Structural Color and Wrinkled Surfaces
Interference on thin films

$\text{difference between optical path lengths of the two reflected rays}$

$OPD = n_2 (AB + BC) - n_1 AD$

$OPD = 2n_2 d \cos(\theta_2)$

$n_1 < n_2 < n_3 \quad n_1 > n_2 > n_3$

no additional phase difference due to reflections

$OPD = m\lambda$

constructive interference of reflected rays

$OPD = (m + \frac{1}{2})\lambda$

$m = 0, \pm 1, \pm 2, \ldots$

destructive interference of reflected rays

$n_1 > n_2 < n_3 \quad n_1 < n_2 > n_3$

additional $\pi$ phase difference due to reflections

constructive interference of reflected rays

$OPD = (m + \frac{1}{2})\lambda$

destructive interference of reflected rays

$OPD = m\lambda$
Structural colors on periodic structures

Single reflected color on structures with uniform spacing

Incoming light

Reflected light

Transmitted light

<table>
<thead>
<tr>
<th>Wavelength $\lambda$ [nm]</th>
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<tbody>
<tr>
<td>400</td>
</tr>
<tr>
<td>500</td>
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<tr>
<td>600</td>
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<tr>
<td>700</td>
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Reflected color depends on the viewing angle!

Morpho butterfly: 1.7µm

Marble berry: 250nm

Chrysochroa raja beetle: 1µm
Silver and gold structural colors

Many colors reflected on structures with varying spacing

Incoming light

Reflected light

Visible spectrum

Wavelength $\lambda$ [nm]

% Reflectance

Gold

Silver

Disordered layer spacing

Chrysina limbata beetle

Chrysina aurigans beetle

Chirped structure

Thicker

Thinner

Bleak fish

Aspidomorpha tecta

Cyprinus carpio

Ficus macrophylla

Lepidoptus caudatus

Aspidomorpha tecta

Ficus macrophylla

Cyprinus carpio

Lepidoptus caudatus

Anoplognathus parvulus

A. parvulus

C. grayanus

The entire body appears equally coloured from every direction (a property required for camouflage). Considering the body appears coloured from any one direction, whereas in a leaf background, and in the structural colour of cuticles, and hence the reflectors, tended to split).
Bragg scattering on crystal layers

Constructive interference for waves with different wavelengths occurs in different crystal planes!

\[ 2d \sin \theta = m \lambda \]
\[ 2d' \sin \theta' = m \lambda' \]
\[ m = 0, \pm 1, \pm 2, \ldots \]
Scattering on disordered structures

Disordered structures with a characteristic length scale. This length scale determines what light wavelengths are preferentially scattered.

The selectively reflected wavelengths are the same in all directions!

This gives rise to blue colors in these birds.

V. Saranathan et al., J. R. Soc. Interface 9, 2563 (2012)
Scattering on disordered structures

Blue jay

keratin cortex
spongy layer (keratin/air matrix)
black melanin layer

https://academy.allaboutbirds.org/how-birds-make-colorful-feathers/
Dynamic structural colors

Chameleons form a highly derived monophyletic group of lizards (Fig. 1a). In chameleons, these D-iridophores, with larger, flatter and somewhat disorganized guanine crystals, which reflect a spectral range is screened in panther chameleons by reflection on the deep layer of iridophores in chameleons further provide them with improved resistance to variable sunlight exposure. It is noteworthy that D-iridophores might not be associated with passive thermal protection, because extant species of the basal chameleons occupy quite open environments where they are exposed to high temperatures (Northern Madagascar and Yemen, 45% decrease in sunlight absorption caused by swelling of cells in excited chameleon. This change. Dashed white line: optical response in numerical simulations (Fig. 3b), indicates that variation of simulated colour photonic response for each vertex of the irreducible first Brillouin zone (colour map, 1 cm, scale bar, 200 nm). This transformation and evolution in comparison with Fig. 1b. (Supplementary Movie 2) have their blue vertical bars covered by red pigment cells. (4, identifying, in this spectral range, to that of water (Fig. 3a).)

