Supplementary appendix to Spatial Models of Delegation

Jonathan Bendor and Adam Meirowitz

This appendix supplements Spatial Models of Delegation and will be made available on the web. Here we more formally define the models and provide proofs for the results in the paper. We do not cover results that hinge on counterexamples discussed in the paper or results that are proven directly in the paper.

Model A

Agents have preferences over the non-empty non-singleton policy space $X \subset \mathbb{R}^n$. Preferences are representable by utility functions of the form:

$$u_i(x) := h(\|x - y_i\|) \tag{1}$$

with the inner-product norm

$$\|x - y_i\| := (x - y_i)(x - y_i)', \tag{2}$$

$y_i \in X$ is agent i's ideal point and $h : R^1 \to R^1$ a strictly decreasing continuous function. Without loss of generality we assume that $h(0) = 0$ and $y_0 = 0$ an interior point of X. We also assume that no decisionmaker other than the boss has $y_m = 0$. A policy is a point $p \in P$ a convex (non-empty, non-singleton) subset of $\mathbb{R}^k$. The relationship between policy and outcomes is expressed by the relationship

$$x = g(p, \varepsilon) \tag{3}$$

where $g : P \times S \to X$ maps the policy p and a random shock ε with support S into outcomes. We make no direct assumptions about the support and law of ε, $F(\varepsilon)$, and instead impose the assumption that for all $\varepsilon' \in S$ and $x' \in X$ there exists a unique $p \in P$ s.t. $x' = g(p, \varepsilon')$. This assumption, termed perfect shock absorption in the body of the paper, ensures that any agent that knows the value of the shock can select a policy so that the final outcome coincides with her ideal point. We denote the coordinates of a point with superscripts as x^j, y^j, p^j ($j \in \{1, 2, ..., n\}$). The relationship in (3) implies that conditional on a policy p the outcome x is a random variable. We denote the conditional distribution function of outcomes by $F(x \mid p)$. We also assume that the distribution of the random outcomes and the function $h(\cdot)$ jointly satisfies the integrability conditions

$$\int \left|x^j\right| dF(x \mid p) < \infty \text{ for all } j \text{ and all } p \in X \tag{4}$$
$$-\infty < \int h(\|x - y_i\|) dF(x \mid p) \text{ for all } y_i \in X \text{ and all } p \in X.$$

The conditions, (4), suffice to ensure that relevant expectations (expected utilities) exist. Finally we assume that the problem of choosing an optimal policy for a boss facing uncertainty is well defined. That is we assume the set

$$\arg\max_P \int h(\|x - y_m\|) dF(x \mid p). \tag{5}$$

is non-empty for every $y_m \in X$. Finally, we assume that $\|y_m\| < \|y_{m+1}\|$.

Results for model A

Proposition 1 *An equilibrium exists and in all equilibria the delegation decision is the same i.e., either not to delegate at all or to delegate to m_1, and the expected utility to the boss is the same.*

Proof: Any agent that is delegated to, knowing ε, will select p so as to minimize $\|y_m - g(p,\varepsilon)\|$. By assumption there exists a p s.t. $y_m = g(p,\varepsilon)$, solving the agent's problem. Thus, the final policy associated with delegation to any agent m is just y_m. By assumption the m which minimizes $\|y_m\|$ is $m = 1$. Thus, if the boss delegates she will delegate to $m = 1$. The boss's utility for not delegating is maximized by selecting a policy p that solves

$$\max_{p\in P} \int h(\|x\|)dF(x \mid p) \tag{6}$$

By assumption a solution to the above problem exists. The boss's choice of whether to delegate rests on an evaluation of the respective utilities. Clearly, it is optimal to delegate iff

$$\max_{p\in P} \int h(\|x\|)dF(x \mid p) =: \underline{v} \leq h(\|y_1\|) \tag{7}$$

where the term $\underline{v}$ is defined by equation (7). Since the solutions to (6) and (7) need not be unique multiple equilibria may exist, but the delegation decision and utilities to the boss are the same in all equilibria.■

Inspection of (7) indicates that the question of whether or not to delegate hinges on the existence of an agent who is closer to the Boss's ideal point than the value

$$d^* := h^{-1}(\underline{v}). \tag{8}$$

Since $h(\cdot)$ is strictly decreasing this inverse exists. An immediate corollary regarding what is termed the delegation set is the following result:

Proposition 2 *If $F(x \mid p)$ is non-degenerate for all p then there exists a subset $D \subset X$ (the delegation set) such that if $y_1 \in D$ then in equilibrium delegation will occur and the final outcome will be y_1 with probability 1. Moreover, the set D is the closure of a non-empty ball around $\mathbf{0}$.*

