Available online at www.sciencedirect.com

SCIENCE DIRECT®

Games and Economic Behavior 51 (2005) 464–489

GAMES and Economic Behavior

www.elsevier.com/locate/geb

Polling games and information revelation in the Downsian framework

Adam Meirowitz

Department of Politics, Corwin Hall, Princeton University, Princeton, NJ 08544, USA

Received 27 June 2003

Available online 8 December 2004

Abstract

We investigate the incentives faced by poll respondents when candidates use polling data to inform their selection of policy platforms. Focusing on models with a unidimensional policy space, single peaked preferences and two office-seeking candidates observing a summary statistic from polls that ask respondents their preferences, we find that for most environments honest poll response cannot occur in a perfect Bayesian equilibrium. However, simple partially-revealing equilibria exist when the poll only asks respondents which party or candidate they prefer. When the candidates learn the sample average or see all the data, there are partially revealing equilibria that mimic those of the binary message game. Interpretation of polling data requires knowledge of the equilibrium played as the meanings of poll responses are endogenously determined. The analysis suggests that naive use of polling data may be misleading.

JEL classification: C72; D72; D82

Keywords: Downsian competition; Elections; Signaling games; Polling; Information revelation; Voting theory

---

1. Introduction

In many democracies public opinion polling is pervasive and political candidates and parties are thought responsive to poll numbers (Gollin, 1987; Sudman and Bradburn, 1987).

E-mail address: ameirowi@princeton.edu.

0899-8256/$ – see front matter © 2004 Elsevier Inc. All rights reserved.
doi:10.1016/j.geb.2004.09.015

These conclusions suggest that public opinion polls are relevant and merit inclusion in theories of elections. We take a first step at analyzing the incentives induced by an electoral institution in which candidates are uncertain about the preferences of the electorate and receive polling information before selecting policy platforms. In such a situation candidates learn from polling data, and, therefore, poll responses may influence candidate platforms and ultimately electoral and policy outcomes. Poll-respondents, recognizing the poll as a means to influence candidate beliefs and actions, may behave strategically. While the possibility of strategic poll response, as a means to manipulate candidate beliefs, has been ignored by theorists at least one early scholar of public opinion seems to have noticed the *temptation to dissimulate*. In Dupeux's (1954) special report he notes a substantial reservation about the use of survey methods to forecast voting.[1]

> There are two difficulties in the way of these investigations by representative sampling, and they are inherent in the technique itself. The first difficulty is the temptation to dissimulate, which may be strong in the case of those questioned, especially when they belong to dependent categories. The second difficulty is certainly the more serious: it is due to the fact that when persons are questioned they are not in exactly the same frame of mind as when they are called upon to vote. They know that their answers will not have any consequence, immediate at least, whereas they are very conscious that their vote can change the political complexion of their country. It is impossible to regard the person questioned and the elector as one and the same person (Quoted in Harris, 1956, p. 383).

Our formulation supports Dupeux's first conjecture. Most poll-respondents, if motivated purely by the desire to influence policy, would have an incentive to dissimulate in responding to pre-election polls. However, we take exception with Dupeux's claim that respondents "know that their answer will not have any consequence." In two party contests, pre-election polls are ripe for strategic behavior as respondents can manipulate the inferences and therefore, the actions of candidates.

Considerable attention has been focused on the analysis of electoral models (in the tradition of Downs, 1957) in which voters face uncertainty (e.g., Ledyard, 1984; Feddersen and Pesendorfer, 1996, 1997; Fey, 1997; Snyder and Ting, 2002). In these papers there is no possibility for learning by the parties/candidates and the focus is on strategic use of information by voters. Bernhardt et al. (2003) consider the candidate location game when candidates receive private polling data. In equilibrium, candidates overweight their private signals and candidate locations are divergent. In the Bernhardt, Duggan and Squintani model poll respondents are not players in the game and polling data is exogenous.

---

[1] In the same year Simon (1954) considered the possible bandwagon and underdog effects stemming from election forecasts. He was concerned with the affect of measurement on voter actions. Our concern is somewhat different, as we are concerned with the actions poll-respondents may take to influence the poll results as a means to influence the actions that candidates take. The current approach differs dramatically from traditional public opinion scholarship (for example Zaller, 1992). We are concerned with the incentives respondents face to manipulate the external democratic process through their poll responses and not the incentives or biases stemming from the internal process of asking individuals their opinion.

McKelvey and Ordeshook (1985, 1986a, 1986b) consider rational expectations equilibria in settings in which candidates learn about voters and uninformed voters learn about candidates. In their 1985 paper, McKelvey and Ordeshook include a model where candidate positions are *fixed* and a sequence of successive polls is considered. The following conclusion is reached.

> Finally, although we do not prove this formally, it appears that the above process, as well as the equilibrium associated with it is nonmanipulable. I.e., given our restrictions on preferences, no voter can gain by adapting strategies different from those prescribed in the above dynamic. Similarly, and more obviously, in equilibrium no one can gain by misrepresenting his preferences (p. 73).

In this paper, we assume candidate positions are not fixed. Rather, they are strategic variables chosen by candidates after observe polling data (as in Bernhardt et al., 2003). In this setting the claim that polls are nonmanipulable is generally false. We consider a class of games—herein termed *polling games*—that have:

(i) an initial polling stage in which a sample of voters announce their ideal points to a polling service,
(ii) an electoral stage where two candidates learn polling statistics and take policy stances, and
(iii) a voting stage where the electorate selects from the candidates.

The analysis suggests that there may be a problem with an explanation of polling that allows for strategic responses and posits that poll respondents are honest. The tension between honest and strategic response arises because, in most polling games, respondents who expect to be unsatisfied with the policy outcome (say a right of center respondent) can manipulate the inferences and policy selections of candidates by lying about their preferences (e.g., claiming to be even more right of center).[2]

To connect the work to polls that have small message spaces we also consider special polling games in which poll respondents are instead asked which party or candidate they prefer. While this message space is too small to support truthful equilibria, in this model partially-revealing perfect Bayesian equilibria do exist. Since the meaning of signals is endogenous, naive interpretation of polling statistics is problematic. This result suggests that polling data may appear biased to a casual observer when in fact candidates can make unbiased inferences. Comparative statics analysis of this model demonstrates the difficulty in interpreting polling data when respondents are strategic. This section is directly applicable to polls in which respondents are asked how they would vote if the election were today. With this interpretation, our finding indicates that the set of people that say they would vote

---

[2] We do not contend that all or even most poll respondents are strategic in their poll responses. Nor do we contend that all respondents view pre-election polls as a medium for communication between themselves and candidates. Nonetheless, in a setting where candidates are responsive to polling numbers, there is an incentive for respondents to be dishonest. The extent to which this opportunity is taken up by real poll respondents is an empirical question not addressed in this paper.

for a particular candidate may not actually be the set of people that would really vote for the candidate if the election were today. Some respondents may misrepresent themselves in an effort to influence the platforms of the candidates. We also show that equilibria in this binary message game can be reinterpreted as partially revealing equilibria in the basic polling game if the candidates observe the raw polling data or the sample average. These equilibria exhibit stark polarization as all respondents claim to be maximally extreme.

While not explicitly focused on polling, four additional papers are related. Piketty (2000) presents a two-period model in which voters are incompletely informed about the desirability of three fixed policy alternatives, and first period voting communicates information to other voters. Piketty offers an interpretation in which the voters are perfectly informed and use first period actions to send messages to a party that moves in the second period. Under this interpretation the party is severely constrained in how it uses the information from the first period. Shotts (2001) explores the informational content of repeated elections, and shows that when candidates use information from previous elections to draw inferences about the distribution of voter preferences, abstention can occur even with costless voting. In the equilibria to Shotts' model voters consider the signaling value of their first period ballots.[3] Razin (2003) considers the informational content of mandates in a model where elected officials enter office uncommitted to particular policy stances. Meirowitz (2001) presents a model of primary and general elections, in which endogenous party affiliation by primary voters reveals information about the electorate's preferences. Relative to the current paper, these models involve severe constraints on either the way candidates can use information provided by voters, or the message space available to subsets of voters, or both.

Finally, while Moulin (1980) does not deal with Bayesian games of elections, his results are closely related. Moulin shows that with single peaked preferences the class of efficient, anonymous and strategy-proof voting schemes corresponds to the set of schemes that add a list of exogenous policies to the list of announced ideal points and then take the median of this longer list. This finding suggests that, in polling games using the sample median, truthful response may be supportable. However, we find that when the polling statistic is the sample median the mapping from poll responses to final outcomes induced by equilibrium play is strategy-proof only in knife-edged cases. In most cases the fact that the polling data is generated from a sample of all voters means that candidate locations are an average of the prior and the data. As such, equilibrium candidate location mappings are not in the class of strategy-proof schemes despite the fact that the observed statistic is the median. If candidates could commit to announcing at the sample median then truthful response would be a best response for voters. But simultaneous location at the sample median is not generally an equilibrium in the candidate location game following truthful response.

---

[3] The equilibrium of the binary-message section should be contrasted with those in the Shotts' model, which can be interpreted as a tri-message model of polling with a particular polling statistic. In the Shotts' model all voters cast ballots for either candidate or abstain in the first election. In the current paper a random sample of the population of voters take actions before candidates commit and poll respondents are required to answer the poll and thus the message space is smaller. In the Shotts' model voters must balance the signaling effect of first stage ballots with the effect that such ballots have on the first period election. In the current paper there is no such tradeoff. In both models the meaning of first period actions is endogenous.

The remainder of this paper is organized as follows. In Section 2 we present the basic model. In Section 3 we develop results from the analysis of this model. In Section 4 we present and analyze the binary message model. In Section 5 we use the results of Section 4 to characterize some partially revealing equilibria in basic polling games. In Section 6 we conclude with a discussion of the findings. Appendix A contains proofs of the lemmas and propositions that are not proven in the body.

2. The polling game

We proceed by first defining a model of elections with a polling stage. We then define the appropriate equilibrium concepts for this class of games.

