Probabilistic Voting and Accountability in Elections with Uncertain Policy Constraints

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Abstract

We consider accountability in repeated elections with two long-lived parties that have distinct policy preferences and different levels of valence. In each period the government faces a privately observed feasibility constraint and selects a publicly observed policy vector. While pure strategy equilibria do not exhibit tight control on government policy making, complete control is possible in mixed strategies. In optimal equilibria voters use reelection functions which depend on policy in a manner that causes the governing party to internalize voter preferences. In these optimal equilibria the voters use different reelection functions for different parties.

1. Introduction

In February 2002, newly elected New York City Mayor Michael Bloomberg faced a public challenge. Financial strain on city resources and a faltering economy exposed the city to a dramatic budgetary problem. Aside from the policy challenge of managing a city under tight conditions Bloomberg faced a difficult but subtle political problem in choosing a budget. Voters could not fully comprehend the complex trade-offs associated with running a city under changing economic conditions. Specifically, a centrist voter would have a hard time discerning if Bloomberg's policy represented a desirable compromise given the complex trade-offs, or a policy that inappropriately

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favored Bloomberg's agenda. The former might merit reelection while the latter would not. In response Bloomberg offered what the New York Times labeled "a budget that hurts everyone" (Cooper 2002).

This paper considers the extent to which informational asymmetries about the set of feasible policies limit the controllability of governments. The model has two distinct features. First, agents have monotone preferences over multiple dimensions of primitive goods that the government can provide. (A classic example is defense spending-guns and social spending-butter; a more timely example is privacy and security.) In this setting preference heterogeneity surfaces in the form of differing marginal rates of substitution.[1] Second, the government faces a feasibility constraint. (For a fixed tax rate buying more guns requires less butter. Increasing security requires that privacy is sacrificed.) The party in office knows the details of the constraint, but the public does not. This assumption is motivated by the fact that most voters devote little effort to information acquisition about the subtleties of policymaking. The approach represents an alternative to classic papers (Barro 1973, Ferejohn 1986) that model elections as settings in which the government selects levels of costly effort and voters observe this choice with noise. Here, shirking describes the selection of a policy vector which the governing party prefers to the voters' constrained-optimum.

In agency models the principal is limited in her ability to determine whether the agent's actions merit sanctions or rewards. Typically, the monitoring problem involves imperfect observability of agent actions. Here the actions are perfectly observable, but the set of alternatives that could have been chosen is hidden knowledge. Equilibria to agency models tend to involve terminal sanctions. In career concerns models a sanctioned employee is fired. Here the pool of potential agents (political parties) is small, and the creation of incentives requires that parties which are sanctioned remain available. Following previous applications of agency models to elections, the set of contracts is severely limited, Voters cannot offer performance based wages, instead they can only determine which agent is given control of policy in the current period. We require that voter actions are themselves best responses and thus the principal does not have the ability to commit to actions in the future. This approach avoids time-consistency problems.

The analysis leads to several conclusions.

(1) In pure strategies, perfect control is not possible.

(2) With mixed strategies, in the form of probabilistic reelection functions, perfect control is possible as the policy and party dependent reelection function causes the government to internalize the public's

[1]This approach is motivated by work in political behavior. With overwhelming regularity scholars of public opinion report that voters "want to have their cake and eat it too" (Zaller 1998).

preferences. Interestingly, voters treat the parties differently—the reelection functions differ across parties.

(3) When parties have valence differences, the optimal equilibria involve randomized reelection rules. In equilibrium the disadvantaged party selects the voters' constrained optimum when in office, but the advantaged party selects compromise polices and thus receives some policy rents.

The sharp difference between mixed and pure strategy results demonstrates that control becomes easier when the set of actions available to the public is enlarged. The model offers an explanation for the finding that voters treat the parties differently (Lowry, Alt, and Ferree 1998). Policies (or outcomes) that are considered acceptable if implemented by one party may be viewed as unacceptable when the other party is in office.