Proof: Assume $F(x \mid p)$ is non-degenerate for all p, so that $\underline{v} < 0$ and thus $d^* > 0$. By the optimality condition in the previous proof delegation occurs iff $\underline{v} \leq h(\|y_m\|)$. This condition is equivalent to $d^* \geq \|y_m\|$. Thus, we define the set $D := \{z \in X : d^* \geq \|z\| \}$ which is non-empty since $d^* > 0$, and note that delegation occurs iff $y_1 \in D$.■

Proposition 3 *The boss's expected utility is non increasing in $\|y_1\|$.*

Proof: The boss's expected utility is $\max\{\underline{v}, h(\|y_1\|)\}$ which is non increasing in $\|y_1\|$.■

Extensions to A

(1) A busy Boss: Let control by the boss involve an additive cost of c.

Proposition 4 *The radius of the delegation set is increasing in c.*

Proof: Now the analogue to (8) is $d^*_c := h^{-1}(\underline{v} - c)$. Since $h(\cdot)$ is decreasing $h^{-1}(\underline{v} - c)$ is increasing in c so d^*_c is increasing in c.■

(2) Informationally imperfect agents: If the boss delegates to agent m then agent m learns ε with probability q_m. The choice of the optimal delegatee is

$$\arg\max_{m\in M} \left\{ q_m h(\|y_m\|) + (1-q_m) \int h(\|x\|)dF(x \mid p(m)) \right\} \tag{9}$$

$$\text{where } p(m) := \arg\max_{p\in X} \left\{ \int h(\|y_m - x\|)dF(x \mid p) \right\}$$

Proposition 5 *(1) With homogenous q, the radius of the delegation set is increasing in q. (2) If $q_i \geq q_j$ and $\|y_i\| \leq \|y_j\|$ and $\int h(\|x\|)dF(x \mid p(i)) \geq \int h(\|x\|)dF(x \mid p(j))$ with at least one inequality strict, than j cannot be delegated to in any equilibrium.*

Proof: (1) Let d_q be the radius of the delegation set for an arbitrary q. Let $\|y_m\| < d_q$. This implies that

$$h(\|y_m\|) \geq \underline{v}. \tag{10}$$

Moreover since $p(m)$ is a feasible policy for the boss and is not shock dependent,

$$\underline{v} \geq \int h(\|x\|)dF(x \mid p(m)). \tag{11}$$

This implies that the utility to delegating to m, $q_m h(\|y_m\|) + (1 - q_m) \int h(\|x\|)dF(x \mid p(m))$ is strictly increasing in q. Thus if $q' > q$, $\|y_m\| < d_q$ implies that for some $\varepsilon > 0$, $\|y_m\| + \varepsilon < d_{q'}$.

(2) Given the assumptions it cannot be the case that j solves problem 9.■

Finally, the equilibrium behavior is continuous at $q_1 = 1$ in the following manner.

Proposition 6 *If $q_1 \geq q_i$ for all i s.t. $y_i \in D$ and delegation occurs for $q = 1$ (i.e. $y_1 \in D$) then there exists a $\overline{q} \in [0, 1)$ s.t. delegation occurs (to agent 1) in the equilibrium to the game if $q_1 \geq \overline{q}$.*

Proof: Assume the hypothesis. Since $q_1 \geq q_i$ and $\|y_1\| < \|y_i\|$ for all i s.t. $y_i \in D$ agent 1 is the best agent to delegate to. Assume delegation occurs. Thus, $y_1 \in D$. By construction this implies that $\|y_1\| < d^*$. Now the expected utility to the boss from delegating to $m = 1$ in the game with parameter q_1 is

$$q_1 h(\|y_m\|) + (1 - q_1) \int h(\|x\|)dF(x \mid p(1)) \tag{12}$$

Clearly for $q_1 = 1$ this value is $h(\|y_m\|)$. It remains only to establish the continuity of (12) in q_1 to establish that for some $q_1 < 1$ the value (12) is also less than $\underline{v}$, the value of control. By inspection, the solution $\arg\max_{p \in X} \left\{ \int h(\|y_1 - x\|)dF(x \mid p) \right\}$ is constant in q_1 so (12) is continuous in q, completing the proof.■

(3) Costly information-gathering: In this extension agents must pay a cost c_m to learn the shock (and therefore the mapping $p \to x$). Let M^* denote the set of agents that are willing to pay cost c_m to enact the outcome y_m as opposed to not learning the shock and selecting $p(m)$.