Changes in osmotic concentration lead to the swelling of cells in excited chameleon. This changes the spacing of periodic structure from which the ambient light is reflected. In vivo (Fig. 1b) and during osmotic pressure corresponds to behavioural relaxation; hence, the reverse order (white arrowhead in CIE colour chart) of red to green to blue time course remains to be determined; however, given that iridophores contain organelles in other types of chromatophores, possibly containing organelles in other types of chromatophores, possibly.

Multiplying the sun radiance by manipulation of white skin osmolarity (from 236 to 1,416 mOsm): (some of which generate structural colours & 123, 1416 mOsm)

Reflectivity of a skin layer is a broad-band reflector in the near infrared region, as the spectral functions returned around 90 million years ago.

Undoubtedly, some species of Beroë cucumis. In

Rainbow color waves are produced by the beating of cilia, which change the orientation of periodic structure from which the ambient light is reflected.


https://www.youtube.com/watch?v=Qy90d0XvJIE

Changes in osmotic concentration lead to the swelling of cells in excited chameleon. This changes the spacing of periodic structure from which the ambient light is reflected.
Structural colors of animals and plants appear due to the selective reflection of ambient light on structural features underneath the surface.


V. Saranathan et al., J. R. Soc. Interface 9, 2563 (2012)
Sound from airplanes that are landing and taking off is reflected from artificial barriers into the atmosphere.
Controllable sound filters

In periodic structures sound waves of certain frequencies (within a “band gap”) cannot propagate. The range of “band gap” frequencies depends on material properties, the geometry of structure and the external load.

**undiformed structure**

**deformed structure**

Waveguides in disordered structures

Channels inside structures can be used as guides for waves with wavelengths that are totally reflected from a complete structure!

Note: channels can have arbitrary bends!

W. Man et al., PNAS 110, 15886 (2013)
Waveguides in periodic structures

In periodic structures waves are completely reflected only at certain angles.

Note: channels with certain bends act as waveguides only for those waves that are completely reflected at these angles!
Further reading about structural colors and photonic crystals

http://ab-initio.mit.edu/book/
In 2008, Cao and colleagues identified four dimensionless parameters which may be combined into a single dimensionless curvature parameter given by:

\[
R_{c} = \frac{t}{s_{f}}\sqrt{\frac{E}{C_{0}}} \cos(k_{x}R_{s})
\]

where \( s_{f} \) and \( s \) refer to the curvature of the surface with thickness \( t \) and substrate, respectively.

This journal is comprised of a biomimetic structured materials. Hierarchical, a key advantage for the design of many specialty paper and devices is described here.
Why do we get wrinkled surfaces?

Fingers after being exposed to water for some time

Old apple

Brain

Rising dough


**Compression of stiff thin sheets on liquid and soft elastic substrates**

For small enough compression, the system deformation is focused within a narrow region of the sheet. The folding can appear downward, i.e. compression leads to the formation of a single fold where all of the deformation is concentrated. As the compression is increased, the amplitude of the initial wrinkles also grows. For larger compression, a dramatic change in the morphology is observed: one wrinkle grows in magnitude at the expense of its neighbors leading to a period-doubling instability with the emergence of a subharmonic. The total energy of the system is then also minimized for some intermediate length-scale. This behavior is observed for both liquid and elastic foundations.


**F. Brau et al., Soft Matter 9, 8177 (2013)**
In compressed thin sheets on liquid and soft elastic substrates global buckling is suppressed, because it would result in very large energy cost associated with deformation of the liquid or soft elastic substrate!
Brief intro to mechanics: Young’s modulus

Hooke’s law (small deformations)

\[
\frac{F}{A} = E \frac{\Delta L_z}{L_z}
\]

- normal stress: \( \sigma = \frac{F}{A} \)
- Young’s modulus: \( E \)
- normal strain: \( \epsilon = \frac{\Delta L_z}{L_z} \)

Elastic energy of deformation

\[
U = \frac{1}{2} V E \epsilon^2
\]

- element volume: \( V = L_x L_y L_z \)
Young’s modulus of materials

http://www-materials.eng.cam.ac.uk/mpsite/physics/introduction/
Poisson’s ratio

Typically material shrinks (expands) in the transverse direction of the axial tension (compression)!