2.1. Basic concepts and definitions

Consider a set of two candidates $C := \{0, 1\}$. Each candidate $c \in C$ will choose a policy platform $x_c \in X := [0, 1]$. The term $-c$ denotes $C \setminus c$. A continuum $V := [0, 1]$ of voters with symmetric single peaked preferences have ideal points located in the policy space X. A generic voter is denoted $v \in V$, and her ideal point is denoted $y_v \in X$. The population of voter preferences is characterized by a distribution function $F(\cdot, \theta)$ with support X that depends on a parameter $\theta \in \Theta := [0, 1]$. For any $\theta \in \Theta$, $F(\cdot, \theta)$ is continuously differentiable and strictly increasing on X (with density $f(\cdot, \theta)$). We assume that the parameter θ is the population median, meaning that for each $\theta \in \Theta$, $F(\theta, \theta) = 1/2$. By $\eta(\cdot)$ on Θ we denote the twice continuously differentiable and strictly increasing prior belief on θ. The distribution $\eta(\cdot)$ induces beliefs over the distribution of voter ideal points $F(\cdot, \theta)$. We further assume that for all $y \in X$, $F(y, \theta)$ is measurable and in fact continuous in θ on Θ. In addition, we sometimes make use of a standard likelihood ratio condition. The *strict monotone likelihood ratio condition* (SMLR) is satisfied if $y < y'$ and $\theta < \theta'$ implies that $f(y', \theta)/f(y', \theta') > f(y, \theta)/f(y, \theta')$.

Candidate priors are given by $\eta(\cdot)$. Voter beliefs about θ are given by a continuously differentiable and strictly increasing conditional distribution on Θ, $\eta(\cdot \mid y)$. In a standard Bayesian game $\eta(\cdot \mid y)$ is interpreted as the posterior belief on θ conditional on realizing y as a single draw from $F(\cdot, \theta)$ and given by Bayes' rule. We do not require that this connection between $\eta(\cdot \mid y)$ and the primitives $F(\cdot, \theta), \eta(\cdot)$ exists. Of particular interest is the case where voters with distinct preferences share beliefs about the location of the median voters, $\eta(\cdot \mid y) = \eta(\cdot)$ (for all y). Aside from results on binary message polling games this assumption is inconsequential. In the binary message case we need $\eta(\cdot \mid y)$ to not vary with y too much.[4]

---

[4] While the assumption that voters with different preferences have common beliefs is inconsistent with the assumption of a common prior and i.i.d. draws from $F(y, \theta)$, this can be attained from non-common priors, or settings in which each voter believes that their type is drawn from a distribution that is different than the typical voter's. See McKelvey and Page (1986) for results on the convergence of beliefs when public information is available.

Voters each have preferences over policy that are representable by the utility function

$$u_v(x) = h\big((x - y_v)^2\big) \tag{1}$$

where $y_v \in X$ is voter v's ideal point and $h : [0, 1] \to \mathbb{R}^1$ is a strictly decreasing loss function. Thus, voter preferences are single peaked, but their risk attitudes are not restricted. Candidates are office seeking and have preferences that are representable by the utility function

$$u_c(x, w) = 1_{\{c=w\}}(w). \tag{2}$$

Here, $1_c(w)$ is the indicator function taking value 1 only if the winning candidate, w, is c and 0 otherwise.[5] Throughout we use the notation $1_A(\cdot)$ to denote the indicator function taking the value 1 if event A is true (more precisely if the argument of the function is in the set $\{A\}$), and 0 otherwise.

The sequence of moves in a *polling game* is as follows:

(i) Nature selects θ by a draw from $\eta(\cdot)$.

(ii) A randomly chosen finite subset $P \subset V$ of voters are asked to simultaneously respond to a poll. This subset is gathered by T (odd) independent draws from the uniform distribution on V.[6] In other words, the vector $\mathbf{y} = (y_1, \dots, y_T)$ of T ideal points are generated by independent draws from $F(\cdot, \theta)$. We use the notation $F^T(\mathbf{y}, \theta)$ to denote the distribution function for the draw of T ideal points. We also use the notation $\mathbf{y}_{-v}$ and $F^{T-1}(\mathbf{y}_{-v}, \theta)$ to denote a vector of $T - 1$ ideal points and its distribution function. The T poll respondents simultaneously announce a message. The message space for each poll respondent is X with a response denoted $m_v \in X$. The vector of poll responses is denoted $\mathbf{m} = (m_1, \dots, m_T) \in X^T$. The set of poll responses is denoted $M := \{m_1, \dots, m_T\}$. By $\mathbf{m}_{-v}$ we denote the vector of $T - 1$ responses that excludes the response by v. With slight abuse of notation we use $\mathbf{m}$ and $(\mathbf{m}_{-v}, m_v)$ interchangeably for any $v \in P$.

(iii) Candidates then learn the summary statistic $s(\mathbf{m})$,where $s(\cdot) : X^T \to \mathbb{R}^n$ $(1 \leqslant n \leqslant T)$, update their beliefs about θ based on the statistic $s(\mathbf{m})$ and simultaneously take stances $x_c \in X$.

(iv) Following the stances (x_0, x_1), all voters cast ballots $b_v(x_0, x_1) \in C$ and the winner, w, is chosen via majority rule. We assume that ties are resolved with a fair coin toss. By having a finite poll sample and a continuum of voters, we capture the notion of drawing a sample from a large population.

---

[5] In a previous version we considered candidates that were also policy motivated. The underlying conclusion that generically equilibria in which poll respondents are truthful do not exist holds under this assumption. However, in this case the existence of pooling equilibria requires appeal to mixed strategies or less general assumptions.

[6] The assumption that the poll sample is finite implies that poll respondents will influence the polling statistic with positive probability. For a continuum of poll respondents a poll respondent's actions are inconsequential and some refinement is needed to make that game non-trivial.

Players have incomplete information about the parameter θ and thus the distribution of ideal points $F(\cdot,\theta)$ is not known. Given an arbitrary belief, $\lambda(\cdot)$ over θ, expectations over an arbitrary $\mathbf{y}_{-v}$—measurable function, $g(\cdot)$, are expressed by the integral

$$\mathbb{E}_{\lambda(\theta)} g(\mathbf{y}_{-v}) = \int\limits_{\Theta} \int\limits_{X^{T-1}} g(\mathbf{y}_{-v})\,\mathrm{d}F^{T-1}(\mathbf{y}_{-v},\theta)\,\mathrm{d}\lambda(\theta). \tag{3}$$

We denote a coordinate of the summary statistic by $s^z(\mathbf{m})$. We denote the inverse image of $s(\cdot)$ by $s^{-1}(s') := \{\mathbf{m} \in X^T\colon\ s(\mathbf{m}) = s'\}$. Three particularly interesting examples are: the identity statistic $s(\mathbf{m}) = \mathbf{m}$; the sample median $s(\mathbf{m}) = \text{median}(\mathbf{m}) := \{x \in M\colon\ \#\{y \in M\colon\ y \leqslant x\} = \#\{y \in M\colon\ y \geqslant x\}\}$; and the sample average $s(\mathbf{m}) = T^{-1}\sum_{i=1}^{T} m_i$.[7] The first and third examples above, are similar in that the polling statistic is always responsive to each poll response. In contrast the sample median, is only responsive to small changes in a response when that response is itself the sample median. Because the incentives present when $s(\cdot)$ is strictly monotone are substantially different than when the polling statistic is sometimes constant in m_v (like the sample median), we will treat the case when $s(\mathbf{m}) = \text{median}(\mathbf{m})$ separately.

We consider several assumptions that a polling statistic can satisfy.

A1 (continuity): The function $s(m_1,\dots,m_T)$ is continuous in each m_v.

A2 (strict monotonicity): If $m'_v \geqslant m_v$ for every $v \in P$ and the inequality is strict for at least one $v \in P$ (denoted $\mathbf{m}' > \mathbf{m}$) then $s^z(\mathbf{m}') \geqslant s^z(\mathbf{m})$ for every z and the inequality is strict for some $z \in \{1,\dots,n\}$ (denoted $s' > s$).

A3: $s(\mathbf{m}) = \text{median}(\mathbf{m})$.

Intuitively, A1 requires that small changes in the messages, m_v, result in small changes in the information observed by the candidates. A2 requires that if a poll respondent raises her response, at least one coordinate of the observed statistic will increase and none will decrease. A3 is incompatible with A2 as if m_v is not the median response then a small change in m_v has no effect on the sample median. However, A3 implies A1. While the union of A1–A2 and A3 do not exhaust the possible polling statistics, we focus on only these cases. A1–A2 is a reasonable axiomatization of standard statistics used by empiricists. Analysis of A3 seems natural given the relevance of the median voter in the Downsian research program and the findings of Moulin.

Example. Before turning to the analysis we present a stylized discontinuous example of the class of models considered. Periodically, we will return to this leading example to make concepts concrete. The example involves players with a uniform prior over the location of the median voter and the belief that voter ideal points are evenly distributed to either side of the median. Suppose that $\eta(\theta) = \theta$ on $[0,1]$ and for $\theta \in (0,1)$, $f(y \mid \theta)$ is a step function taking values $1/2\theta$ if $y \leqslant \theta$ and $1/(2(1-\theta))$ if $y > \theta$. Finally, assume that for each y_v, $u_v(x) = -|x - y_v|$ and that all voters have a common type conditional belief

---

[7] By $\#\{A\}$ we denote the cardinality of set A.

$\eta(\theta \mid y_v) = \eta(\theta)$ for all y_v. It is not difficult to see that a smoother version of the example will satisfy the conditions defined above.

2.2. Strategies, beliefs and equilibria

For convenience we make the technical assumption that $m_v(\cdot)$ is a measurable function. The poll response of respondent v is a function $m_v(y): X \to X$. This function indicates how a voter with ideal point y will answer a poll if she is in the sample P. We consider only equilibria with *symmetric* poll response functions—all poll respondents use the same function $m(\cdot)$.[8] Recall m_v is an individual poll response, $\mathbf{m}$ is a vector of poll responses and $m(\cdot)$ is a measurable poll response function—thus $m_v = m(y_v)$. Candidate stances are measurable functions of the statistic $s(\mathbf{m})$, which we denote $x_c(s): \mathbb{R}^n \to X$, for $c \in C$. Voting defines a ballot function $b(x_0, x_1, y): X^3 \to [0, 1]$ where the range $[0, 1]$ is the set of mixed strategies, with $b = p$ interpreted as voting for candidate 1 with probability p. The first two arguments of $b(\cdot, \cdot, \cdot)$ are the stances of the candidates, the third argument is the voter's ideal point. In principle, voters could condition their vote on the polling statistic, but since voters posses no private information about the candidates at the time of voting the polling statistic (and messages) are payoff irrelevant. In principle this information may be involved in complicated voting strategies that punish some types of poll replies but we ignore this possibility. Accordingly, we have suppressed the dependence of b on $s(\mathbf{m})$.