Proposition 7 *Given costs c_m ($m \in \mathbf{M}$), (1) delegation occurs iff there exists an $m \in M^*$ s.t. $y_m \in D$, (2) if delegation occurs it is to the lowest indexed agent in M^*, (3) as c_m increases for each agent, the set of agents that the boss is willing to delegate to contracts.*

Proof: (1) In order for the boss to delegate to m we need $m \in M^*$ (or else the boss prefers to choose ignorantly over having the agent choose ignorantly) and $y_m \in D$ (or else the boss prefers to choose ignorantly than attain y_m for certain). (2) Since the utility to the boss from delegating to $m \in M^*$ is $\|y_m\|$ the boss would only choose m to minimize this value (recall agents are ordered by $\|y_m\|$). (3)As c_m increases M^* becomes smaller.■

(4a) Ex ante controls and ex post auditing: In this extension the boss selects to control, to delegate to an agent, or to enact a control system (m, d) where m denotes the agent chosen, and $d \geq 0$ denotes the discretion level. Under H-S scheme (m, d) agent m can choose to enact a policy p' s.t. $\|g(p', \varepsilon)\| \leq d$ or the agent can enact a policy p'' s.t. $\|g(p'', \varepsilon)\| > d$. In the latter case with probability q the agent is caught not in compliance and the boss enacts policy in ignorance of the shock. In addition the agent suffers the penalty f. With probability $1 - q$ the boss never catches the agent and the outcome $g(p'', \varepsilon)$ attains. In the former case the outcome $g(p', \varepsilon)$ attains. A given discretion level d and probability of detection q impose a cost $c(d, q)$ on the boss. We assume that this function is differentiable, strictly decreasing in its first coordinate and strictly increasing in the second coordinate. Given q, f, d if agent m complies her maximal utility is

$$h(\|y_m\| - d) \tag{13}$$

If she doesn't comply and selects policy p' knowing the shock ε her expected utility is

$$(1-q)h(\|g(p',\varepsilon) - y_m\|) + q\left[h(\|g(p_0^*,\varepsilon) - y_m\|) - f\right] \tag{14}$$

where p_0^* is the policy that the boss will enact if she catches the agent not in compliance. Clearly if q and f are sufficiently big relative to $d \leq \|y_m\|$, then for every ε the optimal choice will satisfy $\|g(p,\varepsilon)\| = d$. Alternatively, if q and f are sufficiently low the agent will choose $g(p,\varepsilon) = y_m$ and risk being caught. However, for some parameterizations the agent would only comply for some values of ε. In these cases, it is necessary to focus on perfect Bayesian equilibria, in which the boss updates her beliefs about the value of ε when she observes that the agent has not complied. In a PBE of this form the agent will enact $\|g(p,\varepsilon)\| = d$ for some values of ε and he will enact $g(p,\varepsilon) = y_m$ for other values. Note that since detection is not correlated with policy (given that it is not in compliance), the agent will only enact acceptable or optimal (for himself) policies. The boss after observing that the agent did not comply will optimize using the conditional beliefs $F(x \mid p \mid \tilde{}comply)$ where this is the conditional distribution on ε's that induce the agent not to comply. It is easy to construct examples with PBE in which compliance depends on ε.

Proposition 8 *In some H-S schemes there are PBE in which the agent complies for some values of* ε *and not for other values.*

By $p_m(\varepsilon; d, f, q)$ we denote the policy that agent m chooses given shock ε under a particular H-S scheme. By $1_{dfq}(\varepsilon)$ we denote the indicator function taking the value 1 if $\|p_m(\varepsilon; d, f, q)\| \leq d$ and 0 otherwise. We denote the probability that ε takes on a value that makes m comply given d, f, q by $\gamma(d,f,q) = \int 1_{dfq}(\varepsilon)dF(\varepsilon)$. For a given agent m the boss's optimal discretion level d and monitoring rate q solve

$$\arg \max_{d\in[0,\|y_m\|],q\in[0,1]} \left\{ \begin{array}{c} \left[\begin{array}{c} (1-q)h(d)+ \\ q\int h(\|x\|)dF(x \mid p \mid \tilde{}comply) \end{array} \right] [1-\gamma(d,f,q)] \\ +\gamma(d,f,q)h(d) - c(d,q) \end{array} \right\} \tag{15}$$

The optimal H-S scheme for the boss is attained by solving the above problem for each agent m and then selecting the optimal triplet (m, d_m, q_m). If $h(\cdot)$ is bounded from below, then it has bounded slope, and inspection of (15) yields several results about the principal's control variables.