The fact that full control is possible in mixed strategies speaks to the question of whether elections should be viewed as devices for selection or control and counters Fearon's (1999) conjecture that viewing *elections as a means to select high-quality candidates* is more appropriate than viewing *elections as a means to induce accountability by sanctioning poor performance*. Fearon defends this claim by describing conditions under which the selection explanation is more plausible than the control explanation, "(i) repeated elections do not work well as a mechanism of accountability, because [voters] believe that their ability to observe what politicians do and to interpret whether it is in the public interest is so negligible; and (ii) there actually is relevant variation in the types of candidates for political office, and these can be distinguished to some extent, . . ." (p. 68). The current model satisfies the two conditions that are highlighted: difficulty in interpreting whether politician actions are in the public interest and distinguishable variation in candidates. In this model the voters are better off using probabilistic reelection functions, and sometimes having the less desirable party in office, than simply selecting the better party and letting them govern without incentives.

Equilibrium predictions in the case of a valence advantage are consistent with Fiorina's (1973) *marginality hypothesis*—electorally weak incumbents will tend to moderate more than strong incumbents. While empirical support for the hypothesis is mixed, recent formal work involving single period elections with valence and policy commitment (e.g., Groseclose 2001) challenges the hypothesis while the current model supports it.

In Section 2 we present a brief review of related models. In Section 3 we begin by formulating the model. In Section 4 we show that perfect monitoring is impossible in pure strategies. We then establish the existence of mixed strategy equilibria exhibiting perfect control if the value to office is sufficiently high. In Section 5 we extend the analysis to allow for the possibility that there is a policy independent (valence) shock to voter preferences in each period. Section 6 raises a few natural extensions to the basic model and establishes the robustness of the findings to these variations. In Section 7 we conclude with a discussion.

2. Previous Literature

Existing principal-agent models of elections focus on the problem of creating incentives for the government to undertake the optimal amount of costly but publicly desirable effort. Barro (1973), Austen-Smith and Banks (1989), and Ferejohn (1986) consider the moral hazard aspects of this problem, and Banks and Sundaram (1993, 1998) and Ashworth (2005) consider the problem of both moral hazard and adverse selection. Perrson, Roland, and Tabelini (1997) show that separation of powers systems make control easier. While focused on the control of shirking the current paper is conceptually quite distinct. Whereas the above moral hazard models deal with the creation of incentives for the government to not shirk in its effort choice (a variable upon which the principal and agent have diametrically opposed preferences), we deal with the creation of incentives for the government to not shirk in its policy selection (a variable upon which preferences are sometimes aligned). Second, the above models involve imperfect monitoring of agent actions whereas the current model involves perfectly observed actions, but hidden knowledge about the feasible alternative actions. Aside from these critical departures, the current model is similar to the two party model that Ferejohn considers. Both involve informational asymmetries, an infinite horizon, and a pool of two long lived parties. In both models voting serves to constrain governmental action and in an equilibrium in which some control is exerted the voter must be indifferent between having either party in office. The equilibrium results and intuition differ in most other respects.

Banks and Duggan (2000) analyze a repeated elections citizen candidate model in which preelection commitment is not possible.[2] Their model involves a large population of *ex ante* indistinguishable candidates, and uncertainty only about the preferences of candidates. Banks and Duggan bridge the gap between social choice theory and the restrictive Downsian (Downs 1957, Black 1958, Wittman 1977, 1983, Ledyard 1984, Calvert 1985) world while the current model addresses representation and accountability in two party elections with complex or changing political environments. While Banks and Duggan predict convergence to a particular government and policy for reasonable parameterizations we predict stochastic oscillation between the two parties and non-convergence of policy.[3] Caines-Wrone, Heron and Shotts (2001) present a two-period model which explains shirking, in the sense of policy choice that differs from the public optimum, even when preferences

[2]See also Duggan's (2000) unidimensional model.

