Self-folding origami and robots
Shrinky-Dinks

Shrinky-Dinks are sheets made of optically transparent, pre-strained polystyrene that shrink if heated to the glass transition temperature.

https://www.youtube.com/watch?v=m1mCoQFnOGU
Shrinky-Dinks

Shrinky-Dinks are sheets made of optically transparent, pre-strained polystyrene that shrink if heated to the glass transition temperature. Localized heating and shrinking of Shrinky-Dinks can be achieved with patterning of black ink that absorb light.

Folding angle can be controlled with the width of ink and with the exposure time of light.
Shrinky-Dinks origami

size ~ cm

J. Liu et al., Soft Mater 8, 1764 (2012)
Sequential folding of Shrinky-Dinks origami

Different ink colors have different absorption spectra for red, green and blue light.

blue light activates yellow fold

red light activates blue fold

Differential light absorption for sequential folding.

In contrast to black or grayscale inks, which absorb light indiscriminantly, the yellow and cyan inks show an opposite absorption response to red and blue LEDs.
Sequential folding of Shrinky-Dinks origami

The order of folding corresponds to the amount of absorbed blue light (black > red > walnut)

Note: red ink is thicker than the walnut ink!

Sequential folding of Shrinky-Dinks origami

Converting 2D sheets to 3D objects sequentially using only light
Self-folding robots (in 4 min)

S. Felton et al., Science 345, 644 (2014)
Robot assembly

Chemical etching of copper outside the ink mask

Laser cutting of layers

Gluing of layers

Laser cutting

“Shrinky-Dinks”

installment of electrical components, motors, and batteries

S. Felton et al., Science 345, 644 (2014)
Folding of robot

electric current through patterned copper network locally heats up and shrinks the “Shrinky-Dinks” layer

copper

How can we actuate the assembled robot?

S. Felton et al., Science 345, 644 (2014)
Structures with mechanisms
Structures composed of bars and hinges, which have fewer constraints than degrees of freedom, have specific mechanisms (=modes of deformations)

scissor lift

precise robotic surgeries

changing direction of motion

amplifying/reducing amplitude of motion
Crank slider mechanism converts linear to rotary motion!

Crank slider mechanism in car engines

https://en.wikipedia.org/wiki/Crank_(mechanism)
sequential folding enables locking of the crank arm to the robot structure

rotary motor moves the crank arm, which controls the movement of robot legs via a specific structure mechanism

S. Felton et al., Science 345, 644 (2014)
Helices in plants

How are helices formed?
Differential growth or differential shrinking produces spontaneous curvature

Differential growth (shrinking) of the two layers produces spontaneous curvature

Filaments that are longer than \( L > 2\pi R \) form helices to avoid steric interactions.
Helix

Mathematical description

\[ \vec{r}(s) = \left( r_0 \cos(\frac{s}{\lambda}), r_0 \sin(\frac{s}{\lambda}), \frac{p}{2\pi \lambda} s \right) \]

Set \( \lambda \) to fix the metric in natural parametrization:

\[ \vec{t}(s) = \frac{d\vec{r}}{ds} = \left( -\frac{r_0}{\lambda} \sin(\frac{s}{\lambda}), \frac{r_0}{\lambda} \cos(\frac{s}{\lambda}), \frac{p}{2\pi \lambda} \right) \]

\[ g = \vec{t} \cdot \vec{t} = \frac{r_0^2}{\lambda^2} + \frac{p^2}{4\pi^2 \lambda^2} = 1 \]

\[ \lambda = \sqrt{r_0^2 + \left(\frac{p}{2\pi}\right)^2} \]
### Helix

#### Mathematical description

\[
\vec{r}(s) = \left( r_0 \cos\left(\frac{s}{\lambda}\right), r_0 \sin\left(\frac{s}{\lambda}\right), \frac{p}{2\pi \lambda} s \right)
\]

\[
\lambda = \sqrt{r_0^2 + \left(\frac{p}{2\pi}\right)^2}
\]

#### Tangent and normal vectors

\[
\vec{t}(s) = \frac{d\vec{r}}{ds} = \left( -\frac{r_0}{\lambda} \sin\left(\frac{s}{\lambda}\right), \frac{r_0}{\lambda} \cos\left(\frac{s}{\lambda}\right), \frac{p}{2\pi \lambda} \right)
\]

\[
\vec{n}_1(s) = \left( -\cos\left(\frac{s}{\lambda}\right), -\sin\left(\frac{s}{\lambda}\right), 0 \right)
\]

\[
\vec{n}_2(s) = \left( \frac{p}{2\pi \lambda} \sin\left(\frac{s}{\lambda}\right), -\frac{p}{2\pi \lambda} \cos\left(\frac{s}{\lambda}\right), \frac{r_0}{\lambda} \right)
\]

#### Helix curvatures

\[
\vec{n}_1 \cdot \frac{d^2\vec{r}}{ds^2} = \frac{r_0}{\lambda^2} = \frac{r_0}{r_0^2 + \left(\frac{p}{2\pi}\right)^2} = K
\]

\[
\vec{n}_2 \cdot \frac{d^2\vec{r}}{ds^2} = 0
\]
Cucumber tendrils want to pull themselves up above other plants in order to get more sunlight.