Proposition 9 *Assume that* $h(\cdot)$ *is bounded from below. (1) For any* $d \geq 0$ *and every* $q \in (0,1]$ *there exists an* $f < \infty$ *s.t.* $\gamma(d,f,q) = 1$. *(2) For any* $f > 0$ *and* $q \in (0,1]$ *there exists a* $d < \|y_m\|$ *s.t.* $\gamma(d,f,q) = 1$.

Proof: Part 1 is driven by the fact that with $q > 0$ (14) is unbounded as f grows, while (13) is finite even at $d = 0$. Part 2 hinges on the fact that if $f > 0$ and $q > 0$ then $h(\|y_m\| - d) > (1-q)h(\|0\|) + q\left[h(\|g(p_0^*,\varepsilon) - y_m\|) - f\right]$ for every ε for some $d < \|y_m\|$.■

A corollary to the theorem partially characterizes the optimal decision for the boss when the cost function $c(d,q)$ is sufficiently flat, small or both and a scheme in which the agent is in full compliance is chosen.

Corollary 1 *(1) If* $\frac{\partial c(d,q)}{\partial d}$ *is sufficiently close to 0,* f *is sufficiently big and an H-S scheme is used then it will involve* $d = 0$. *(2) If in addition* $c(d,q)$ *is sufficiently close to 0 then the optimal decision for the boss will involve implementation of an H-S scheme with* $d = 0$. *(3) If* $c(d,q)$ *and* $\frac{\partial c(d,q)}{\partial d}$ *is sufficiently close to 0 (but* f *may be small) then the boss will choose to enact an H-S scheme with some agent.*

Parts 1 and 2 follow from part 1 of the theorem. Part 3 follows from part 2 of the theorem and the fact that if $c(d,q)$ is small enough getting $\gamma(d,f,q) = 1$ beats control or simple delegation. While it may not be optimal for the boss to utilize an H-S scheme which induces full compliance we will nevertheless characterize several comparative statics about the full compliance H-S scheme.

Proposition 10 *If $q(f)$ and $d(f)$ are optimal control choices with $\gamma(d(f), f, q(f)) = 1$ and if $f' > f$ then either $q(f') < q(f)$ or $d(f') < d(f)$ or both.*

Proof: Consider a solution $q(f)$,$d(f)$ to (15) with $\gamma(d(f), f, q(f)) = 1$. Assume that $f' > f$ and the optimal solutions satisfy $q(f') \geq q(f)$ and $d(f') \geq d(f)$. Clearly $\gamma(d(f), f', q(f)) = 1$ for $f' > f$. Since (14) is decreasing in f and q and (13) is constant in f and q there is a strictly positive scaler δ_q s.t. for $q' \in [q(f) - \delta_q, q(f)]$ we have $\gamma(d, f', q') = 1$. This and the fact that $c(d, q)$ is strictly increasing in q means that the boss does strictly better under d, q',f' than she does under d, q, f'. This means that the optimum under f'' must yield a higher utility to the boss. This is possible only if monitoring is less costly or the final policy is closer. Thus at least one of the conditions in the proposition must hold.■

(4b) Reasserting control: In this section agent m is informed with probability q_m, and after delegating to an agent and observing that they are not informed the boss retakes control and selects policy in ignorance.

Proposition 11 *(1) If $y_m \in D$ then the boss is willing to delegate to m (regardless of q_m). (2) With homogenous q if delegation occurs than agent 1 is chosen. (3) With heterogenous q_m's if the boss delegates to i then there is not another agent j with $q_j \geq q_i$ and $\|y_j\| \leq \|y_i\|$ (with at least one inequality strict).*

Proof: (1) The utility to delegation to agent m in this case is $q_m h(\|y_m\|) + (1 - q_m)\underline{v}$. Meanwhile the utility to control is $\underline{v}$. Thus the former is larger than the latter iff $\underline{v} \leq \int h(\|x\|)dF(x \mid p(m))$, which is equivalent to $y_m \in D$. (2) With homogenous q the utility of delegating to agent i is decreasing in $\|y_i\|$. (3) Suppose the boss delegates to i in an equilibrium but there is an agent j with $q_j \geq q_i$ and $\|y_j\| \leq \|y_i\|$ (with at least one inequality strict). Since delegation occurs to i it must be that $y_i \in D$. Since $h(\|y_j\|) \geq h(\|y_i\|) \geq \underline{v}$ the assumed ordering implies that $q_j h(\|y_j\|) + (1 - q_j)\underline{v} > q_i h(\|y_i\|) + (1 - q_i)\underline{v}$, contradicting the assumption that delegation to i was a best response.■