[3]We do not necessarily predict that each party is in office with equal probability.

are perfectly aligned. The explanation hinges on the executive's incentive to convince the voter that it is competent and should thus be retained. Given the preference alignment the question is not why do candidates act on the behalf of the voter. Rather the puzzle is why do they sometimes shirk. In contrast, the presence of non-shirking behavior needs an explanation in the current paper as the incumbent has an ideological/preference motivation not to select policies desirable to the electorate. Maskin and Tirole (2004) and Alesina and Tabellini (2003) analyze models of control and policy making. Both papers highlight factors under which accountability and non-accountability relationships are advantageous. The structure of these models is quite distinct from that of the current paper. Rogoff and Sibert (1988) analyze a dynamic model of macroeconomic policy in which the government has temporarily private information about its fitness and find that this short-term informational asymmetry can generate political business cycles. While there are similarities in terms of the number of agents and the sequence of play, in Rogoff and Sibert the uncertainty pertains to the competency of the government, and the policy is unidimensional—the provision of a good. These differences result in quite different equilibrium behavior.

3. The Basic Model

3.1. Players and Preferences

To make the incentives clear we focus on just two political parties and a representative voter, each infinitely lived and concerned with discounted streams of per period payoffs. The policy space is two-dimensional, with parties each seeking to maximize one of the two issues. Voters have well-behaved preferences in which each issue is a "normal good." Feasibility constraints are represented by linear constraints, and information asymmetry about the constraint is captured by assuming that the slope and the resource level are observed only by the in-government party. Since the structure of the model is distinct from existing agency and election theories, in several places we have foregone generalizations, which do not alter the qualitative properties of the results but do complicate the exposition/notation. For example there are ways to: allow parties to care about both issues; relax the assumption that the constraint surface is linear; consider an arbitrary odd number of voters; or consider an arbitrary number of policy dimensions. These points are taken up in Section 6.

We consider three players interacting in an infinite number of periods. A set of two parties $P = \{l, r\}$ compete for office in each period, and the representative voter m selects between the parties. We sometimes denote parties with the subscripts p and $-p$. The policy space is $X = \mathbb{R}^2_+$. We use bold letters to denote a policy which is a vector, and non bold typeface with subscripts 1 or 2 to denote the coordinates of a policy. Thus $\mathbf{x} = (x_1, x_2)$. We introduce a convenient partial ordering on X. For two policies $\mathbf{x}, \mathbf{y} \in X$ we say $\mathbf{x} \nwarrow \mathbf{y}$ if

$x_2 > y_2$ and $x_1 < y_1$. Intuitively, $\mathbf{x} \nwarrow \mathbf{y}$ means that $\mathbf{x}$ is to the northwest of $\mathbf{y}$. We assume that each party cares about the policy enacted and values holding office. If policy $\mathbf{x}^t$ is chosen in period t and party p is in office during period t the period t payoff to party p is

$$u_p(\mathbf{x}^t) + \eta_p. \tag{1}$$

The party specific term $\eta_p \geq 0$ measures the non-policy rents associated with holding office. In contrast if party p is not in office but policy $\mathbf{x}^t$ is chosen the t period payoff to party p is $u_p(\mathbf{x}^t)$. We assume that the policy-specific utility function $u_p(\cdot) : X \to \mathbb{R}^1$ is twice differentiable.

Voter m cares only about policy and has twice differentiable utility function $u_m(\cdot)$. Players l, r, m are assumed to have globally non-satiated preferences. We are interested in the case where party l is a high demander of dimension 2, party r is a high demander of dimension 1 and m likes more of each dimension. This approach represents a departure from standard models of politics which usually assume that agents have ideal points. These "bliss-point" or spatial preferences are generally motivated as stemming from required trade-offs and traditional economic (non-satiated) preferences over goods. In standard models both of these features are unmodeled and the analysis starts with spatial preferences over policy. Here, the feasibility constraint is explicitly modeled, so the right starting point is preferences over primitive issues. Endogenous to the model is how policy should and will balance competing desires.