We assume that voters vote *sincerely*, so that

$$b(x_0, x_1, y) = \begin{cases} 1 & \text{if } |x_0 - y| > |x_1 - y|, \\ 0 & \text{if } |x_0 - y| < |x_1 - y|, \\ \frac{1}{2} & \text{otherwise.} \end{cases} \tag{4}$$

Since no voter has influence in the election any voting strategy is a best response to any other profile of voting strategies. The assumption of sincere voting selects the equilibrium in a voting game which is the limit of a sequence of equilibria (in weakly undominated strategies) to finite population games. The assumptions that (1) voting does not depend on $\mathbf{m}$ or $s(\mathbf{m})$, and (2) sincere voting strategies are used, are potentially circular and do limit the equilibrium set. We make these assumptions for one technical and one substantive reason. First, while refinements to continuum games are poorly defined, in a finite one-shot voting game involving just two fixed candidate positions weak dominance requires that voters ignore everything except the candidate locations and vote sincerely. Accordingly, in a finite population game, equilibria in which polling behavior has a direct effect on voting (aside from the indirect effect on candidate stances) involves weakly dominated behavior. Second, a defense of truthful polling that hinges on complicated voting strategies that select less desirable candidates in response to peculiar realizations of polling data seems underwhelming.

---

[8] We require polling strategies to be symmetric for two reasons. First, requiring that all identical agents take the same actions is a reasonable if not focal starting point for any analysis. Second, with many polling statistics the candidates will not be able to infer individual responses, and so an equilibrium in which poll response functions are not the same introduces inefficiency by making the polling statistic less informative.

We dispense with the technical details of constructing a continuum of random variables, and directly assume that if $b(x_0, x_1, y) = 1/2$ for a.e. y the probability, $\phi_c(x_c, x_{-c}, \theta)$, that c is the winner is $1/2$.[9] Given sincere voting and an arbitrary belief $\lambda(\theta)$ on Θ it follows from (2), (3) and (4) that candidate expected utility functions over policy platforms, $\mathbb{E}_{\lambda(\theta)} U_c(x_0, x_1)$, are of the form:

$$\mathbb{E}_{\lambda(\theta)} U_c(x_0, x_1) = \begin{cases} \lambda(\frac{x_0+x_1}{2}) & \text{if } x_c < x_{-c}, \\ 1 - \lambda\big(\frac{x_0+x_1}{2}\big) & \text{if } x_c > x_{-c}, \\ \frac{1}{2} & \text{otherwise.} \end{cases} \tag{5}$$

A poll respondent faces several forms of uncertainty. Uncertainty about θ is characterized by the belief $\eta(\theta \mid y)$. Uncertainty about the other poll respondent's ideal points, $\mathbf{y}_{-v} \in X^{T-1}$ is induced by uncertainty about θ and uncertainty about the draws from $F^{T-1}(\mathbf{y}_{-v}, \theta)$. For a fixed poll response function employed by $P \backslash v$ and mappings $x_c(s)$ a poll respondent, v, with ideal point y_v, offering response m_v has expected utility:

$$\begin{aligned} &\mathbb{E}_{\eta(\theta|y)} u(m_v, y_v) := \\ &\int\limits_{\Theta} \int\limits_{X^{T-1}} \begin{bmatrix} u(x_0(s(\mathbf{m}_{-v}(\mathbf{y}_{-v}), m_v)), y_v)\phi_0(x_0(s(\mathbf{m}_{-v}(\mathbf{y}_{-v}), m_v)), x_1(s(\mathbf{m}_{-v}(\mathbf{y}_{-v}), m_v), \theta) \\ +u(x_1(s(\mathbf{m}_{-v}(\mathbf{y}_{-v}), m_v)), y_v)\phi_1(x_0(s(\mathbf{m}_{-v}(\mathbf{y}_{-v}), m_v)), x_1(s(\mathbf{m}_{-v}(\mathbf{y}_{-v}), m_v), \theta) \end{bmatrix} \\ &\quad \times \mathrm{d}F^{T-1}(\mathbf{y}_{-v} \mid \theta)\, \mathrm{d}\eta(\theta \mid y). \end{aligned} \tag{6}$$

We now focus on the Bayesian updating of candidate beliefs about the location of the median voter. Let $\operatorname{prob}\{\mathbf{m} \in s^{-1}(\mathbf{A}) \mid \theta < k\}$ denote the probability that $s \in A$ is observed given that $\theta < k$, when respondents use strategy $m(\cdot)$. Formally this term is

$$\begin{aligned} &\operatorname{prob}\{\mathbf{m} \in s^{-1}(\mathbf{A}) \mid \theta_1 < k\} := \\ &\frac{\int_{\Theta} \int_{X^T} 1_{\{s(m_1(y_1),\ldots,m_T(y_T)) \in A'\}}(\mathbf{y}) \times 1_{\{\theta_1 < k\}}(\theta)\, \mathrm{d}F^T(\mathbf{y} \mid \theta)\, \mathrm{d}\eta(\theta)}{\int_{\Theta} 1_{\{\theta_1 < k\}}(\theta)\, \mathrm{d}\eta(\theta)}. \end{aligned} \tag{7}$$

Given the prior, $\eta(\cdot)$, a poll response strategy, $m(\cdot)$, and an observed statistic, s', the posterior distribution (termed a *belief*), $\pi(k \mid s)$, of the median ideal point θ is given by Bayes' rule,

$$\pi(k \mid s) = \frac{\eta(k)\operatorname{prob}\{\mathbf{m} \in s^{-1}(\mathbf{s}') \mid \theta < k\}}{\eta(k)\operatorname{prob}\{m \in s^{-1}(\mathbf{s}') \mid \theta < k\} + (1 - \eta(k))\operatorname{prob}\{\mathbf{m} \in s^{-1}(\mathbf{s}') \mid \theta > k\}}. \tag{8}$$

If poll respondents use the mapping $m_v(s)$ then uncertainty about θ and the randomness associated with selecting poll respondents induces a measure on the profile of poll responses $\mathbf{m}$. We use the shorthand s -a.e. to describe a statement that holds on the image $s(A)$ with A a full measure set of possible poll responses.

---

[9] Judd (1985) addresses the question of constructing a continuum of independent random variables and the possibility that a law of large numbers holds. The assumption here, that mixing by almost everyone results in a fair lottery over the two candidates is consistent with possible constructions in which a law of large numbers does or does not hold.

Definition 1. A candidate belief $\pi(k \mid s)$ is *consistent for a poll response strategy* $m(\cdot)$ if it is equivalent to (8) s -a.e.

Combining the concepts yields a straightforward equilibrium definition.

Definition 2. A symmetric strategy profile $\langle m^*(y), x_0^*(s), x_1^*(s), b^*(x_0, x_1, y)\rangle$ and a belief $\pi(k \mid s)$ are a *perfect Bayesian equilibrium* (PBE) if:

(1) For all s' in the image of $s(\cdot)$

$$\begin{aligned} x_0^*(s') &\in \arg\max_{x \in X} \mathbb{E}_{\pi(k|s')} U_0\bigl(x, x_1^*\bigr), \\ x_1^*(s') &\in \arg\max_{x \in X} \mathbb{E}_{\pi(k|s')} U_1\bigl(x_0^*, x\bigr). \end{aligned} \tag{9}$$

(2) Given that everyone other than v uses the mapping $m^*(\cdot)$ if sampled,

$$m^*(y) \in \arg\max_{m_v \in X} \bigl\{\mathbb{E}_{\eta(\theta|y)} u(m_v, y)\bigr\} \quad \text{for all } y \in X. \tag{10}$$

(3) Voting is sincere.

(4) $\pi(k \mid s)$ is consistent for $m^*(\cdot)$.

In the remainder of the paper we examine the extent to which poll respondents may tell the truth in a symmetric PBE.

Definition 3. A symmetric PBE is *truthful* if $m(y) = y$.

Since the main result establishes the non-existence of truthful PBE, there is no need to consider refinements to the concept of PBE. We now turn to the analysis.

3. Results

In this section we first establish several intermediate results and then turn to the main results: if A3 is satisfied then generically a truthful PBE does not exist; if A1–A2 and SMLR are satisfied then a truthful PBE does not exist.

3.1. Intermediate results

We first show that in every PBE the candidate stances will be $x_0 = x_1 = x^{\text{med}}(s)$ where this term solves $\pi(x^{\text{med}}(s) \mid s) = 1/2$. This result is equivalent to one of the extensions of the median voter theorem established by Calvert (1985).

Lemma 1 (Probabilistic median voter theorem). *For a given $s(\cdot)$, $m(\cdot)$ and posterior $\pi(\cdot \mid s)$, if voting is sincere the unique profile of candidate best responses is $x_0^*(s) = x_1^*(s) = x^{\text{med}}(s)$.*

The logic supporting the conclusion that this profile is a simultaneous best response is straightforward. When candidate 0 locates at $x^{\text{med}}(s)$ candidate 1 believes (based on her posterior belief) that the probability that the median voter is to either side of x_0 is $1/2$. Accordingly, if candidate 1 were to locate to the left (right) of x_0 the median voter would be closer to x_0 than x_1 with probability greater than a half, and thus 1 would win with probability less than a half. On the other hand, if voters vote for each candidate with equal probability when the candidates locate at the same point, then under the profile $x_0 = x_1 = x^{\text{med}}(s)$ candidate 1 wins with probability $1/2$. Since voters are indifferent between the two candidates, this voting strategy is optimal for them. Clearly when voters resolve ties in this manner, this profile of stances is the only pair or simultaneous best responses. In the appendix we complete the proof by showing that no other profile is supported by a different tie-break rule.

Combining Lemma 2 and condition 2 of Definition 2 yields a useful corollary—providing a simplified expression of the poll respondents' optimization problem.

Corollary 1. *The strategy profile* $\langle m^*(y), x^{\text{med}}(s), x^{\text{med}}(s), b^*(x_0, x_1, y)\rangle$ *with* $b^*(x_0, x_1, y)$ *sincere is supportable as a symmetric PBE iff for a.e.* $y \in X$, $m^*(y)$ *solves*:

$$m^*(y) \in \arg\max_{m_v \in X} \left\{ \int\limits_{\Theta} \int\limits_{X^{T-1}} u\big(x^{\text{med}}\big(s\big(\mathbf{m}_{-v}(\mathbf{y}_{-v}), m_v\big)\big), y_v\big)\, \mathrm{d}F^{T-1}(\mathbf{y}_{-v} \mid \theta)\, \mathrm{d}\eta(\theta \mid y) \right\}. \tag{11}$$

Another useful intermediate result demonstrates that with truthful strategies the function $x^{\text{med}}(s)$ inherits continuity from A1.