(5) Agents can precommit:

Definition 1 *Preferences satisfy diversity if either (i) there does not exist a vector $s \in X$ s.t. for all $m \in \mathbf{M}$, $x_m = \lambda_m s$ for some $\lambda_m \in R^1$ or (ii) if such a vector does exist then there must be two agents $m, m' \in \mathbf{M}$ s.t. for the λ_m and $\lambda_{m'}$ described above $\lambda_m < 0$ and $\lambda_{m'} > 0$.*

Proposition 12 *If there are at least two agents in $\mathbf{M}$ and preferences satisfy diversity, then in every subgame perfect equilibrium at least two agents in $\mathbf{M}$ offer the outcome 0 (the boss's ideal point) and the boss accepts one such offer. Moreover, all such equilibria yield the boss the same utility.*

Proof: By way of a proof by contradiction, assume that diversity is satisfied and the claim does not hold. This implies that either at most one agent promises $x_m = 0$ or at least two agents make this offer and the boss does not delegate to one of these agents. If the latter occurs then with positive probability the final outcome is not 0, but since a lottery that assigns probability 1 to the final outcome 0 maximizes the boss's expected utility and is available to the boss this cannot occur in equilibrium. This implies that it must be the case that we have a subgame perfect equilibrium in which at most one agent announces $x_m = 0$. There are two sub-cases to consider: (a) no agent offers $x_m = 0$ or (b) exactly one agent offers $x_m = 0$. We treat these cases in turn.

-(a) Let $x' = \arg\min_M \|x_m\|$. There are three separate sub-sub cases that we consider to derive the contradiction for case (a). (1) If the boss does not accept x' which is the best available offer then it must be the case that no offer is in D, delegation does not occur, and the final outcome is non-degenerate. But a deviation by any agent m to offer the policy $\frac{h^{-1}(\underline{v})}{\|y_m\|} y_m$ which is in the delegation set D and which he prefers to the lottery induced by the boss's uninformed best response, dominates m's offer in the conjectured equilibrium, contradicting the assumption that no offer is in the delegation set in an equilibrium. (2) If the boss does accept an offer (which must be x' by subgame perfection) and part (i) of diversity is satisfied then we know that the set $\{z \in X : \|z - y_m\| < \|x' - y_m\|$ and $\|z\| < \|x'\|\}$ is not empty. This is the set of points that both the boss and m prefer to the policy x'. Since this set is non-empty there exists a deviation by m which dominates her offer under the conjectured equilibrium. (3) If the boss does accept an offer (x') and part (i) of diversity is not satisfied but part (ii) is, then the result follows from the median voter theorem.

-If (b) attains then we know that the boss will accept this offer. However since $y_m \neq 0$ and $\min_{n \in M \setminus m} \|x_n\| := \gamma > 0$ a deviation by agent m to, say, $x'_m = \frac{\gamma}{2\|y_m\|} y_m$ results in delegation by the boss to m but a final outcome that m prefers to the outcome 0. This contradicts the fact that $x_m = 0$ is a best response for m when no other agent offers 0.■

Proposition 13 *If there are at least two agents in* **M** *and preferences satisfy diversity, then any profile of offers in which at least two agents offer $x_m = 0$ is supportable in a subgame perfect equilibrium.*

Proof: By arguments in the previous proof, we know that given any profile of offers with at least one offer of 0 the boss will accept an offer of 0. Given this any deviation by one agent in M will not change the final outcome. Thus, no such deviation is desirable.■

(6) Multiple bosses: Now there are also n (odd) bosses $\{b_1, b_2, ..., b_n\}$. In period 1 boss b_1 proposes which agent to delegate authority to (or to control). In period 2 all of the bosses vote to accept or reject the offer. If the offer is accepted by a winning coalition $L \in \Lambda$ (a proper, monotone class of sets) then the delegated agent enacts policy knowing the shock. If the offer is not accepted then boss b_1 must select policy in ignorance. Letting

$$p_1 := \arg\max_{p \in P} \int h(\|x - y_{b_1}\|) dF(x \mid p) \tag{16}$$

boss b_j is willing to delegate to any agent with ideal point in the closed ball

$$D_{b_j} = \{y : \int h(\|x - y_{b_j}\|) dF(x \mid p_1) \leq h(\|y - y_{b_j}\|)\}. \tag{17}$$