\[ \nu = - \frac{\varepsilon_x}{\varepsilon_z} = - \frac{\varepsilon_y}{\varepsilon_z} \]

\[ \epsilon_z = \frac{\sigma_z}{E} \]

normal strains: \[ \epsilon_i = \frac{\Delta L_i}{L_i} \]
Effective negative Poisson’s ratio for structures

Certain structures behave like they have effective negative Poisson’s ratio, even though they are made of materials with positive Poisson’s ratio!
Bulk modulus

undeformed material element

hydrostatic stress

Hooke’s law (small deformations)

\[
\frac{\Delta V}{V} = -\frac{p}{K}
\]

hydrostatic stress: \( p \)

bulk modulus: \( K = \frac{E}{3(1-2\nu)} \)

volumetric strain: \( \frac{\Delta V}{V} \approx 3 \frac{\Delta L}{L} \)

Elastic energy of deformation

\[
U = \frac{1}{2} V K \left( \frac{\Delta V}{V} \right)^2 \approx V E \left( \frac{\Delta L}{L} \right)^2
\]
Shear

undeformed material element

\[ A = L_x L_y \]

Hooke's law
(small deformations)

\[ \frac{F}{A} = G \gamma \]

shear stress:
\[ \tau = F/A \]

shear modulus:
\[ G = \frac{E}{2(1 + \nu)} \]

shear strain:
\[ \gamma = \arctan \left( \frac{\Delta}{L_z} \right) \]
\[ \gamma \approx \Delta/L_z \]

Elastic energy of deformation

\[ U = \frac{1}{2} V G \gamma^2 \sim V E \left( \frac{\Delta}{L_z} \right)^2 \]

Note: shear does not change the volume of material element!

Element volume:
\[ V = L_x L_y L_z \]
Arbitrary deformation of 3D solid element

Arbitrary deformation can be decomposed to the volume change and the shear deformation.

\[
U = U_{\text{bulk}} + U_{\text{shear}}
\]
In plane deformations of thin sheets

undetormed square patch of thin sheet

patch area $A = L^2$

sheet thickness $t$
Young’s modulus $E$
Poisson’s ratio $\nu$

isotropic deformation

$U = \frac{B}{2} \left( \frac{\Delta A}{A} \right)^2 \approx \frac{B}{2} \left( \frac{2\Delta L}{L} \right)^2$

$2D$ bulk modulus

$B = \frac{Et}{2(1 - \nu)}$

shear deformation

$U = \frac{\mu \gamma^2}{2}$

$2D$ shear modulus

$\mu = Gt = \frac{Et}{2(1 + \nu)}$

anisotropic stretching

$U = \frac{B}{2} (\epsilon_1 + \epsilon_2)^2 + \frac{\mu}{2}(\epsilon_1 - \epsilon_2)^2$

$\epsilon_1, \epsilon_2 \ll 1$

(shearing can be interpreted as anisotropic stretching)
Curvature of surfaces

(A) curvature of space curves

\[ \frac{1}{R} = \frac{h''}{(1 + h'^2)^{3/2}} \approx h'' \]

R. Phillips et al., Physical Biology of the Cell

(B) curvature tensor for surfaces

\[ K_{ij} \approx \begin{pmatrix} \frac{\partial^2 h}{\partial x^2} & \frac{\partial^2 h}{\partial x \partial y} \\ \frac{\partial^2 h}{\partial x \partial y} & \frac{\partial^2 h}{\partial y^2} \end{pmatrix} \]

maximal and minimal curvatures (principal curvatures) correspond to the eigenvalues of curvature tensor
Surfaces of various principal curvatures

\[ \frac{1}{R_1} = \frac{1}{R_2} = \frac{1}{r} \]

sphere

\[ \frac{1}{R_1} = \frac{1}{r} \]
\[ \frac{1}{R_2} = 0 \]

cylinder

potato chips = “saddle”

\[ \frac{1}{R_1} > 0 \]
\[ \frac{1}{R_2} < 0 \]
Bending energy cost for thin sheets

undeformed thin sheet
(thickness \( t \))

deformed thin sheet

\[ U = \int dA \left[ \frac{\kappa}{2} \left( \frac{1}{R_1} + \frac{1}{R_2} \right)^2 + \kappa_G \frac{1}{R_1 R_2} \right] \]

\[ U \approx \int dx dy \left[ \frac{\kappa}{2} \left( \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right)^2 + \kappa_G \det \left( \frac{\partial^2 h}{\partial x_i \partial x_j} \right) \right] \quad x_i, x_j \in \{ x, y \} \]

bending rigidity (flexural rigidity) \( \kappa = \frac{Et^3}{12(1 - \nu^2)} \)