Lemma 2. *In a PBE with truthful strategies, if* A1 *is satisfied then* $x^{\text{med}}(s)$ *is a continuous function.*

Proof. Assume A1 and that messages are truthful. By truthfulness, since $F(y, \theta)$ is continuously differentiable in both y and θ the joint density $f(m, \theta)$ exists and is continuous in m and θ. Let the relation $\ll$ on $[0, 1]^T$ be defined as $a \ll b$ if $a_i < b_i$ for each $i = 1, \ldots, T$. For each $s' \in (0, 1]^T$ define the set $D(s') := \{m \in [0, 1]^T\colon\ s(m) \ll s'\}$ which is measurable by A1. Consider the function

$$L(s', k) = \int\limits_0^k \int\limits_{D(s')} f(m, \theta)\, \mathrm{d}m\, \mathrm{d}\theta. \tag{12}$$

To see that this is continuous in the first argument, fix k and consider a sequence s^n converging to s'. Consider the random variable $1_{m \in D(s^n)}(m)$. By A1 and the fact that $f(m, \theta)$ is atomless, as $s^n \to s'$ this random variable converges to $1_{m \in D(s')}(m)$ in measure. This and the fact that $|1_{m \in D(s^n)}(m)| \leqslant 1$ (is bounded) implies, by the bounded convergence theorem (Durrett, 1996), that

$$\lim_{s^n \to s'} \int_0^k \int 1_{m \in D(s^n)}(m) f(m,\theta)\,\mathrm{d}m\,\mathrm{d}\theta$$

$$= \int_0^k \int 1_{m \in D(s')}(m) f(m,\theta)\,\mathrm{d}m\,\mathrm{d}\theta = L(s',k). \tag{13}$$

Thus for each k, $L(s',k)$ is continuous in its first argument. Moreover, for each s', $L(s,k)$ is continuous in k because $f(\cdot,\cdot)$ is continuous in its second argument. Thus $L(\cdot,\cdot)$ is continuous in both arguments. Applying Bayes' rule, we have

$$\pi(k \mid s') = \frac{L(s',k)}{\eta(k)} \tag{14}$$

which as the ratio of continuous functions is continuous in both arguments on the support of θ. By this, the fact that $\pi(0 \mid s) = 0 < 1/2 < 1 = \pi(1 \mid s)$ and the intermediate value theorem, at least one value $x^{\mathrm{med}}(s)$ solving

$$\pi\bigl(x^{\mathrm{med}}(s) \mid s\bigr) = \frac{1}{2} \tag{15}$$

exists for each s. Moreover, since the prior has full support and the posterior is strictly increasing in k for any $s \in \{s'\colon \exists y \in [0,1]^T, s(y) = s'\}$, this value is unique. Finally, since we have shown that $\pi(k \mid s)$ is continuous in its second argument, the function $x^{\mathrm{med}}(s)$ is continuous in s. This result follows from the Theorem of the Maximum (Berge, 1963) by noting that $x^{\mathrm{med}}(s) = \arg\min_{x \in [0,1]} |\pi(k \mid s) - 1/2|$. □

As a demonstration that the non-existence of truthful PBE is not the result of specifying a model possessing no PBE we establish the existence of pooling PBE. In these equilibria poll respondents of every type send the same message. Accordingly, candidates do not learn anything from the poll, and simultaneously announce policies corresponding to the middle of their priors. Given that candidates ignore the poll, poll respondents have no incentive to send any message other than the uninformative message called for by the equilibrium.

Proposition 1. *For any $x \in X$, the following is a symmetric PBE*:

$$m^*(y) = x,$$

$$x_c^*(s) = \pi_{\eta(\theta)}^{-1}\left(\frac{1}{2}\right) \quad \text{for } c \in C,$$

$$b^*(x_0, x_1, y) \text{ is sincere},$$

$$\pi(k \mid s) = \eta(k) \quad \text{for any } s.$$

The class of pooling equilibria exhibited in Proposition 1 may not be robust to refinements in the spirit of Cho and Kreps (1987) and Banks and Sobel (1987). However, if one allows for completely mixed strategies by polling respondents, then robust pooling equilibria can be constructed.

3.2. The median

We now turn to the analysis of polling games when A3 is satisfied, i.e., $s(\mathbf{m}) = \text{median}(\mathbf{m})$. We show that generically, truthful PBE do not exist. The logic is as follows. A poll respondent's message only influences the statistic if her message is the median of the reported messages. It is usually the case that the middle of the posterior belief (conditioned on the sample median), $x^{\text{med}}(s)$, does not coincide with the sample median and small changes in one's response will have small (but real) changes in $x^{\text{med}}(s)$. This means that respondents that act as if they will be influential usually have an incentive to report a value of m that is more extreme than y. We now formalize this argument.

If poll respondents use truthful strategies, then for any $s \in [0, 1]$ the density (conditional on the candidate's information) of a candidate observing s given that $\theta < k$ is given by the expression

$$\begin{aligned}&\text{prob}_y\bigl\{\mathbf{y} \in s^{-1}(s) \mid \theta < k\bigr\}\\&\quad= \begin{pmatrix} T(T-1) \\ \frac{T-1}{2} \end{pmatrix} f(s \mid \theta_1 < k) F(s \mid \theta < k)^{\frac{T-1}{2}} \bigl(1 - F(s \mid \theta < k)\bigr)^{\frac{T-1}{2}},\end{aligned} \tag{16}$$

where we let

$$F(s \mid \theta < k) := \frac{\int_\Theta 1_{\{s' < s\}}(\theta) 1_{\{\theta < k\}}(\theta)\, \mathrm{d}F(s', \theta)\, \mathrm{d}\eta(\theta)}{\int_\Theta 1_{\{\theta < k\}}(\theta)\, \mathrm{d}\eta(\theta)}. \tag{17}$$

The term $f(s \mid \theta < k)$ is defined as the limit in the usual way. The existence of this density is ensured by the differentiability of $F(\cdot, \theta)$. Similarly, the terms which condition on $\theta > k$ are defined in the obvious manner. The relevant conditional probabilities for the poll respondents differ as they must also condition on the respondent's ideal point, y. Substituting the right-hand side of (16) into (8), setting the posterior equal to $1/2$ and rearranging yields

$$\left(\frac{(1 - \eta(k))}{\eta(k)}\right) = \left(\frac{f(s \mid \theta < k)}{f(s \mid \theta > k)}\right) \left[\left(\frac{F(s \mid \theta < k)}{F(s \mid \theta > k)}\right)\left(\frac{(1 - F(s \mid \theta < k))}{(1 - F(s \mid \theta > k))}\right)\right]^{\frac{T-1}{2}}. \tag{18}$$

Thus, if a candidate believes that poll-responses are truthful, Lemma 1 implies that she will locate at a policy $x^{\text{med}}(s) = k$ where k solves the above equation. Lemma 2 ensures that this location is a continuous function of the public messages. We can use Corollary 1, to express a local incentive compatibility condition for truthful response—that no agent has an incentive to send a slightly dishonest response.

Proposition 2. *If* A3 *is satisfied then a truthful PBE exists only if for a.e.* $s \in (0, 1)$ *either* $x^{\text{med}}(s)$ *is constant on a neighborhood of* s *or*

$$\left(\frac{(1 - \eta(s))}{\eta(s)}\right) = \left(\frac{f(s \mid \theta < s)}{f(s \mid \theta > s)}\right) \left[\left(\frac{F(s \mid \theta < s)}{F(s \mid \theta > s)}\right)\left(\frac{(1 - F(s \mid \theta < s))}{(1 - F(s \mid \theta > s))}\right)\right]^{\frac{T-1}{2}}. \tag{19}$$

Proof. Since A3 satisfies A1, Lemmas 1 and 2, imply that as a function of s the final policy $x^{\text{med}}(s)$ is continuous. By Corollary 1 in a truthful equilibrium with A3 it must be the case that for each $s \in (0, 1)$ either $x^{\text{med}}(s) = s$ or $x^{\text{med}}(s)$ is constant on a neighborhood of s. The equation corresponds to the case of former. □

One conjecture is that when the prior beliefs are diffuse (i.e. uniformly distributed, $\eta(k) = k$ on [0, 1]) as in the set up of our recurring example, truthful PBE will exist. This conjecture is based on the supposition that when the prior is uninformative candidates will form beliefs which have as their midpoint the observed sample median. This argument, however, turns out to not be true in general. To demonstrate this point we first consider the example of only one poll-respondent, and ask will the candidates' locate at the respondent's response (i.e., is the condition satisfied). In this case the condition simplifies to

$$\left(\frac{1-z}{z}\right) = \left(\frac{f(z \mid \theta < z)}{f(z \mid \theta > z)}\right). \tag{20}$$

But the ratio of conditional densities need not be identical to the prior hazard-rate. We demonstrate this in the recurring example with $f(y \mid \theta)$ a step function taking values $1/(2\theta)$ if $y \leqslant \theta$ and $1/(2(1-\theta))$ if $y > \theta$. Integrating over θ (which is assumed to be uniform) yields the following two conditional densities $f(z \mid \theta < z) = -(1/2)\ln(1-z)$ and $f(z \mid \theta > z) = -(1/2)\ln z$. In this case the ratio on the right-hand side of (20) is $\ln(1-z)/\ln z$ which is not equivalent to the left-hand side, $(1-z)/z$.

This example demonstrates that even with a single poll respondent assuming a diffuse prior is not sufficient to insure that a truthful PBE exists. In fact if we were to pick a parameterization for which the condition were true, small changes in the parameterization would cause the condition to fail. More precisely the condition is not satisfied on a generic subset of the parameter space. We now develop this argument.

Let $\Lambda_\Theta := \{\eta\colon\ \eta$ is a differentiable density on $\Theta\}$. We endow this space with the metric $d(\eta, \eta') := \max_{z\in[0,1]} |\eta(z) - \eta'(z)|$. Let $\Psi_\Theta := \{F\colon\ F(\cdot, \theta)$ is a continuous distribution with support [0, 1] that is continuous in $\theta\}$. We endow this space with the metric $\delta(F, F') := \max_{z\in[0,1]}\{\sup_{\theta\in\Theta} |F(z, \theta) - F'(z, \theta)|\}$. Let $\Gamma_\Theta := \Lambda_\Theta \times \Psi_\Theta$ endowed with the product topology Υ.