Accordingly, the set of ideal points that b_1 can successfully delegate to is

$$A = \bigcup_{L \in \Lambda} \bigcap_{b_j \in L} D_{b_j} \tag{18}$$

which is a compact (but not necessarily convex or non-empty) set. Thus boss b_1 will delegate to the best agent in the set $D_{b_1} \cap A$ if this set is non-empty and she will control otherwise. If we assume that bosses have $h(\cdot)$ functions that are strictly concave (i.e., the bosses are risk averse) then we know that as long as ε is not degenerate for any p then for some open ball $B(y_{b_j}, \delta)$ around y_{b_j} we have $B(y_{b_j}, \delta) \subset D_{b_1} \cap A$ and thus with agents of enough types delegation will occur.

Instead now consider a game with at most n periods. In period $t = 1, ..., n$ the boss with index $n + 1 - t$ proposes an agent, and there is a vote. If a winning coalition supports this delegation the game ends and the proposed agent is given control, otherwise it continues to the next period with a new proposal. If none of the n offers is accepted then b_1 must select policy in ignorance. Since control by b_1 is the default, any accepted offer must satisfy $y_m \in A$. Additionally, as long as A is non-empty some proposer will be able to and willing to propose an acceptable offer. The following conclusion is then immediate.

Proposition 14 *With proposing by b_1 only, delegation occurs iff $D_{b_1} \cap A$ is non-empty. If all bosses get to propose then delegation occurs iff A is non-empty.*

The following corollary applies.

Corollary 2 *Delegation is more likely if the rule is less restrictive (Λ is larger) or the ideal points of the bosses are closer.*

Model A'

The modifications from the basic assumptions A to those of A' are the following: 1. $S \subset P = X = \mathbb{R}^n$. 2. The outcome mapping is $g(p, \varepsilon) = p + \varepsilon$. 3. The distribution $F(\cdot)$ of ε is symmetric about the mean vector $\mathbf{0}$. By symmetric we mean the distribution $F^j(\varepsilon^j)$ of each coordinate of ε is symmetric about 0.

Comparative statics for A'

(2) More variable shocks:

Definition 2 *Random variable a with distribution function $A(\cdot)$ is a mean preserving spread of b having distribution $B(\cdot)$ if (i) $\int adA(a) = \int b(dB(b)$ (we denote this mean as z) and (ii) for any $\delta > 0$, $\int_{\{\|a-z\|>\delta\}} dA(a) \geq \int_{\{\|b-z\|>\delta\}} dB(b)$ with strict inequality if the the left hand side is not zero.*

The intuition behind this concept is that random variable a is a mean preserving spread of b if they have the same mean and the tails of $A(\cdot)$ are fatter than the tails of $B(\cdot)$. Given such random variables a and b with mean z the random variables $a^* := \|a - z\|$ and $b^* := \|b - z\|$ are well defined. Note that these two random variables are real valued.

Lemma 1 *If random variable a with distribution function $A(\cdot)$ is a mean preserving spread of b having distribution $B(\cdot)$, then the distribution functions $A^*(\cdot)$ of a^* and $B^*(\cdot)$ of b^* satisfy the ordering $A^*(\cdot) \leq B^*(\cdot)$ with strict inequality on the interior of the union of the supports of a^* and b^*.*

Proof: Assume that a is a mean preserving spread of b. By definition we have

$$\int_{\{\|a-z\|>\delta\}} dA(a) \geq \int_{\{\|b-z\|>\delta\}} dB(b). \tag{19}$$

for any $\delta > 0$ with strict inequality if the left hand side is non-zero. But by definition

$$A^*(\delta) = 1 - \int_{\{\|a-z\|>\delta\}} dA(a) \tag{20}$$

and

$$B^*(\delta) = 1 - \int_{\{\|b-z\|>\delta\}} dB(b) \tag{21}$$

for $\delta \geq 0$. Combining these three lines yields

$$A^*(\cdot) \leq B^*(\cdot) \tag{22}$$

with strict inequality on the interior of the union of the supports of a^* and b^*.■

Proposition 15 *Assume the assumptions of A', that a is a mean preserving spread of b and that under both lottery the boss's optimal choice is $-z$, then under random shock a the delegation set is a strict superset of the delegation set for shock b.*

