

Statistical mechanics and the asymmetry of counterfactual dependence*

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Abstract

In “Counterfactual dependence and time’s arrow” ([5]), D. Lewis defends an analysis of counterfactuals intended to yield the *asymmetry of counterfactual dependence*: that later affairs depend counterfactually on earlier ones, and not the other way around. I argue that careful attention to the dynamical properties of thermodynamically irreversible processes shows that in many ordinary cases, Lewis’s analysis fails to yield this asymmetry. Furthermore, the analysis fails in an instructive way: one that teaches us something about the connection between the asymmetry of overdetermination and the asymmetry of entropy.

1

The *asymmetry of counterfactual dependence* is (roughly) that later affairs depend counterfactually on earlier ones, and not the other way around. This asymmetry seems to be a feature of our counterfactual talk (at least in many contexts), which is reason enough to seek an analysis of counterfactuals that reproduces it. There are other reasons. Many analyses of causation rely on counterfactuals that exhibit such an asymmetry. Causal decision theory (as formulated, for example, in [4]) does, too. And one might invoke the asymmetry of counterfactual dependence to explain the

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asymmetry of *openness*: our feeling that while the one true past is fixed, many different alternative futures lie before us. ([5]:38)

In “Counterfactual dependence and time’s arrow” ([5]), D. Lewis defends an analysis of counterfactuals intended to yield this asymmetry. Lewis is loath to rule out backward causation and future-to-past counterfactual dependence a priori. So his analysis doesn’t have any time asymmetry built in. Instead, it is designed to yield the desired asymmetry when combined with a contingent feature of the world he calls the *asymmetry of overdetermination*.

This paper applies some reasoning from statistical mechanics to Lewis’s analysis. It turns out that in many cases that involve thermodynamically irreversible processes, Lewis’s analysis fails. Furthermore, the analysis fails in an instructive way: one that teaches us something about the connection between the asymmetry of overdetermination and the asymmetry of entropy.

2

For present purposes we can take Lewis’s analysis of counterfactuals to be the following:¹

The counterfactual “If A were true, then C would be true” is true if and only if C is true at the A -world that is most similar to the actual world. (An A -world is a world at which A is true.)

To make the discussion concrete, focus on an example. At 8:00, Greta cracked open an egg onto a hot frying pan. Are the following counterfactuals true?

- (1) If Greta hadn’t cracked the egg onto the pan, then at 8:05 there wouldn’t have been a cooked egg on the pan.

¹The analysis given isn’t Lewis’s analysis exactly (Lewis complicates matters to deal with the case in which there is no most similar A -world), but the added complications don’t matter for present purposes. So I’ll use Stalnaker’s (simpler) analysis. [5] gives some reasons outside the scope of the present discussion for preferring Lewis’s analysis.

- (2) If Gretta hadn't cracked the egg onto the pan, then at 7:55 she wouldn't have taken an egg out of her refrigerator.

To answer, we must first ask: of the *no-crack worlds* (worlds in which Gretta doesn't crack the egg onto the pan), which is *closest* (i.e., which is most similar to the actual world)? In order for the analysis to yield the asymmetry of counterfactual dependence for this choice of antecedent, it has to turn out that the closest no-crack world is one in which

- (A) history before 8:00 is almost exactly like actual history before 8:00; and
(B) history after 8:00 differs significantly from actual history after 8:00.

If the closest no-crack world meets these conditions, then counterfactuals such as (1)—ones describing how matters after 8:00 would be different if matters at 8:00 were different—will often turn out true, but counterfactuals such as (2)—ones describing how matters *before* 8:00 would be different if matters at 8:00 were different—will almost never turn out true.

So the crucial question is: Does the closest no-crack world meet conditions (A) and (B)?

3

Lewis stipulates that the following criteria² determine how similar a given world is to the actual world:

- (I) "It is of the first importance to avoid big, widespread, diverse violations of [actual] law."
(II) "It is of the second importance to maximize the spatio-temporal region throughout which perfect match of particular fact [with the actual world] prevails." [5]

²I list only the first two members of Lewis's list of criteria since the other two play no role in the following discussion.

Let us follow Lewis in assuming that the laws of nature are *deterministic*, in order to explore how the asymmetry of counterfactual dependence might arise even under such laws. In other words, let us assume that the state of the world at one time,³ together with the laws, determines the state of the world at all other times.