Proposition 3. *Fix T, Θ and assume that* A3 *is satisfied. The set of parameterizations $\langle \eta(\cdot), F(\cdot\,,\cdot)\rangle$ for which a truthful PBE exists has an empty interior in Υ.*

Proof. Assume there is a truthful PBE when A3 is satisfied at parameterization $\langle \eta(\cdot), F(\cdot\,,\cdot)\rangle \in \Gamma_\Theta$ In any open set (relative to Υ) there exists a parameterization $\langle \eta'(\cdot), F'(\cdot\,,\cdot)\rangle$ for which $F(z, \theta) = F'(z, \theta)$ for every $z \in [0, 1]$ (and therefore the distributions are identical) but $\eta'(z) = \alpha\eta(z) + \beta z^2 + (1 - \alpha - \beta)z \neq \eta(z)$ for $\alpha, \beta > 0$ and sufficiently small. Accordingly, the following conclusions are true

$$\frac{1-\eta(z)}{\eta(z)} \neq \frac{1-\eta'(z)}{\eta'(z)}, \tag{21}$$

$$\mathrm{d}\left(\frac{1-\eta(z)}{\eta(z)}\right)\Big/\mathrm{d}z \neq \mathrm{d}\left(\frac{1-\eta'(z)}{\eta'(z)}\right)\Big/\mathrm{d}z, \tag{22}$$

for each $z \in (0, 1)$. Since a truthful PBE exists at $\langle \eta(\cdot), F(\cdot\,,\cdot)\rangle$ for each $s \in (0, 1)$ either the equality condition is satisfied at this parameterization or else the solution is constant on a neighborhood of s. But the above two conclusions demonstrate that neither condition is satisfied at $\langle \eta'(\cdot), F'(\cdot\,,\cdot)\rangle$ and thus the subset of Γ_Θ for which truthful PBE exist has an empty interior. □

Moreover, there are no parameterizations for which truthful PBE exist for more than one poll sample size. The following result is proven in Appendix A.

Proposition 4. *If* A3 *is satisfied and there is a truthful PBE with the parameterization* $\langle \eta(\cdot), F(\cdot,\cdot), T\rangle$ *then if* $T' \neq T$ *there is not a truthful PBE in the parameterization* $\langle \eta(\cdot), F(\cdot,\cdot), T'\rangle$.

The intuition behind these results is as follows. When the condition in Proposition 2 is satisfied, a candidate will form a posterior belief that has as its midpoint the observed polling statistic. However this phenomena is rare. In most cases the candidates will locate at a policy that is somewhere between the midpoint of their prior $\eta(\cdot)$ and the observed statistic. The interpretation of these two results is straightforward. When the sample median is the polling statistic truthful PBE can exist. However, their existence is knife-edged. Any change in the polling sample size will result in the disappearance of the truthful PBE and holding the size fixed there are arbitrarily small changes in the distributions that result in a game with no truthful PBE.

3.3. Strictly monotone statistics

If a strictly monotone statistic (say the sample average) is used, truthful PBE are even harder to support. If $s(\cdot)$ is the sample average and respondents are truthful, the median of the posterior (and therefore the final policy) is strictly increasing in every poll respondent's response. A poll respondent with an extreme ideal point would have an incentive to claim to be even more extreme. Returning to our recurring example, suppose that all respondents are truthful and candidates believe that all participants will be truthful. Given a response of m_v the expected value of s is $(T-1)/(2T) + m_v/T$ which is between m_v and $1/2$. Moreover, given m_v the expected location of x^m is between $(T-1)/(2T) + m_v/T$ and $1/2$. In the proof of the next result we show that if messages are truthful then the expected value of x^{med} is a continuous and monotone function of m_v. This means that for a respondent with y_v close to 0, truthful response results in expected utility $-\mathbb{E}x^{\text{med}}(y) + y_v$ and a deviation to $m_v < y_v - \varepsilon$ (for ε small) results in expected utility $-\mathbb{E}x^{\text{med}}(y) + \delta + y_v$ with δ small. Thus the deviation is desirable. While quite simple, this logic is used to show that in any polling game satisfying SMLR and A1–A2 there cannot be a truthful PBE.

Proposition 5. *If* A1–A2 *are satisfied and SMLR holds then there is not a truthful PBE.*

Proof. By way of contradiction assume that A1–A2 and SMLR hold and there is a truthful PBE. Given A2 this implies that truthful response is optimal for every ideal point,

$$y \in \arg\max_{m_v \in X} \left\{ \int_{\Theta} \int_{X^{T-1}} u\big(x^{\text{med}}\big(s\big(\mathbf{m}_{-v}(\mathbf{y}_{-v}), m_v\big)\big), y\big)\, \mathrm{d}F^{T-1}(\mathbf{y}_{-v} \mid \theta)\, \mathrm{d}\eta(\theta \mid y) \right\}. \tag{23}$$

Step 1. By A1 and Lemma 2, $x^{\text{med}}(s)$ is a continuous function of s.By A1 again this means that $x^{\text{med}}(s(m))$ is a continuous function of m_v for each $\mathbf{m}_{-v}$. This and the continuity of $u(\cdot,\cdot)$ means that the objective function (23) is continuous in m_v and y.

Step 2. We now establish that for some $\varepsilon > 0$ if $y < \varepsilon$ then

$$\operatorname{prob}\big\{x^{\mathrm{med}}\big(s\big(\mathbf{m}_{-v}(\mathbf{y}_{-v}), m_v\big)\big)r < y\big\} := \int\limits_{\Theta}\int\limits_{X^{T-1}} 1_{\{\mathbf{y}_{-v}: x^{\mathrm{med}}(s(\mathbf{m}_{-v}(y_{-v}), y_v)) < y\}}(y_{-v})\,\mathrm{d}F^{T-1}(\mathbf{y}_{-v} \mid \theta)\,\mathrm{d}\eta(\theta \mid y) = 0. \tag{24}$$

Since $\eta(\theta \mid y)$ has full support, and T is finite $x^{\mathrm{med}}(s(\mathbf{m})) > 0$ if $\mathbf{m} = \mathbf{0}$. Since $x^{\mathrm{med}}(s(\mathbf{m}))$ is continuous in $\mathbf{m}$ the result of this step is established.

Step 3. We now establish that for $y < \varepsilon$ (the same ε as in step 2) and some $\delta > 0$, $x^{\mathrm{med}}(s(\mathbf{m}_{-v}(\mathbf{y}_{-v}), y - \delta)) < x^{\mathrm{med}}(s(\mathbf{m}_{-v}(\mathbf{y}_{-v}), y))$ with probability 1. First, since the y_v are conditionally independent draws from $f(y, \theta)$, SMLR implies that $(y_1, \ldots, y_T, \theta)$ are affiliated random variables (also termed multivariate totally positive of order 2, see Karlin and Rinott, 1980). Given this, Theorem 4.1 of Karlin and Rinott implies that for any increasing function $g(\mathbf{m}) : [0, 1]^T \to [0, 1]$, the conditional expectation, $\mathbb{E}(g(\mathbf{m}) \mid \theta)$ is increasing in θ. Fix the mapping $s(\cdot)$ satisfying A2. The composition of each coordinate of s and any increasing function is an increasing function of the form $g(\mathbf{m}) : [0, 1]^T \to [0, 1]$. Accordingly, this theorem implies that the random vector $(s^1(\mathbf{m}), \ldots, s^n(\mathbf{m}), \theta)$ is affiliated. In this case it is well known that for the posteriors on θ given $s(\mathbf{m})$ (denoted $F(\theta \mid s(\mathbf{m}))$, if $s(\mathbf{m}') > s(\mathbf{m})$ then $F(\theta \mid s(\mathbf{m}'))$ dominates $F(\theta \mid s(\mathbf{m}))$ in terms of likelihood ratio (and thus first order stochastic dominance). This implies that in a truthful equilibrium, $x^{\mathrm{med}}(s(\mathbf{m})) < x^{\mathrm{med}}(s(\mathbf{m}'))$ if $\mathbf{m} < \mathbf{m}$. Thus, $x^{\mathrm{med}}(s(\mathbf{m}_{-v}(\mathbf{y}_{-v}), y - \delta)) < x^{\mathrm{med}}(s(\mathbf{m}_{-v}(\mathbf{y}_{-v}), y))$ for all $\mathbf{y}_{-v}$ as $(\mathbf{m}_{-v}(\mathbf{y}_{-v}), y - \delta) < (\mathbf{m}_{-v}(\mathbf{y}_{-v}), y)$.

Step 4. We now establish the contradiction—if $y < \varepsilon$ (the same ε as in step 2):

$$y \notin \arg\max_{m_v \in X}\left\{\int\limits_{\Theta}\int\limits_{X^{T-1}} u\big(x^{\mathrm{med}}\big(s\big(\mathbf{m}_{-v}(\mathbf{y}_{-v}), m_v\big)\big), y\big)\,\mathrm{d}F^{T-1}(\mathbf{y}_{-v} \mid \theta)\,\mathrm{d}\eta(\theta \mid y)\right\}. \tag{25}$$

By steps 1–3 there exists a $\delta > 0$ s.t.

$$\begin{aligned}&\int\limits_{\Theta}\int\limits_{X^{T-1}} u\big(x^{\mathrm{med}}\big(s\big(\mathbf{m}_{-v}(\mathbf{y}_{-v}), y - \delta\big)\big), y\big)\,\mathrm{d}F^{T-1}(\mathbf{y}_{-v} \mid \theta)\,\mathrm{d}\eta(\theta \mid y)\\ &\quad > \int\limits_{\Theta}\int\limits_{X^{T-1}} u\big(x^{\mathrm{med}}\big(s\big(\mathbf{m}_{-v}(\mathbf{y}_{-v}), y\big)\big), y\big)\,\mathrm{d}F^{T-1}(\mathbf{y}_{-v} \mid \theta)\,\mathrm{d}\eta(\theta \mid y).\end{aligned} \tag{26}$$

This follows from $\operatorname{prob}\{x^{\mathrm{med}}(s(\mathbf{m}_{-v}(\mathbf{y}_{-v}), y)) < y\} = 0$, $x^{\mathrm{med}}(s(\mathbf{m}_{-v}(\mathbf{y}_{-v}), y - \delta)) < x^{\mathrm{med}}(s(\mathbf{m}_{-v}(\mathbf{y}_{-v}), y))$ with probability 1, and the fact that the respondents utility function is continuous and strictly decreasing in x for $x > y$. Thus we have exhibited a non-null set of ideal points for which truthful response is not a best response when all other respondents are truthful. □

A2 (strict monotonicity) and the strict monotone likelihood ratio condition are used to establish that if poll respondents use truthful strategies, respondents with types corresponding to 0 (or alternatively 1) would like to pull the candidates' inferences in a particular direction. A1 (continuity) is used to show that this result holds for an open set of extreme types. We conjecture however, that this result holds on a larger set of polling statistics, Specifically, a weakening of strict monotonicity, to a condition like strict monotonicity with positive probability seems possible. Moreover, without SMLR, the presence of a profitable local deviation hinges only on the conditional expectation $\mathbb{E}(u_v(x^{\text{med}}(s(\mathbf{m}_{-v}(y_{-v}), y-\delta)) - u_v(x^{\text{med}}(s(\mathbf{m}_{-v}(y_{-v}), y-\delta)) \mid y)$ not being constant on an open set near $y=0$ for small values of δ. Just as Proposition 3 replaces SMLR with a local non-constant condition, it seems possible to establish a version of the above result that replaces SMLR with an analogue to the local non-constant condition in Proposition 3.