Already studied by Charles Darwin in 1865:

S. J. Gerbode et al., Science 337, 1087 (2012)
Coiling in older tendrils is due to a thin layer of stiff, lignified gelatinous fiber cells, which are also found in wood.
Helical coiling of cucumber tendril

Drying of fiber ribbon increases coiling

Drying of tendril increases coiling

Rehydrating of tendril reduces coiling

During the lignification of g-fiber cells water is expelled, which causes shrinking.

The inside layer is more lignified and therefore shrinks more and is also stiffer than the outside layer.
Coiling of tendrils in opposite directions

Ends of the tendril are fixed and cannot rotate. This constraints the linking number.

\[ \text{Link} = \text{Twist} + \text{Writhe} \]
Twist, Writhe and Linking numbers

Ln = Tw + Wr  
linking number: total number of turns of a particular end

Tw  
twist: number of turns due to twisting the beam

Wr  
writhe: number of crossings when curve is projected on a plane

360°
Twist = -1, Writhe = 0.

360°
Twist = +1, Writhe = 0.

720°
Twist = -2, Writhe = 0.

720°
Twist = +2, Writhe = 0.

Twist = 0, Writhe = -2.

Twist = 0, Writhe = +2.

Toroidal  Pleatonemic  
Toroidal  Pletronemic
Coiling of tendrils in opposite directions

Ends of the tendril are fixed and cannot rotate. This constraints the linking number.

\[
\text{Link} = \text{Twist} + \text{Writhe}
\]

Coiling in the same direction increases Writhe, which needs to be compensated by the twist.

In order to minimize the twisting energy tendrils combine two helical coils of opposite handedness (=opposite Writhe).

Note: there is no bending energy when the curvature of two helices correspond to the spontaneous curvature due to the differential shrinking of fiber.
Overwinding of tendrils coils

Old tendrils overwind when stretched. Rubber model unwinds when stretched.
Overwinding of tendril coils

Preferred curved state

In tendrils the red inner layer is much stiffer than the outside blue layer.

High bending energy cost associated with stretching of the stiff inner layer!

Tendrils try to keep the preferred curvature when stretched!

In rubber models both layers have similar stiffness.

Small bending energy.

S. J. Gerbode et al., Science 337, 1087 (2012)
Overwinding of rubber models with an additional stiff fabric on the inside layers when pulled, adding turns on both sides of the perversion (Fig. 2A, right, and movie S5). Eventually though, under high enough tension the fiber ribbon unwinds, returning to a flat, uncoiled state as expected (movie S5).

Inspired by our observations of asymmetric lignification in fiber ribbons, which suggest that the inner layer is less extensible, we added a relatively inextensible fabric ribbon to the inside of a coiled physical model. To mimic lignified cells that resist compression, we added an incompressible copper wire to the exterior of the helix. The internal fabric ribbon prevents elongation, whereas the external copper wire prevents contraction. Together, these modifications increase...
Overwinding of helix with infinite bending modulus

Mathematical description

\[
\vec{r}(s) = \left( r_0 \cos\left(\frac{s}{\lambda}\right), r_0 \sin\left(\frac{s}{\lambda}\right), \frac{p}{2\pi \lambda} s \right)
\]

\[
\lambda = \sqrt{r_0^2 + \left(\frac{p}{2\pi}\right)^2}
\]

\[
Z = pN = p\left(\frac{L}{2\pi \lambda}\right)
\]

Infinite bending modulus fixes the helix curvature during stretching

\[
K = \frac{r_0}{r_0^2 + \left(\frac{p}{2\pi}\right)^2}
\]

Helix pitch and radius

\[
r_0 = \frac{1}{K} \left(1 - \frac{Z^2}{L^2}\right)
\]

\[
p = \frac{2\pi Z}{KL} \sqrt{1 - \frac{Z^2}{L^2}}
\]

Number of loops

\[
N = \frac{Z}{p} = \frac{KL}{2\pi \sqrt{1 - (Z/L)^2}}
\]

length of the helix backbone

number of loops

diameter

pitch

overwinding of helix backbone

number of loops
Overwinding of helix with infinite bending modulus

Helix pitch and radius

- Radius of helix backbone:
  \[ r_0 = \frac{1}{K} \left( 1 - \frac{Z^2}{L^2} \right) \]

- Pitch of helix:
  \[ p = \frac{2\pi Z}{KL} \sqrt{1 - \frac{Z^2}{L^2}} \]

Number of loops

- Number of loops:
  \[ N = \frac{Z}{p} = \frac{KL}{2\pi \sqrt{1 - (Z/L)^2}} \]

Length of the helix backbone:

- Diameter:
  \[ 2r_0 \]

Number of loops:

- \[ N = \frac{Z}{p} \]

Overwinding:

- \[ 2\pi N/(KL) \]
- \[ pK/(2\pi) \]
Further reading

ON GROWTH AND FORM
The Complete Revised Edition
D'Arcy Wentworth Thompson

Elasticity and Geometry
From Hair Curls to the Non-linear Response of Shells
B. Audoly Y. Pomeau

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