To see how criteria (I) and (II) work under deterministic laws, imagine that you are a god looking down at all of actual history. Your job is to perform the minimal modification that results in a world in which Gretta does not crack the egg.

One strategy is to make modifications so that in the resulting world, (1) Gretta doesn't crack her egg, and (2) no actual laws are violated. Let w_1 be the resulting world. Since w_1 differs from the actual world at 8:00, and since no laws are violated at w_1 , it follows from the assumption of determinism that w_1 differs from the actual world at all times.

Another strategy is to introduce a tiny *miracle* (violation of actual law) shortly before 8:00. The strategy is to leave everything before the miracle untouched but to have the state just after the miracle evolve (via the laws) into a future in which Gretta doesn't crack her egg. (Perhaps the miracle is that a few extra neurons fire in Gretta's brain, getting her to put the egg back in her refrigerator rather than crack it.) Let w_2 be the resulting world.

How do these first two strategies compare? w_1 and w_2 are on a par as far as (I) goes: neither contains big, diverse violations of actual law. But w_2 beats w_1 as regards (II): while no spatio-temporal region of w_1 matches the actual world, the whole region before the miracle in w_2 matches the actual world.

A third strategy is to introduce a miracle shortly *after* 8:00. The strategy is to leave everything after the miracle untouched but to have the state just before the miracle evolve backwards (via the laws) into a past in which Gretta doesn't crack her egg. Let w_3 be the resulting world. Whether this third strategy is promising

³By a "state" here, I don't mean an instantaneous state. Instead I mean what D. Albert calls a *dynamical condition*—an instantaneous state along with the rates of change of certain dynamical quantities (such as particle position). I owe this point to [2].

depends on how big of a miracle is required.

Lewis thinks that a very big, diverse, widespread miracle is required to implement the third strategy. Here's his idea. Suppose that the miracle in w_3 occurs at 8:05. Then w_3 matches the actual world perfectly after 8:05. In the actual world after 8:05, there are many *traces* of Gretta's having cracked an egg: Light waves streak away from her house bear images of her cracking an egg, Gretta has memories of cracking an egg, there are bits of cooked egg stuck to the pan, and so on. We may even suppose that Gretta's voyeuristic neighbor videotaped the egg cracking. So after 8:05, w_3 also contains all of those varied traces.

That's what w_3 looks like after 8:05. What about before 8:05? w_3 is a world in which Gretta *doesn't* crack the egg. So in w_3 right before 8:05, there *aren't* any traces of her having cracked the egg. (In §7 we'll see that the argument commits a crucial error at this step.) Yet we just saw that in w_3 right after 8:05 there are *tons* of (seeming⁴) traces of her having cracked the egg. So the miracle in w_3 has to take care of *making* all of those (misleading) traces, and that requires doctoring the light waves, Gretta's memories, the bits of egg stuck to the pan, the neighbor's videotape, and so on. That's enough doctoring to require a big, widespread, diverse miracle.

If all of this is right, then the second strategy (ensuring that Gretta doesn't crack the egg by putting in a miracle before 8:00) has a giant advantage over the third strategy (ensuring that Gretta doesn't crack the egg by putting in a miracle after 8:00). The purported advantage is an instance of what Lewis calls the *asymmetry of miracles*:

Ensuring that Gretta doesn't crack the egg by putting in a miracle *before* 8:00 requires only a *tiny* miracle, but ensuring that Gretta doesn't crack the egg by putting in a miracle *after* 8:00 requires a *huge* miracle.

⁴For convenience I use a non-factive sense of "trace". That is, I use "trace" in such a way that from the fact that there are traces of an explosion it does *not* follow that there was an explosion. I use "memory" similarly. Nothing is at stake here; if you are worried, feel free to read "trace" and "memory" as "q-trace" and "q-memory" respectively.

If there is such an asymmetry, then w_2 counts as closer than w_3 because w_2 contains only a tiny miracle while w_3 contains a huge one. Granting for the sake of argument that there are no other likely candidates, it follows that the closest no-crack world is a world such as w_2 —one whose past perfectly matches the actual world's past up until shortly before 8:00.

Recall that this is just the result Lewis needs in order for his analysis to yield the asymmetry of counterfactual dependence. For then it will turn out that if Gretta hadn't cracked the egg, history after 8:00 would have been different (potentially very different) than it actually is. And it will turn out that if Gretta hadn't cracked the egg, history before 8:00 would have been exactly the same as it actually is, except for a small transition period immediately preceding 8:00.