4. A binary message space

While the above analysis highlights the incentives for dissemination, its predictive applicability is limited because in practice polls tend to limit the number of responses open to respondents. To focus on the incentives in this case we now consider a modified polling game in which poll respondents are asked a question with two possible answers (e.g. which candidate do you prefer, or which party do you identify with). Thus, the message space is now $m_v \in \{0,1\}$. We assume that the statistic is $s(\mathbf{m}) = T^{-1}\sum_{v\in P} m_v$. One interpretation is that the question asks respondents which party they support. Such a game cannot possess a fully-revealing PBE because the message space is too small relative to the type space. Nonetheless we show that this game has a reasonable partially revealing PBE. In the next section the analysis of the binary message game is used to characterize a partially revealing equilibrium in the basic model in which $m_v \in [0,1]$ and either $s(\mathbf{m}) = \mathbf{m}$ or $s(\mathbf{m}) = T^{-1}\sum_{v\in P} m_v$.

The fact that respondents each have distinct posterior beliefs $\eta(\cdot \mid y)$ introduces a difficulty here. It is possible that two respondents with different ideal points $y < y'$ have such different posterior beliefs on θ that the agent with ideal point y thinks that most voters are to the left of her and the voter with ideal point y' thinks most voters are to the right of her. Technically, we will require that the posterior $\eta(\cdot \mid y)$ is close to constant in y. This requires a joint condition on $\eta(\cdot)$ and $F(\cdot,\theta)$ for which $\eta(\cdot \mid y)$ and $\eta(\cdot \mid y')$ are very similar even if y and y' are distinct. The appropriate notion of closeness here is captured by the metric

$$d\big(\eta(\cdot \mid y), \eta(\cdot \mid y')\big) = \max_{z\in[0,1]} \left| \int F(z \mid \theta)\,\mathrm{d}\eta(\theta \mid y) - \int F(z \mid \theta)\,\mathrm{d}\eta(\theta \mid y') \right|. \tag{27}$$

Our recurring example represents a stylized version of the case where this distance is always small– $\eta(\cdot \mid y)$ does not depend on y at all. Substantively, the assumption that this distance is small is quite defensible. One would expect voters to have similar views about the shape of the distribution of voter ideal points regardless of their individual preferences. The limiting case where one assumes that $\eta(\cdot \mid y)$ is constant in y captures the case where a room full of voters (with non-identical preferences), would agree over the odds on any bet about the location of the median voter (Auman, 1976). If this distance is always small then there is a particularly simple “cutpoint” equilibrium.

Proposition 6. *If SMLRis satisfied and $d(\eta(\cdot \mid y), \eta(\cdot \mid y'))$ is sufficiently small for every y, y' then there exists a $z^* \in (0, 1)$ for which the following is a PBE*:

$$m^*(y) = \begin{cases} 0 & \textit{if } y < z^*, \\ 1 & \textit{otherwise}, \end{cases}$$

$$x_c^*(s) = \pi_{\text{post}}^{-1}\left(\frac{1}{2} \,\Big|\, s\right) \quad \textit{for } c \in C,$$

$$b^*(x_0, x_1, y) \textit{ is sincere},$$

$$\pi_{\text{post}}(k \mid s) \textit{ satisfies } (8) \quad \textit{for any } s.$$

The endogenous language equating $m = 0$ with a low ideal point is of course arbitrary, and the alternative case in which $m = 0$ is identified with a high ideal point can also be supported. In contrast, the cutpoint z^* is not arbitrary. Any such cutpoint must satisfy an indifference condition which is derived in the proof (in Appendix A). Intuitively, if poll respondents use the equilibrium strategy then a respondent with ideal point z^* will be indifferent between sending either message when she compares her expected utility over each of the two possible messages she can send. Recall that these expected utilities integrate over uncertainty about the other $T - 1$ poll respondents' ideal points.

An interesting question is how the cutpoint(s) z^* are related to the underlying primitives of the environment. It is straightforward to show that when the prior beliefs on θ are skewed to say the left (right) the cutpoint(s) z^* are to the left (right) of the midpoint of the policy space.

Proposition 7. *If $\eta(\cdot)$ is skewed to the left (right), i.e., $\eta(1/2) > (<)\ 1/2$, then for T sufficiently large any equilibrium cutpoint satisfies, $z^* < (>)\ 1/2$.*

The intuition behind this result is as follows. When it is more likely that the population median is left of the midpoint of the policy space, poll respondents anticipate that policy is likely to be to the left of the midpoint as well. Accordingly, a poll respondent with an ideal point close to the midpoint of the policy space will anticipate being to the right of policy, and accordingly she will want to move policy to the right. In equilibrium the endogenous cutpoint needs to be sufficiently leftist so that a respondent with an ideal point corresponding to the cutpoint won't anticipate being right or left of policy. The prediction of Proposition 6 is consistent with a degree of efficient information aggregation. When the population is skewed to the left, a cutpoint in the middle of the policy space results in a statistic that is an inefficient discriminator. More information is gained by moving the cutpoint to the left. This is exactly the phenomena that occurs in equilibrium.

This last finding demonstrates a potentially important problem in interpreting polling data. Since the messages (observed) have endogenous meaning, interpretation requires knowledge of the equilibrium (unobserved). Proposition 6 states that to the extent that prior beliefs are not static over time, intertemporal comparison of polling data may require some degree of filtering. If the cutpoint changes from one poll to the next then the comparison of summary statistics across polls will be misleading. Finally, it should be noted that the analysis does not suggest that candidates are fooled by respondents. In equilibrium candidates understand the incentives and know the cutpoint z^*. Thus, they correctly filter the data. We use the analysis to forward the conclusion that scholars might also want to think about how to interpret the polling messages and filter the data.

5. Partial revelation in basic polling games

In this section we return to basic polling games—respondents are asked to select $m_v \in [0,1]$. We consider two different statistics, showing that simple partially revealing PBE exist. If the statistic $s(\mathbf{m}) = \mathbf{m}$ is revealed to the candidates a partially revealing equilibria that mimics the cutpoint equilibrium characterized above will exist. Specifically, if candidates think that respondents are using a mixed strategy that puts probability one on the interval $[0, z^*)$ when $y < z^*$ and probability one on the interval $[z^*, 1]$ when $y \geqslant z^*$ then a respondent faces exactly the same calculus in deciding whether to select $m_v > (<)\ z^*$ in this game as a respondent in the binary message model in deciding whether to select m_v equal to 0 or 1. Moreover, since any m is feasible under these message strategies there are no off-the path beliefs. Accordingly given consistent beliefs (relative to this message strategy) the candidate response characterized in Proposition 6 is sequentially rational. A similar pure strategy PBE exists (types to the left of z' announce 0 and types to the right announce 1) if candidates use off the path beliefs that do not create an incentive for moderate responses. One way to do this is to let candidates interpret messages in the interior of the message space as coming from a $y_v = 0$ type. Accordingly, we are left with the following result.

Corollary 2. *Consider a polling environment. If the equilibrium characterized in Proposition* 6 *exists for the cutpoint value z' in the binary polling game then in the basic polling game with message space* $[0, 1]$, *statistic $s(\mathbf{m}) = \mathbf{m}$ and this polling environment*

(1) *for any two distributions G_1 with support* $[0, z']$ *and G_2 with support* $[z', 1]$ *there is a partially revealing mixed strategy PBE with mixed message strategy*

$$pr(m_v \leqslant m \mid y_v) = \begin{cases} G_1(m) & \text{if } m \leqslant z' \text{ and } y_v \leqslant z', \\ 1 & \text{if } m > z' \text{ and } y_v \leqslant z', \\ G_2(m) & \text{if } m > z' \text{ and } y_v > z', \\ 0 & \text{if } m \leqslant z' \text{ and } y_v > z'; \end{cases}$$

(2) *there is a pure strategy equilibrium with the message function*

$$m_v(y) = \begin{cases} 0 & \text{if } y < z', \\ 1 & \text{otherwise.} \end{cases}$$

A similar construction can be used to characterize a partially revealing equilibrium when the sample average is used. Consider a basic polling game with the sample average, and suppose the message function is as in part 2 of the corollary. On the path the set of supportable values of s is $S = \{0, 1/T, 2/T, \dots, q/T, \dots, 1\}$ where q is an integer between 1 and T. For $n \in [0, 1]$ let

$$\lfloor n \rfloor := \sup\Bigl\{\arg\min_{j \in S} |j - n|\Bigr\} \tag{28}$$

denote the result of rounding n to the closest element of S. Consider beliefs that interpret s as evidence that $T\lfloor s \rfloor$ respondents have ideal points to the left of z' and $T - T\lfloor s \rfloor$ respondents have ideal points to the right of z'. Given these beliefs if all respondents other than v use the message strategy

$$m_v(y) = \begin{cases} 0 & \text{if } y < z', \\ 1 & \text{otherwise,} \end{cases} \tag{29}$$

every possible deviation has the same effect as either a message of 0 or 1, and thus if the characterized equilibrium exists in the binary message case then the conjectured message strategy is sequentially rational given the described beliefs. Moreover, given this message profile only values of s in S occur on the path, and thus the beliefs are consistent.