We define the marginal rate of substitution at $\mathbf{x}$ for player i as

$$MRS_i(\mathbf{x}) = \frac{\dfrac{\partial u_i(x)}{\partial x_1}}{\dfrac{\partial u_i(x)}{\partial x_2}} \tag{2}$$

and assume that for any $\mathbf{x} \in X$, $MRS_l(\mathbf{x}) < MRS_m(\mathbf{x}) < MRS_r(\mathbf{x})$. For simplicity we take the extreme case where $MRS_l(\mathbf{x}) = 0$ and $MRS_r(\mathbf{x}) = \infty$ for every $\mathbf{x} \in X$. This holds when $u_l(\mathbf{x}) = h_l(x_2)$ and $u_r(\mathbf{x}) = h_r(x_1)$ with $h_l(\cdot)$ and $h_r(\cdot)$ strictly increasing functions.[4] We assume that the voter, m, has strictly convex preferences.

We consider an infinite sequence of elections. In period t nature, a non-strategic player, randomly selects a constraint set $B^t \subset X$ and the government, $g^t \in P$ after observing B^t, selects a policy point $\mathbf{x}^t \in B^t$. The voter knowing $\mathbf{x}^t$ but not B^t then casts a ballot $v^t \in \{0, 1\}$ where a vote of 1 is a vote to keep the incumbent and a vote of 0 is a vote to replace the incumbent with party $P \backslash g^t$. In period $t + 1$ a new constraint B^{t+1} is realized, a new policy $\mathbf{x}^{t+1} \in B^{t+1}$ is selected by g^{t+1} and a new election occurs. Without loss of generality we assume that period 1 involves selection by government $g^1 = l$. This game form necessitates that we extend the period utility functions to preferences

[4]In Section 6 we discuss how this assumption can be relaxed.

over an infinite horizon. Accordingly, for a sequence $\{\mathbf{x}^t, g^t\} = \{(\mathbf{x}^1, g^1), \ldots, (\mathbf{x}^t, g^t), \ldots\}$ party p's utility is

$$U_p(\{\mathbf{x}^t, g^t\}) = (1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}\big[u_p(\mathbf{x}^t) + \eta_p 1_p(g^t)\big], \tag{3}$$

where $\delta \in (0, 1)$ is a common discount rate[5] and $1_p(g^t)$ is an indicator taking the value 1 if $g^t = p$ and 0 otherwise. Similarly, the voter's utility over such a sequence is

$$U_m(\{\mathbf{x}^t, g^t\}) = (1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}u_m(\mathbf{x}^t). \tag{4}$$

Initially, we assume that any feasible constraint is of the form $B^t = \{\mathbf{x} \in X : b^t x_1 + (1-b^t)x_2 \leq c^t\}$ where $b^t \in [\gamma, 1-\gamma]$ and $c^t \in [\underline{\varsigma}, \bar{\varsigma}]$ with $0 < \gamma < \frac{1}{2}$ and $0 < \underline{\varsigma} \leq \bar{\varsigma} \leq 1$. In other words a constraint is given by a relative price b^t and a resource level c^t. To make clear the dependence of the constraint on the parameters we sometimes denote a constraint by $B(b, c)$. The set of possible constraints is isomorphic to $\mathbb{B} = [\gamma, 1-\gamma] \times [\underline{\varsigma}, \bar{\varsigma}]$. The set of policies which are feasible for some constraint is denoted $\beta = \cup_{b,c\in\mathbb{B}} B(b, c)$.[6]

We assume that the common belief is that for every t the parameters b^t and c^t are given by an independent draw from the continuous and strictly increasing distribution function $F_{bc}(\cdot)$ on support $\mathbb{B}$. The parties l and r only observe the values b^t, c^t if they are in office at period t. The parameters b^t, c^t are never revealed to players other than g^t. The game, thus, involves hidden knowledge about the period state variable b^t, c^t.