Note that this whole account rests on the asymmetry of miracles. If the boxed statement above is false—if somehow the third strategy can be implemented with a small miracle—then there is no reason to think that w_2 is closer than w_3 , and hence no reason to think that Lewis's analysis yields the asymmetry of counterfactual dependence.

The boxed statement above *is* false.

The third strategy *can* be implemented with a small miracle.

It will take a little statistical mechanics to see why.

4

4.1

To keep things simple, pretend that the laws of nature are the laws of Newtonian mechanics.⁵ Then to specify the state of the world at a time it is sufficient to specify

⁵The discussion remains in relevant respects the same if we consider more sophisticated (deterministic) laws. When such laws are in play, a more complicated transformation plays the role that velocity-reversal plays in the upcoming discussion.

the positions and momenta of all of the particles that exist at that time. The set of physically possible states is called *phase space*. A specification of how the particles move over time corresponds to a trajectory through phase space. (Each point on the trajectory corresponds to the state the system is in at a particular time.)

Let s_0 be the state of the world at 8:00—a state in which Gretta is about to crack the egg into the pan. Over the course of five minutes, s_0 evolves into s_1 , a state in which the egg is sitting on the pan, cooked.

The s_0 -to- s_1 process The egg oozes out of the cracked shell and drops down towards the pan, where it splats on the pan, making a noise, slightly heating up the surrounding air, and setting up some vibrations in the pan. Then the egg cooks by absorbing heat from the pan.

So: starting with s_0 and running the laws forwards for five minutes results in s_1 . We can also look at things in the other temporal direction: starting with s_1 and running the laws *backwards* for five minutes results in s_0 .

The aim of this section is to show that the process that gets from s_1 to s_0 by running the laws backwards is extremely sensitive to certain small changes in s_1 .

We've assumed that the laws are deterministic, and so assumed that any given state has a unique lawful past just as it has a unique lawful future. Nevertheless, it is easier to think about running the laws forward than it is to think about running them backward. So we will investigate the process of running the laws backwards to get from s_1 to s_0 indirectly, by investigating the following closely related process.

Let z_1 be the *velocity-reverse* of s_1 —the result of reversing the velocities of all of the particles in s_1 . Like s_1 , z_1 is a state in which the cooked egg sits on the pan. But z_1 has an unusual future: the particle motions that result from starting with z_1 and running the laws *forwards* are exactly the motions that result from starting with s_1 and running the laws *backwards*. In other words, the five minutes that lawfully follow z_1 involve the egg uncooking and then jumping back up into its shell. The resulting state (at the end of the five minutes) is z_0 , the velocity-reverse of s_0 .

The z_1 -to- z_0 process The cooked egg uncooks itself by giving up heat to the (already very hot) pan. Meanwhile, molecules in the pan start to coordinate to form a pattern of vibration converging on the center of the pan. Air molecules around the room begin to form a series of spherical waves that converge on the pan. Just as the egg finishes uncooking, the coordinated action of these inward-directed air waves and pan vibrations congeals the egg into a round shape and propels it vertically towards the waiting open shell, which then seals around it.

As far as positions of particles go, the process that gets from s_1 to s_0 by running the laws *backwards* is exactly like the process that gets from z_1 to z_0 by running the laws *forwards*. So in order to show that the first process is sensitive to certain small changes in s_1 , it is enough to show that the second process is sensitive to certain corresponding changes in z_1 .

4.2

A process in which a cooked egg sits on a pan and gradually cools is unremarkable. In contrast, a process in which an egg spontaneously uncooks and jumps back up into its shell (such as the z_1 -to- z_0 process) is *amazing*. We would be shocked if such a process were to occur. Yet both processes are perfectly in accord with the (fundamental dynamical) laws.

Let COOKED be the set of states that are exactly like z_1 with respect to macroscopic parameters (such as temperature and pressure distribution). All of the states in COOKED are ones in which a cooked egg sits on a pan; these states differ from each other only in microscopic respects.

Some of the states in COOKED (such as s_1) have futures in which the egg acts in ways that we would consider thermodynamically normal: for example, futures in which the egg just sits there and cools. The rest of the states in COOKED (such as z_1) have futures in which the egg acts in ways that we would consider thermody-

namically abnormal: for example, futures in which the egg uncooks and jumps into the air. Let AB be the set of members of COOKED with abnormal futures.⁶

There is a tradition dating back to Boltzmann of trying to explain, for example, the scarcity of instances of egg-uncooking-and-jumpings by appeal to the scarcity of states in COOKED with abnormal futures. More precisely, the tradition appeals to the following fact:

AB occupies only a tiny part of the phase space volume occupied by COOKED (on a natural way of measuring such volumes⁷).