Proof: Assume the hypothesis. Given $p_a^* = p_b^* = -\int adA(a) = -\int bdB(b) = -z$ the expected utility to the boss from not delegating in each case is

$$\int h(\|a - z\|)\, dA(a) = \int h(a^*) dA^*(a^*) \tag{23}$$

$$\int h(\|b - z\|)\, dB(b) = \int h(b^*) dB^*(b^*) \tag{24}$$

But, by the previous lemma we have $A^*(\cdot) \leq B^*(\cdot)$ with strict inequality on the interior of the union of the supports of a^* and b^*. Since $h(\cdot)$ is strictly decreasing it is well known that a (first order) stochastically dominated random variable is preferred to the random variable that dominates it, so that we have

$$\int h(a^*)dA^*(a^*) < \int h(b^*)dB^*(b^*). \tag{25}$$

But, this means that the boss's expected utility from not delegating is lower under a than under b. Since the utility to delegation is the same under a and b the result follows.■

Extensions to A'

(1a) Reasserting control and costly information gathering, revisited: We assume

$$h(0) - \int h(\|\varepsilon\|)dF(\varepsilon) < c. \tag{26}$$

Proposition 16 *Delegation occurs iff $A = \{m : d^1 \leq \|y_m\| \leq d^2\}$ is not empty, for some $d^1, d^2 > 0$.*

Proof: In any subgame perfect equilibrium if the boss chooses to delegate and then the delegatee does not specialize, the boss will enact $p = \mathbf{0}$. Thus, a delegatee specializes iff

$$\int h(\|\varepsilon - y_m\|)dF(\varepsilon) \leq h(0) - c. \tag{27}$$

But by equation (27) there exists a $\delta > 0$ s.t. for an agent with $\|y_m\| < \delta$ (28) does not hold. Thus, there exists a positive

$$d^1 := \min_y \{\int h(\|\varepsilon - y\|)dF(\varepsilon) - h(0) + c\} \tag{28}$$

and delegation to an m with $\|y_m\| < d^1$ is not a best response as it results in no specialization. On the other hand if $\|y_m\| \geq d^1$ then delegation to m results in the final policy y_m. The boss prefers delegation on this set if

$$\int h(\|\varepsilon\|)dF(\varepsilon) \leq h(\|y_m\|) \tag{29}$$

Thus, as long as

$$\min_{\{\|y_m\| \geq d^1\}} \|y_m\| \leq h^{-1}(\int h(\|\varepsilon\|)dF(\varepsilon)) =: d^2 \tag{30}$$

delegation to an agent who will specialize is a best response. Combining implies that delegating to m is a best response iff

$$d^1 \leq \|y_m\| \leq d^2. \blacksquare \tag{31}$$

(2) Imperfectly competent agents, revisited. Let $\overline{q}(y_i)$ denote the minimal level of competency such that the boss prefers delegation to an agent with ideal point y_i and competency $\overline{q}(y_i)$ (without the possibility of retaking control) to choice in ignorance.

Proposition 17 *The threshold $q(y_i)$ is increasing in $\|y_i\|$.*

Proof: The value $\overline{q}(y_i)$ solves

$$\overline{q}(y_i)h(\|y_i\|) + (1 - \overline{q}(y_i)) \int h(\|x\|)dF(x \mid p(i)) = \underline{v}. \tag{32}$$

If $\overline{q}(y_i) < 1$ we must have $h(\|y_i\|) > \underline{v} > \int h(\|x\|)dF(x \mid p(i))$. The first term is decreasing in $\|y_i\|$ and since $x = p + \varepsilon$ the third term is also decreasing in $\|y_i\|$. This implies that $q(y_i)$ is increasing in $\|y_i\|$.■

(3) Multiple principals: We denote the set of bosses as $\mathbf{L}$ and the profile of their ideal points as $(y_1, ..., y_L)$. Assume: (1) In the initial period the bosses simultaneously vote whether or not to delegate. A weakly-paretian voting rule is used to determine whether or not delegation occurs. (2) If the decision is to not delegate then a finite period Baron-Ferejohn bargaining game ensues in which a policy $p \in P$ is chosen without knowledge of ε . Then ε is realized and payoffs attain. (3) If delegation is chosen then a finite period Baron-Ferejohn bargaining game ensues over which agent to delegate to. Once an agent is chosen ε is realized and learned by the agent; he then selects a policy and payoffs attain.