Corollary 3. *Consider a polling environment . If the equilibrium characterized in Proposition* 6 *exists for the cutpoint value z' in the binary polling game then in the basic polling game with message space* $[0, 1]$, *statistic $s(\mathbf{m}) = T^{-1} \sum_v m_v$ and this polling environment there exists an equilibrium in which*

$$m_v(y) = \begin{cases} 0 & \textit{if } y < z', \\ 1 & \textit{otherwise.} \end{cases}$$

While this equilibrium involves beliefs in which candidates sometimes ignore potential information, a reasonable interpretation can be offered. Candidates expect that respondents are giving polarized responses and they round the polling statistics. Respondents on the other hand anticipate that candidates are going to be rounding in their interpretation of polling statistics. This approach utilizes off the path beliefs to flatten the relationship between messages and policy locations and represents and example of a more general point that the PBE concept sometimes allows off the path beliefs to mimic a form of commitment in signaling games (Baron and Meirowitz, 2004). A similar construction can be used to characterize cutpoint equilibria when the median is used (albeit a different cutpoint will be used).

The equilibrium in this last corollary is similar to the equilibria in De Sinopoli and Iannantuoni (2003). They find that in proportional rule games with a continuum of voters and many candidates, all votes go to one of the two extreme parties. In finite population games they find that nearly all voters cast ballots for one of the extreme voters. Here, we characterize an equilibria to a finite respondent game in which all respondents are polarized. In principal respondents with types in the middle of the space may have an incentive to be less polarized, but the beliefs in the above equilibrium flatten things out enough to get rid of this potential incentive problem. It is possible that other equilibria exist in which respondents near the cutpoint send messages in the interior of the message space. The similarity between this equilibrium and the one in De Sinopoli and Iannantuoni demonstrates the presence of a somewhat pervasive equilibrium force for polarized communication between masses and the elites. While richer partially revealing equilibria might exist we only address this simple type of equilibrium to demonstrate the relationship between binary polling games and basic polling games with larger message spaces.

A note about the polarization results is in order. In the polling games considered here (as well as the model of De Sinopoli and Iannantuoni) the policy space is bounded and polarization is interpreted as reporting to have a type on the boundary of the policy space. If instead the policy space were unbounded (say the real line), then the notion of polarization is less well defined. Given the conjecture that, in a partially revealing equilibrium, strategies map into a bounded subset of the message space a suitably extreme respondent might have an incentive to send a message outside of the specified subset. This type of argument can be used to show that the equilibrium supporting Corollary 3 cannot be directly extended to the case of an unbounded policy space.[10] However, with $s(\mathbf{m}) = \mathbf{m}$, beliefs that interpret messages outside a compact set to be uninformative can be used to extend Corollary 2 to the case of an unbounded policy space.

6. Discussion

The motivating assumptions of the analysis are:

(i) polls are of value to candidates because they resolve some form of uncertainty about voter preferences;
(ii) candidates use the information provided by polls to select policy stances.

The analysis suggests that there may be a problem with an explanation of polling that allows for strategic response and posits that poll respondents are honest as there is an incentive compatibility problem. Poll respondents have an incentive to misrepresent their preferences if candidates are selecting policies in a way that is responsive to the polling statistic and the respondent would like to pull the expected policy in a particular direction. Put simply, some poll respondents have an incentive to lie. In the case of polls that ask respondents their preferences this incentive incompatibility generally results in the non-existence of truthful PBE. In contrast to Moulin's result that the median is incentive compatible, here the presence of a finite polling sample and a prior belief means that even if the statistic is the sample median the final policy will not correspond to the sample median and thus respondents have an incentive to be dishonest.

In considering the incentive to be dishonest (when it is conjectured that everyone else is truthful) we can make clear statements regarding the affects of increasing the poll sample size, T. In the case of the sample median, generically the amount by which agents wish to lie will vanish as T grows (e.g., $\max_{y_v \in [0,1]} |m_v(y_v) - y_v|$ tends to 0). This is because the larger is T, the more heavily the posterior weights the data $s(m)$. Accordingly, the gap between candidate stances and the polling statistic vanishes as T grows. In contrast, when the polling statistic is strictly monotone the incentive to be dishonest does not decrease as T grows. As an example consider the case where the polling statistic is the sample average. While the influence of any one response m_v on $s(m)$ vanishes as T grows the incentive to manipulate $s(m)$ does not vanish. It is easy to construct examples where as T grows the optimal response (if everyone else is truthful) for a respondent that is left (right) of center

---

[10] In the reality of pre-election polls, time and technology constraints seem sufficient to justify the assumption that the message space is bounded. Moreover, the assumption that the policy space is itself bounded seems no more objectionable than the assumption that it is unidimensional.

tends to $m_v = 0$ (1). Moreover as Corollary 3 demonstrates, even with a finite number of poll respondents there are equilibria in which every poll response is 0 or 1 when s is the sample average.

Analysis of the binary message case and applications of Proposition 6 to basic polling games (Corollaries 2 and 3) demonstrate that partially revealing PBE exist. It is important to note that the information revealed by poll respondents needs to be properly interpreted. In the model, the meaning of left and right messages is determined in equilibrium. More precisely, the cutpoint partitioning respondents into *leftists* and *rightists* is endogenous. Accordingly, analysts using polling data should be careful in interpreting the polling statistic, as a naive interpretation can lead to incorrect inferences. Imputation of the meaning of a message (and thus a polling statistic) requires consideration of the equilibrium strategies played. These strategies will vary with the information available to poll respondents. This last fact suggests the possibility of testing comparative statics of the model and determining which polls are more likely to be influenced by the incentives to be strategic. Loosely speaking, in polls involving prior beliefs about the location of the median voter that are symmetric and candidates that have close to equal ex-ante prospects of winning the equilibrium cutpoint will closely approximate the middle of the policy space. In contrast asymmetries in the parameters of the model result in cutpoints that are not in the center of the policy space and greater care needs to be taken in interpreting polling data.

Despite the negative perspective this paper takes, showing that truthful response to polls is not consistent with strategic behavior, there is also a positive conclusion. Actual polls tend to involve small message spaces (questions with 2, 3, and 7 categories are common). The theory developed here suggests that this artificial smallness of the message space may not be inefficient as very rich message spaces cannot actually be effectively used in equilibrium. Second, Corollaries 2 and 3 demonstrate how equilibrium behavior can convert a setting with a rich message space to one that effectively has a binary message space. Finally, while we have made some progress in considering partially revealing equilibria in polling games, this analysis is far from complete. Future work establishing bounds on the amount of information conveyed in equilibria and comparisons of these bounds across different statistics may further our understanding of information transmission and aggregation in elections.

Acknowledgments

I thank Chris Achen, David Baron, Jonathan Bendor, Mark Fey, John Duggan, Tim Groseclose, Keith Krehbiel, and Thomas Palfrey for helpful discussions and the anonymous reviewers for helpful comments. An early version of this paper was presented at the First Game Theory Congress in Bilbao, Spain and the Midwest Political Science Association's annual meeting in Chicago, IL in 2000.

Appendix A

Proof of Lemma 1. Fix g, s and $m_v(y)$.

Existence. Assume that $x_1^*(s) = x^{\mathrm{med}}(s)$. By (5) we have

$$\mathbb{E}_{\pi(k)} U_0\big(x_0, x_1^*(s)\big) = \begin{cases} \pi\big(\frac{x_0 + x^{\mathrm{med}}(s)}{2} \mid s\big) & \text{if } x_0 < x^{\mathrm{med}}(s), \\ 1 - \pi\big(\frac{x_0 + x^{\mathrm{med}}(s)}{2} \mid s\big) & \text{if } x_0 > x^{\mathrm{med}}(s), \\ \frac{1}{2} & \text{otherwise.} \end{cases} \tag{30}$$

But by the definition of $\pi(\cdot \mid s)$ we have

$$\pi\left(\frac{x_0 + x^{\mathrm{med}}(s)}{2} \,\bigg|\, s\right) < \frac{1}{2} \quad \text{if } x_0 < x^{\mathrm{med}}(s), \quad \text{and}$$

$$1 - \pi\left(\frac{x_0 + x^{\mathrm{med}}(s)}{2} \,\bigg|\, s\right) < \frac{1}{2} \quad \text{if } x_0 > x^{\mathrm{med}}(s).$$

Thus 0's best response is $x_0^*(s) = x^{\mathrm{med}}(s)$. The proof that 1's best response to $x_0^*(s) = x^{\mathrm{med}}(s)$ is $x_1^*(s) = x^{\mathrm{med}}(s)$ is analogous.

Uniqueness. By way of a contradiction suppose $x_0^*(s) = x_1^*(s) = x^{\mathrm{med}}(s)$ does not hold for a profile of best responses. There are 4 cases:

(i) If $x_c^*(s) \leqslant x_{-c}^*(s) < x^{\mathrm{med}}(s)$ then $\mathbb{E}_{\pi(k|s)} U_c(x_0^*(s), x_1^*(s)) \leqslant 1/2$ and a deviation to $x_c^*(s) = x^{\mathrm{med}}(s)$ would result in $\mathbb{E}_{\pi(k|s)} U_c(x_0^*(s), x_1^*(s)) > 1/2$. This is a contradiction.
(ii) The case of $x_c^*(s) \geqslant x_{-c}^*(s) > x^{\mathrm{med}}(s)$ is analogous.
(iii) If $x_c^*(s) < x^{\mathrm{med}}(s) \leqslant x_{-c}^*(s)$ then a deviation by c to $\gamma x_c^*(s) + (1-\gamma) x_{-c}^*(s)$ for $\gamma \in (0, 1)$ would increase $\mathbb{E}_{\pi(k|s)} U_c(x_0^*(s), x_1^*(s))$. This is a contradiction.
(iv) The case of $x_c^*(s) \leqslant x^{\mathrm{med}}(s) < x_{-c}^*(s)$ is analogous. □

Proof of Corollary 1. $\Longleftarrow$: Assume that (11) is satisfied by the function $m^*(\cdot)$. Since a version of conditional probability exists a posterior $\pi_{\mathrm{post}}(k \mid s)$ that is consistent for $m^*(\cdot)$ exists. By Lemma 1 the n-tuple $\langle m^*(\cdot), x^{\mathrm{med}}(s), x^{\mathrm{med}}(s), b^*(x_0, x_1, y)\rangle$ with $b^*(x_0, x_1, y)$ sincere is a strategy profile satisfying conditions (1)–(3) of Definition 2.

