Stochastic Filtering via Reweighted-\( l_1 \)
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Introdution
Sparsity Models have proven indispensable in estimating single signals in measurement poor applications [1]. However, many applications require recovery of correlated sparse signals (e.g. [2]);

- Recover: a set of signals \( x_k \in \mathbb{R}^N \) where \( k \in K \subseteq \mathbb{Z}^D \)
- Given: measurements \( y_k \in \mathbb{R}^M \), \( M < N \); 
  \[ y_k = \Phi \cdot x_k + \epsilon_k \]
  \( \Phi \): measurement matrix \( \epsilon_k \): noise

Reweighted-\( l_1 \) Framework

We propose stochastic filtering using second order moments native to sparse signal estimation. Based on reweighted-\( l_1 \) optimization [3], using weights to propagate information, e.g. [4].

Reweighted-\( l_1 \) optimization can be viewed as a second order model for sparse signals.

- Gaussian Measurements:
  \[ p(y_k | \phi) = \mathcal{N}(\Phi \cdot \phi_k, \sigma_k^2 I) \]
- Laplacian Conditional:
  \[ p(\phi_k | y_k) = \mathcal{L}(\|y_k - \Phi \cdot \phi_k\|^2_2 + \gamma \|\phi_k\|_1) \]
- Gamma Hyperprior on Variance:
  \[ p(\sigma_k^2) = \mathcal{G}(\|\gamma_k\|^2_2) \epsilon_k^2 \]
- Marginal Prior
  \[ p(\phi_k) = \mathcal{L}(\|y_k - \Phi \cdot \phi_k\|^2_2 + \gamma \|\phi_k\|_1) \]
- Variance parameters can be correlated via \( \theta_k \): 
  \[ \theta_k = \xi (\sigma_k \epsilon_k) + \eta \]
- Expectation Maximization approach to optimization
  Algorithm: iterate until convergence

M-step:
  \[ G_k^{(t)} = \arg \min_{\theta_k} \| y_k - \Phi \cdot \theta_k \|^2_2 + \frac{\epsilon_k^2}{\gamma_k} \sum \| \phi_k^{(t)} \|_1 \]

E-step:
  \[ \lambda_k^{(t+1)} = \sum \Phi \cdot \theta_k^{(t)} + \epsilon_k \]

Propagating information in the second order moments allows for more robustness to model mismatch

Special Cases: RWL1-DF and RWL1-SF

Reweighted-DF Spatial Filtering (RWL1-DF):
- Spatially located correleated signals (\( K \subseteq \mathbb{Z} \))
- Example application: Hyperspectral imagery (HSI)
- Spatial correlations defined by a kernel \( \mathbb{K} \)
- Spatially Dependent Variance:
  \[ \theta_k[i] = \sum_{j \in K} \| y_k[i] - \Phi \cdot \theta_k[j] \|^2_2 \epsilon_j + \eta \]
- Exploit correlation for spatial filtering

Algorithm: E-step becomes

\[ \lambda_k^{(t+1)} = \sum_{j \in K} \Phi \cdot \theta_k[j] \]

Pixel at \( (i,j) \)
Kernel Centered at \( (i,j) \)

RWL1-DF: Compressive Video Recovery

Foreman Video Recovery:

\[ \text{RWL1-DF} \]

\[ \text{BPDN} \]

\[ \text{RWL1} \]

\[ \text{DCS-AMP} \]

\[ \text{BDPN} \]

\[ \text{BPDN-DF} \]

\[ \text{RWL1-DF} \]

The Foreman video sequence recovered from randomly selected noiselet measurements \( N = 0.25N \) [7]. Comparisons shown to BPDN, BPDN-DF [6], RWL1 [3], DCS-AMP [8], and modCS [9].

Conclusions
Second order models allow for joint recovery of correlated, sparse signals.

- Improved estimation quality (RMSE)
- Robustness to innovations' statistics
- Computational complexity no more than a number of BPDN solutions

Future directions:
- Understand theoretical limits of the algorithm

References:

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