Lemma 2 *The subgame that begins with a decision to delegate has a subgame perfect Nash equilibrium in weakly undominated strategies.*

Proof: We establish the claim by backwards induction. At any history in which delegation has occurred to agent m and ε is revealed, the best response for agent m is to select policy $y_m - \varepsilon$. Given this the decision to delegate to any m results in a degenerate lottery of attaining outcome y_m with probability 1. Since the bosses have complete, reflexive, and transitive preferences over this set of outcomes, the finite bargaining game over these outcomes has a subgame-perfect equilibrium in weakly undominated strategies.■

Definition 3 *Given a multiple bosses game, a strategy profile satisfies condition α if at any information set in which any agent i has a binary choice between action a and b in which the expected utility resulting from every profile of strategies that involve the choice of a by agent i is less than the expected utility resulting from the play of best responses by agents at histories following i's choice of b, agent i plays action b.*

Proposition 18 *For fixed boss and agent ideal points, there exists a distribution $F(\cdot)$ of the shock ε for which any strategy profile satisfying condition α results in unanimous votes to delegate by the bosses.*

Proof: The proof is constructive. Given fixed boss and agent ideal points and arbitrary distribution $F(\cdot)$ we consider L translated games constructed as follows. For boss $l \in \mathbf{L}$ the l-translated single boss game has shock $\varepsilon^l = \varepsilon - y_l$, and agent ideal points $y^l_m = y_m - y_l$ for each $m \in M$, and boss ideal point $\mathbf{0}$. Clearly, each l-translated game satisfies the assumptions of the basic game of Model A'. In each l-translated game either all agent ideal points are in the delegation set or they are not. If not then by the risk-comparative static we can take mean preserving spreads of ε^l until the delegation set covers all agent ideal points in the l-translated game. Thus, a mean preserving spread of ε^l exists for which all agent ideal points are in the delegation set. Denote this shock as ε^{l+}. Now, in the multiple boss game with random shock $\varepsilon^{l+} + y_l$ the boss with ideal point y_l prefers to delegate to any agent than to select policy herself (facing uncertainty about shock $\varepsilon^{l+} + y_l$). Repeating these operations for each $l' \in \mathbf{L} \backslash l$ we can construct mean preserving spreads of $\varepsilon^{l+} + y_l$ such that every boss in $\mathbf{L}$ prefers delegating to any agent in $\mathbf{M}$ to selecting policy herself (under uncertainty). Since the best expected utility that a boss can attain in the non-delegation subgame is from selecting policy herself, this implies that if a strategy profile satisfies condition α then delegation must occur. Thus, we have constructed the relevant shock distribution.■

Model A"

In addition to the assumptions of A', model A" assumes that. $h(\cdot)$ is twice differentiable with $h''(\cdot) < 0$.

Results for model A"

Proposition 19 *Not delegating is ex-ante inefficient.*

Proof: Not delegting results in a lottery over outcomes, $L(x)$. Consider the degenerate lottery $L^*(x)$ that places probability 1 on the outcome $x^* = \int x dL(x)$. Given that each agent is strictly risk averse the expected utility of $L^*(x)$ is strictly higher than the expected utility of lottery $L(x)$. Since any agent could enact the lottery $L^*(x)$ (because of perfect shock absorption) the claim is established.■

Comparative statics for A"

A more risk averse boss: The Arrow-Pratt measure of absolute risk aversion is $a(x) = \frac{-h''(x)}{h'(x)}$

Proposition 20 *Consider two loss functions $h_1(\cdot)$ and $h_2(\cdot)$ with $a_1(x) < a_2(x)$ for all x. The delegation set under h_1, is contained in the delegation set under h_2.*

Proof: Let $p_1 = \arg\max_{p \in P} \int h_1(\|x\|)dF(x \mid p)$ and $p_2 = \arg\max_{p \in P} \int h_2(\|x\|)dF(x \mid p)$. Assume that $y_m \notin D_2$. This implies that

$$\int h_2(\|x\|)dF(x \mid p_2) =: \underline{v_2} > h(\|y_m\|). \tag{33}$$

It is well known that if $a_1(x) < a_2(x)$ then

$$\int h_2(\|x\|)dF(x \mid p_2) > h(\|y_m\|) \tag{34}$$

implies

$$\int h_1(\|x\|)dF(x \mid p_1) > h(\|y_m\|) \tag{35}$$

which implies that

$$\int h_1(\|x\|)dF(x \mid p_2) > h(\|y_m\|). \tag{36}$$

Since

$$\int h_1(\|x\|)dF(x \mid p_1) \geq \int h_1(\|x\|)dF(x \mid p_2). \tag{37}$$

we have

$$\int h_1(\|x\|)dF(x \mid p_1) > h(\|y_m\|) \tag{38}$$

and thus $y_m \notin D_1$. Thus $y_m \notin D_2$ implies that $y_m \notin D_1$ so $D_1 \subset D_2$.■