For present purposes, it doesn't matter whether the fact that AB is so tiny can serve the purposes of the Boltzmannian tradition. What matters is simply that AB is so tiny.⁸ Even more important than the size of AB is its shape. AB does *not* consist of a single compact blob. Instead it consists of a dispersed pattern of miniscule specks and thin strands. z_1 is a member of AB, and so it sits on one of those specks or strands. Since the specks are so miniscule and the strands so thin, almost all of the states near z_1 in phase space are *not* members of AB.⁹

A small change in phase space location (for example, a change that gets from z_1 to one of z_1 's neighbors) might correspond to two types of changes to particles. It might correspond to slight changes in the states of many particles. Or it might correspond to slight changes to the states of just a small, localized bunch of particles. Call this second sort of change a *small-miracle change*. The purpose of the preceding discussion is to make plausible the following empirical claim:

Since so little of the phase-space neighborhood of z_1 is within AB, some small-miracle change of z_1 results in a point outside of AB.

This claim tells us something about the sensitivity of the z_1 -to- z_0 process. While z_1 itself has an *abnormal* future (one in which the egg uncooks and jumps back into its

⁶Here my terminology follows the terminology in [1].

⁷For the details on how to measure the volumes of regions of phase space, see [3] or [9].

⁸See, for example, [7], [8], [9].

⁹D. Albert puts this fact to different use in [1].

shell), most of the states *near* z_1 have *normal* futures—ones in which the egg just sits on the pan, cooling. And some of these neighboring states differ from z_1 merely by a small-miracle change. We already knew that starting with z_1 and running the laws forwards yields a process in which the egg uncooks and jumps back into its shell. What we just learned is that starting with z_1 , *making the right sort of small-miracle change*, and *then* running the laws forwards yields a very different process—one in which the egg just sits on the pan, and never jumps back into its shell.

4.3

It is worth making vivid the manner in which such a small difference in initial state can lead to such dramatically different processes. To do so we'll compare the z_1 -to- z_0 process with a modified process whose starting state differs from z_1 by a small-miracle change. For concreteness, suppose that the starting state of the modified process can be gotten from z_1 by slightly changing the positions of a small bunch of molecules in the pan. Suppose also that the starting state of the modified process has a normal future.

The two processes start out much the same: in both, a cooked egg begins to uncook itself and air molecules engage in complicated preliminary motions that in the z_1 -to- z_0 process will lead to them forming converging spherical waves. But there is a tiny difference between the processes in the motion of a few molecules of the pan.

In the modified process, the pan molecules whose positions were changed bump into neighboring molecules, making the trajectories of those neighbors differ from the trajectories of their counterparts in the z_1 -to- z_0 process. The complicated patterns that in the z_1 -to- z_0 process lead to the formation of inwardly directed vibrational waves are, in the modified process, disrupted by these changed trajectories. The disruption spreads: In the modified process, air molecules bump into the surface of the pan in slightly different ways than they do in the z_1 -to- z_0 process, making

them move in ways that increasingly differ from the ways their counterparts move in the z_1 -to- z_0 process. These disrupted air molecules alter the trajectories of their neighbors, messing up the coordinated motion needed to form inwardly directed air waves.

The upshot is that in the modified process, the inwardly directed air waves and pan vibrations never form. So while in the z_1 -to- z_0 process the uncooked egg ends up being propelled back up into its shell, in the modified process the egg just sits on the pan.

4.4

The whole point of investigating the process that gets from z_1 to z_0 by running the laws forwards is to shed light on the process that gets from s_1 to s_0 by running the laws backwards. The main lesson—that the z_1 -to- z_0 process is very sensitive to certain small changes in z_1 —leads immediately to a corresponding lesson about the s_1 -to- s_0 process.

Suppose that a small-miracle change gets from z_1 to z'_1 , a state with a future in which the egg just sits on the pan. Then a corresponding change (namely, the spatial reflection of that change) gets from s_1 to s'_1 , a state with a *past* in which the egg just sits on the pan. In other words, the past history of s'_1 is one in which the egg was never cracked onto the pan.