$\Longrightarrow$: By way of contradiction assume that $\langle m^*(\cdot), x^{\mathrm{med}}(s), x^{\mathrm{med}}(s), b^*(x_0, x_1, y)\rangle$ and $\pi_{\mathrm{post}}(k \mid s)$ is a PBE and $m^*(\cdot)$ does not satisfy (11) for a non-null subset of X. This implies that condition (2) of Definition 2 is not satisfied—contradicting the assumption that this profile is a PBE. □

Proof of Proposition 1. Clearly (3) of Definition 2 is satisfied. Given $\pi(k \mid s)$ and Lemma 2, $x_c^*(s)$ satisfies condition (1) of Definition 2. Given $\pi(k \mid s)$ and Corollary 1 any mapping $m(\cdot)$ satisfies condition (2) of Definition 2. Finally given $m^*(\cdot)$ on the path of play $\pi(k \mid s) = \eta(k)$. Thus $\pi(k \mid s)$ is consistent. □

Proof of Proposition 4. Without loss of generality assume $T' > T$. Assume there is a truthful PBE at $\langle \eta(\cdot), F(\cdot\,, \cdot), T\rangle$. Then by Proposition 2 it must be the case that for each

$z \in (0,1)$ either

$$\left(\frac{(1-\eta(z))f(z \mid \theta > z)}{\eta(z)f(z \mid \theta < z)}\right) = \left[\left(\frac{F(z \mid \theta < z)}{F(z \mid \theta > z)}\right)\left(\frac{(1-F(z \mid \theta < z))}{(1-F(z \mid \theta > z))}\right)\right]^{\frac{T-1}{2}} \tag{31}$$

or $x^{\mathrm{med}}(s)$ is constant on a neighborhood of z. But since the left-hand side is less than 1 for some z', this implies that the right-hand side is less than 1 at z'. Since the left-hand side is the same when the poll size is T or T', and the right-hand side is strictly smaller at T' than T when evaluated at z', we have shown that equality condition cannot be true at z' with T'. Now on any neighborhood of z' the right-hand side function is flatter at T' than at T. Thus it cannot be the case that the conditions in Proposition 2 are satisfied at the parameterization $\langle \eta(\cdot), F(\cdot,\cdot), T' \rangle$. □

Proof of Proposition 6. Given Corollary 1 it is sufficient to verify that there exists a $z^* \in (0,1)$ for which the specified poll-strategy constitutes a simultaneous best response. We first assume that $\eta(\theta \mid y) = \eta(\theta)$ for all y, and then extend the result to the case where $d(\eta(\theta \mid y), \eta(\theta \mid y'))$ is sufficiently small. Let $x_c^*(k,z) = x_c^*(s)$ when the strategy uses cutpoint z and $k = Ts(\mathbf{m})$ (the number of 1 responses). If the poll respondents $j \in P \backslash i$ use the strategy

$$m(y) = \begin{cases} 0 & \text{if } y < z, \\ 1 & \text{otherwise,} \end{cases} \tag{32}$$

then the difference in expected utility between a response of $m = 1$ and $m = 0$ is

$$\delta(y,z) := \int_{\Theta} \sum_{k=0}^{T-1} \binom{T-1}{k} \begin{bmatrix} u(x_c^*(k+1,z),y) \\ -u(x_c^*(k,z),y) \end{bmatrix} F(z \mid \theta)^{T-1-k}\big[1 - F(z \mid \theta)\big]^k \,\mathrm{d}\eta(\theta). \tag{33}$$

Step 1. We First show that at some z^* we have $\delta(z^*, z^*) = 0$. By SMLR $x_c^*(k+1,z) > x_c^*(k,z)$ if $z \in (0,1)$ and $k < T$, so for some $\varepsilon > 0$, $\delta(\varepsilon, z) < 0 < \delta(1-\varepsilon, z)$ for every $z \in (0,1)$. This and the continuity of $\delta(\cdot,\cdot)$ in both arguments, imply by the intermediate value theorem that there exists a z^* s.t. $\delta(z^*, z^*) = 0$.

Step 2. We now show that $\delta(y, z^*) < (>)\ 0$ if $y < (>)\ z^*$. Since $x_c^*(k+1, z^*) > x_c^*(k, z^*)$ with probability 1 (by SMLR), if $y < (>)\ z^*$ it is the case that $\|x_c^*(k+1,z^*) - y\| > (<)\ \|x_c^*(k,z^*) - y\|$ with probability 1. Given the structure of $u(\cdot,\cdot)$ this implies that the claim of step 2 is true.

Step 3 Combining these results, we have established that if $T-1$ respondents use the strategy with z^* then a poll respondent with ideal point $y = z^*$ is indifferent between each response, a respondent with ideal point less than z^* strictly prefers $x_c^*(k,z)$ to $x_c^*(k+1,z)$ (and vice versa).

Step 4. Now, $\eta(\theta \mid y)$ is continuous in y and the case at $y = 1$ and $y = 0$ is as in step 1 thus step 1 holds for $\eta(\theta \mid y)$ not constant in θ. As long as $d(\eta(\theta \mid y), \eta(\theta \mid y'))$ is sufficiently close to 0 the fact that strict inequalities hold in step 2 and the continuity of $\delta(y, z)$ in $\eta(\theta)$ (relative to the topology induced by the metric $d(\cdot, \cdot)$) implies that the conclusion of step 3 generalizes.

Thus the claim is established. □

Proof of Proposition 7. Assume $\eta(1/2) > (<)\ 1/2$. By Proposition 6, a z^* exists. Suppose by way of a contradiction that $z^* > (<)\ 1/2$. Since by step 2 (above), $\delta(y, z^*) > 0$ if $y > z^*$ and $\delta(y, z^*) < 0$ if $y < z^*$. The assumption implies that $\delta(1/2, z^*) < (>)\ 0$. But since $\eta(1/2) > (<)\ 1/2$ and $x_c^*(k, z^*) = x^{\mathrm{med}}(s)$ when the posterior calculation uses the cutpoint z^*, the expected policy resulting from use of the cutpoint z^* is

$$\int_{\Theta} \sum_{k=0}^{T} \binom{T}{k} x_c^*(k, z) F(z \mid \theta)^{T-k} \big[1 - F(z \mid \theta)\big]^k \, \mathrm{d}\eta(\theta \mid y) < (>)\ \frac{1}{2}. \tag{34}$$

Moreover, for every $\varepsilon > 0$ there exists a T sufficiently large that for a population of T poll respondents the Bayesian posterior is such that $x_c^*(k+1, z) - x_c^*(k, z) < \varepsilon$. This implies that for T sufficiently large the expected policy resulting from the deviation to $m = 1$ $(m = 0)$ is also less (greater) than 1/2. Thus it cannot be the case that $\delta(1/2, z^*) < (>)\ 0$, and we have attained the contradiction. □

References

Auman, R., 1976. Agreeing to disagree. Ann. Statist. 4, 1236–1239.

Baron, D., Meirowitz, A., 2004. Fully revealing equilibria of multiple-sender signaling and screening games. Soc. Choice Welfare. In press.

Banks, J., Sobel, J., 1987. Equilibrium selection in signaling games. Econometrica 55, 647–661.

Berge, C., 1963. Topological Spaces. Macmillan Co., New York (translated by E.M. Patterson).

Bernhardt, D., Duggan, J., Squintani, F., 2003. Private polling in elections. Typescript. University of Rochester.

Calvert, R., 1985. Robustness of the multidimensional voting model: Candidate motivations, uncertainty, and convergence. Amer. J. Polit. Sci. 28, 127–146.

Cho, I.K., Kreps, D., 1987. Signaling games and stable equilibria. Quart. J. Econ. 102, 179–221.

De Sinopoli, F., Iannantuoni, G., 2003. A spatial voting model where proportional rule leads to two-party equilibria. Working paper No. 31. Centre for International Studies on Economic Growth, Tor Vergata.

Downs, A., 1957. An Economic Theory of Democracy. Harper Collins, New York.

Dupeux, M., 1954. Special monograph for Political Commission.

Durrett, R., 1996. Probability: Theory and Examples, second ed. Duxbury, Belmont.

Feddersen, T., Pesendorfer, W., 1996. The swing voters curse. Amer. Polit. Sci. Rev. 86, 408–424.

Feddersen, T., Pesendorfer, W., 1997. Voting behavior and information aggregation in elections with private information. Econometrica 65, 1029–1058.

Fey, M., 1997. Stability and coordination in Duverger's Law: A formal model of pre-election polls and strategic voting. Amer. Polit. Sci. Rev. 91, 135–147.

Gollin, A., 1987. Polling and the news media. Public Opinion Quart. 51, 86–94.

Harris, L., 1956. Some observations on election behavior research. Public Opinion Quart. 20, 379–391.

Judd, K., 1985. The law of large numbers with a continuum of i.i.d. random variables. J. Econ. Theory 35, 19–25.

Karlin, S., Rinott, Y., 1980. Classes of orderings of measures and related correlation inequalities: 1. Multivariate totally positive distributions. J. Multivariate Anal. 10, 467–498.

Ledyard, J., 1984. The pure theory of large two-candidate elections. Public Choice 44, 7–41.

McKelvey, R., Ordeshook, P., 1985. Elections with limited information: A fulfilled expectations model using contemporaneous poll and endorsement data as information sources. J. Econ. Theory 36, 55–85.

McKelvey, R., Ordeshook, P., 1986a. Sequential elections with limited information. Amer. J. Polit. Sci. 29, 480–512.

McKelvey, R., Ordeshook, P., 1986b. Information, electoral equilibria, and the democratic ideal. J. Politics 48, 909–937.

McKelvey, R., Page, T., 1986. Common knowledge, consensus, and aggregate information. Econometrica 54, 109–128.

Meirowitz, A., 2001. Informational party primaries and strategic ambiguity. J. Theoretical Politics. In press.

Moulin, H., 1980. On strategy-proofness and single peakedness. Public Choice 35, 437–455.

Piketty, T., 2000. Voting as communicating. Rev. Econ. Stud. 67, 169–192.

Razin, R., 2003. Signaling and election motivations in a voting model with common values and responsive candidates. Econometrica 71, 1083–1119.

Sudman, S., Bradburn, N., 1987. The organizational growth of public opinion research in the United States. Public Opinion Quart. 51, 57–78.

Snyder, J., Ting, M., 2002. An informational rationale for political parties. Amer. J. Polit. Sci. 46, 90–110.

Shotts, K., 2001. A signalling model of repeated elections. Typescript. Stanford University.

Simon, H., 1954. Bandwagon and underdog effects and the possibility of election predictions. Public Opinion Quart. 18, 245–253.

Zaller, J., 1992. The Nature and Origins of Mass Opinion. Cambridge Univ. Press, New York.