Gauss bending rigidity \( \kappa_G = -\frac{Et^3}{12(1 + \nu)} \)
Compression of stiff thin sheets on liquid and soft elastic substrates

Liquid substrate

- 10 µm thin sheet of polyester on water
  - \( \lambda_0 = 1.6 \text{ cm} \)

Elastic substrate

- ~10 µm thin PDMS (stiffer) sheet on PDMS (softer) substrate
  - \( \lambda_0 = 70 \mu\text{m} \)

Before

- Wrinkles grow uniformly across the sheet

After

- Wrinkles grow leading to a period change in the morphology

Compression of stiff thin membranes on liquid substrates

Consider the energy cost for two different scenarios:

1.) thin membrane is compressed (no bending)

2.) thin membrane is wrinkled (no compression) + additional potential energy of liquid
Compression of stiff thin membranes on liquid substrates

compression energy of thin membrane

\[ U_c \sim A \times E_m d \times \epsilon^2 \]

membrane area
\[ A = WL \]

membrane 3D Young’s modulus
\[ E_m \]

strain
\[ \epsilon = \frac{\Delta}{L} \]

liquid density
\[ \rho \]

Note: upon compression the liquid surface also raises, but we will measure the potential energy relative to this new height!
Compression of stiff thin membranes on liquid substrates

**assumed profile**

\[ h(s) = h_0 \cos(2\pi s/\lambda) \]

**projected length assuming that membrane doesn’t stretch**

\[
L - \Delta = \int_0^L ds \sqrt{1 - h'(s)^2} \approx \int_0^L ds \left(1 - h'(s)^2/2\right) \approx L \left(1 - \frac{\pi^2 h_0^2}{\lambda^2}\right)
\]

**amplitude of wrinkles**

\[ h_0 = \frac{\lambda}{\pi} \sqrt{\frac{\Delta}{L}} = \frac{\lambda \sqrt{\varepsilon}}{\pi} \]

**bending energy of stiff membrane**

\[ U_b \sim A \times \kappa \times \frac{1}{R^2} \sim A \times E_m d^3 \times \frac{h_0^2}{\lambda^4} \sim \frac{A E_m d^3 \varepsilon}{\lambda^2} \]

**potential energy of liquid**

\[ U_p \sim m \times g \times \Delta h \sim \rho \times A h_0 \times g \times h_0 \sim A \rho g \lambda^2 \varepsilon \]

minimize total energy \((U_b + U_p)\) with respect to \(\lambda\)

\[ \lambda \sim \left(\frac{E_m d^3}{\rho g}\right)^{1/4} \]

\[ U_b, U_p \sim A \varepsilon \sqrt{E_m d^3 \rho g} \]
Compression of stiff thin membranes on liquid substrates

\[ U_c \sim A \times E_m d \times \varepsilon^2 \]

\[ U_b, U_p \sim A \varepsilon \sqrt{E_m d^3 \rho g} \]

wrinkles are stable above the critical strain

\[ \varepsilon > \varepsilon_c \sim \sqrt{\frac{\rho g d}{E_m}} \]

wavelength of wrinkles

\[ \lambda \sim \left( \frac{E_m d^3}{\rho g} \right)^{1/4} \]

amplitude of wrinkles at the critical strain

\[ h_0^* \sim \lambda \sqrt{\varepsilon_c} \sim d \]
Compression of stiff thin membranes on liquid substrates

Scaling analysis

\[ \lambda \sim \left( \frac{E_m d^3}{\rho g} \right)^{1/4} \]

Exact result

\[ \lambda = 2\pi \left( \frac{\kappa}{\rho g} \right)^{1/4} \]

\[ \kappa = \frac{E_m d^3}{12(1 - \nu^2_m)} \]