5

Go back to being a god looking at all of actual history. We know that you can guarantee that Gretta doesn't crack the egg by inserting a small miracle before 8:00, and then evolving that modified state forward according to the laws. The crucial question is: can you guarantee that Gretta doesn't crack the egg by inserting a small miracle *after* 8:00, and then evolving that modified state backwards according to the

laws? At the end of §2, we saw that if the answer is yes, there is no reason to believe that Lewis's analysis yields the asymmetry of counterfactual dependence.

But because the s_1 -to- s_0 process is so sensitive to small changes in s_1 , the answer is yes. Take the actual state of the world at 8:05. Modify it by appropriately changing the positions of a few molecules in the pan as was illustrated in the previous section (i.e., make the small-miracle change that gets from s_1 to s'_1). Now evolve this changed state backwards according to the laws. The result is a past history in which the egg never fell onto the pan, and hence is a past history in which Gretta never cracked the egg onto the pan.

Therefore, in this case there *is* no asymmetry of miracles, and hence in this case Lewis's analysis fails to yield the asymmetry of counterfactual dependence.

6

The history of the actual world, from a time-reversed point of view, is quite amazing. Eggs uncook, congeal themselves together and get propelled up into waiting shells. Coordinated waves in the earth conspire to eject meteorites out into space. Air and ground waves push rubble up the sides of a mountain in time-reversed landslides. The processes that look so amazing in reverse are the so called *thermodynamically irreversible* processes—processes that are associated with increases in entropy.

In §4 we saw that the egg-uncooking-and-jumping process is *fragile*. If you are watching the history of the world backwards and see an egg starting to uncook, all it takes is a small miracle to mess up the finely coordinated action required for the egg to completely uncook and be propelled upwards. But the point is general: many thermodynamically irreversible processes are fragile in this way.

Every thermodynamically irreversible process is sensitive to changes in its final conditions that correspond to small changes in phase space location. Whether an irreversible process is sensitive to a change in its final condition that corresponds to a *small-miracle* change depends on the degree of coupling between the parts of

the system that undergoes the process. Make a small-miracle change to the end state of a process and run the laws backwards. Certainly the change messes up the coordinated movement of the process in the near neighborhood of the change. If the parts of the system are strongly coupled, then the region “infected” by the change (i.e., the region containing particles whose trajectories are greatly altered from what they would have been without the change) will grow rapidly.¹⁰

Many ordinary thermodynamically irreversible processes are strongly coupled in this way, and so are sensitive to small-miracle changes in their final conditions. (Examples include the processes of milk mixing into coffee, glasses shattering, water boiling, and balloons popping.) So the violation of the asymmetry of miracles described in §5 is no fluke—similar violations arise in many other mundane cases.

7

The world that we’ve used to make trouble for Lewis’s analysis is w_3 , a world gotten from the actual world by inserting an appropriate small miracle at 8:05 and evolving the changed state backwards according to the laws. w_3 makes trouble for Lewis’s analysis because it is a no-crack world that (1) contains only a tiny violation of actual law and (2) matches the actual world perfectly after 8:05.

In §3 I gave some (faulty) reasoning concluding that there is no such world. We’re now in a position to see how that reasoning goes wrong. Here is the reasoning:

1. At 8:05 the actual world contains traces of Gretta’s having cracked the egg.
2. So immediately after 8:05 w_3 also contains those traces.
3. But since w_3 is a world in which Gretta doesn’t crack the egg, immediately before 8:05 w_3 does *not* contain those traces.
4. Therefore a large trace-manufacturing miracle occurs in w_3 at 8:05.

¹⁰I borrow “infected region” terminology from T. Maudlin, who puts it to different use in [6].

The error is in step 3. Though Gretta doesn't crack the egg in w_3 , at 8:05 w_3 is filled with traces as though she had done so.

Let's look at how those extremely misleading traces are formed in w_3 .

We'll start by seeing what w_3 looks like from a future-to-past perspective. Then we'll take the time-reverse to see what w_3 looks like from an ordinary (past-to-future) perspective.

Start with the state of the actual world at 8:05. The cooked egg sits on the pan. Light waves from Gretta's window bear images of an egg being cracked.

