_d, X_c)$, where all the elements in $X_d \in \mathbb{R}^{L_d}$ are discrete random variables and all the elements in $X_c \in \mathbb{R}^{L_c}$ are continuous random variables. Take $y \in \mathbb{R}$. Construct an estimator for $\Pr(Y \leq y | X = x)$, provide conditions under which your proposed estimator is consistent as well as conditions under which there exists a normalizing function $\varphi(N)$ such that $\varphi(N)(\widehat{\Pr}(Y \leq y | X = x) - \Pr(Y \leq y | X = x)) \xrightarrow{d} \mathcal{N}(0, \Sigma)$. Find Σ .
3. Consider a discrete-choice model in which $\Pr(Y = 1 | X) = F(X'\beta)$, where F is unknown and $X \in \mathbb{R}^L$ is continuously distributed. Suppose β has all the appropriate normalizations needed for identification (i.e., no intercept and the coefficient of an element in X is normalized to one). Take $x \in \mathbb{S}(X)$ —an element in the support of X —. Suppose that we choose an arbitrary functional form $\bar{F}(\cdot)$ (for example, logistic or normal) and for this “impostor” $\bar{F}(\cdot)$, we estimate β by solving:

$$\hat{\beta}(x) = \operatorname{argmax}_{\beta} \sum_{i=1}^N K\left(\frac{X_i - x}{h}\right) \left[Y_i \log(\bar{F}(X_i'\beta)) + (1 - Y_i) \log(1 - (\bar{F}(X_i'\beta))) \right]$$

Characterize the properties of $\widehat{\beta}(x)$. Let β_0 denote the true parameter value. Can you find conditions under which $\widehat{\beta}(x) \xrightarrow{p} \beta_0$? Can you find conditions under which $\varphi(N)(\widehat{\beta}(x) - \beta_0)$ is asymptotically normal? for some scaling function $\varphi(N)$?