Apply an appropriate small-miracle change by altering the positions of a small bunch of molecules of the pan. The resulting state is the state of w_3 immediately before 8:05. Now run time backwards. We already saw that the egg just sits on the pan (since the miracle interferes with the formation of the coordinated waves required to congeal the egg together and propel it upward). The history of the egg as we continue to run time backwards looks increasingly like the (past-to-future) history of an ordinary egg. The egg cools and eventually rots.

What does the history of the trace-bearing light look like in reverse? As the time approaches 8:00, the light streaks into Gretta's kitchen, heading for the place in the air where, in the actual world, Gretta cracked the egg. In the actual world, the light is perfectly deflected into light bulbs by the surfaces of the egg and of Gretta's skin. In w_3 , however, Gretta and the egg aren't in the path of the light when it reaches the kitchen. So it travels on and is scattered and absorbed haphazardly by walls and furniture.

More generally, the situation is as follows. At 8:05, w_3 matches the actual world except for a tiny infected region (the region in which the miracle occurs). As we run time backwards, the infected region rapidly expands. Within that region, what we see looks (from our backwards-in-time vantage point) thermodynamically typical. (For example, eggs get more and more rotten as time gets earlier and earlier.) Outside of that region, events look thermodynamically *reversed*. (For example, eggs get less and less rotten as time gets earlier and earlier.)

Now look at w_3 in the ordinary (past-to-future) direction. In the distant past, the infected region is huge. Within that region are events that look thermodynamically reversed. Events outside the infected region look thermodynamically typical.

At 8:00 the infected region includes a cooked egg sitting on a pan in Gretta's kitchen. Over the next five minutes, the egg warms up, and by 8:05 is in a state that would suggest that it had been recently cracked on that pan and cooked. This suggestion is entirely misleading. The egg was never raw and was never cracked onto the pan. Long ago, the egg formed onto the pan as an extremely rotten slime, and got into its 8:05 state by a process of reverse-rotting.

What about the trace-bearing light? Shortly before 8:00, the walls and furniture of Gretta's kitchen began emitting light that deflected around the room. The light deflected in a coordinated fashion so that it traveled out of Gretta's window bearing (highly misleading) images of Gretta cracking an egg.

All of the traces in w_3 of Gretta having cracked the egg were formed in such ways. Each such trace was formed by an anti-thermodynamic process—by what would seem to be a finely tuned conspiracy in the motions of large numbers of microscopic particles.

8

At w_3 at 8:05, all signs point to Gretta having cracked the egg. The cooked egg sits on the pan. Light waves bear images of Gretta cracking the egg. Gretta remembers having cracked the egg onto the pan. Offhand it might seem as though the only lawful way for all of those traces to be formed is by Gretta's cracking the egg at 8:00. More generally, one might agree with Lewis that there is an asymmetry of *overdetermination*—that “very many simultaneous disjoint combinations of traces of any present fact are determinants thereof; there is no lawful way for the combination to have come about in the absence of the fact.” ([5]:50)

But that's all wrong. It only takes a small miracle to make the difference between

the actual world (a world with many veridical traces of the egg-cracking) and w_3 (a world in which those same traces are all highly misleading). In general, the existence of traces seemingly of an event (together with the laws, and together with the absence of evidence that those traces have been faked) falls far short of entailing that the event occurred.¹¹

References

- [1] David Z Albert. Quantum mechanics and the approach to thermodynamic equilibrium. *British Journal for the Philosophy of Science*, 45:669–677, 1994.
- [2] David Z Albert. Time and chance. Manuscript, 1999.
- [3] Kerson Huang. *Statistical mechanics*. Wiley, 1928.
- [4] David Lewis. Causal decision theory. *Australasian Journal of Philosophy*, 59:5–30, 1981.
- [5] David Lewis. Counterfactual dependence and time’s arrow. In *Philosophical Papers*. Oxford University Press, Oxford, 1986.
- [6] Tim Maudlin. A modest proposal concerning laws, counterfactuals, and explanations. Manuscript, 1998.
- [7] Roger Penrose. *The emperor’s new mind*. Oxford University Press, Oxford, 1989.
- [8] Huw Price. *Time’s arrow and Archimedes’ point*. Oxford University Press, New York, 1996.
- [9] Lawrence Sklar. *Physics and chance : philosophical issues in the foundations of statistical mechanics*. Cambridge University Press, New York, 1993.

¹¹D. Albert makes this observation in a slightly different context in [2].

- [10] Tommaso Toffoli and Norman Margolus. *Cellular automata machines*. The MIT Press, Cambridge, MA, 1991.