Compactness and convexity of the constraint and continuity and strict convexity of the preferences ensure that for any given constraint b, c the set of induced ideal policies for agent $i \in \{m, l, r\}$

$$\mathbf{x}_i^*(b, c) = \arg\max_{\mathbf{x}\in B(b,c)} u_i(\mathbf{x}) \tag{5}$$

contains exactly one point. Moreover, by the theorem of the maximum (Berge 1963) the function mapping constraints into induced ideal points (defined in (5)) is continuous. We impose one additional assumption on the preferences of m.

ASSUMPTION 1: *If $b < b'$ and $c' \geq c$ then the second coordinate of $\mathbf{x}_m^*(b', c')$ is strictly larger than the second coordinate of $\mathbf{x}_m^*(b, c)$ and if $b > b'$ and $c' \geq c$ then the first coordinate of $\mathbf{x}_m^*(b', c')$ is strictly larger than the first coordinate of $\mathbf{x}_m^*(b, c)$.*

This assumption states that both dimensions are normal goods for m—if a dimension becomes cheaper and total resources do not decrease than demand for the dimension will increase. A consequence of this condition is

[5]The assumption of common discount rates can be easily relaxed.
[6]In Section 6 we discuss convex constraints with nonlinear boundaries.

that for some feasible constraint, m's optimal choice from the constraint is not a corner solution (and is thus suboptimal for both parties). A natural example to keep in mind is, the commonly studied, case of Cobb-Douglas utility functions, $u_m(\mathbf{x}) = x_1^{\alpha} x_2^{1-\alpha}$. Here the solution $\mathbf{x}_m(b, c) = (\frac{\alpha c}{b}, \frac{(1-\alpha)c}{1-b})$ involves positive levels of both policy dimensions and Assumption 1 is satisfied.

3.2. Interpretations

One stylized interpretation of the model is to think about issue 1 as defense spending and issue 2 as welfare or redistributive spending. In this interpretation c^t represents the available revenue (from taxing and deficit spending). Party l then has the label *Democrats* and r has the label *Republicans*.[7] An alternative interpretation is closer, in spirit, to the public finance literature. Define $x_1 = 1 - \tau$ were τ is the tax rate, and let x_2 denote the amount of government redistribution. The constraint is $b(1 - \tau) + (1 - b)x_2 \leq c$, so the production technology on redistribution requires $x_2 \leq \frac{c - b(1-\tau)}{(1-b)}$ which is stochastic with random parameters b and c. Party l seeks the maximization of welfare spending, x_2, and party r seeks the minimization of taxation. The representative voter seeks to balance the marginal cost and benefit of redistribution when she has increasing and strictly convex preferences over $(1 - \tau)$ and x_2. The model is also applicable to areas of regulatory politics in which the executive or congress can select from a set of feasible agencies in defining discretion. Additionally, internal agency decision making, may be described in this manner with a principal choosing between different departments each staffed with agents that have certain policy biases.

3.3. Strategies and Equilibria

We focus on stationary perfect Bayesian equilibria (SPBE). An SPBE consists of a government policy function $\psi_p(\cdot) : \mathbb{B} \to B(b, c)$ for each $p \in P$, a ballot function $v(\cdot, \cdot) : \beta \times P \to \{0, 1\}$ for the voter, m, and a voter belief $\pi(\cdot \mid \mathbf{x}, g)$ about the constraint faced by the government conditional on the policy chosen, $\mathbf{x}$, and the government identity, g. This belief mapping is a distribution function on $\mathbb{B}$ conditional on a policy $\mathbf{x} \in \beta$ and identity $p \in P$. The policy function of p needs to be optimal given the ballot function and the policy function of $-p$, the ballot function needs to be sequentially rational relative to the belief mapping and the policy functions, and the belief mapping needs to satisfy Bayes' rule when it is defined. Equilibria that maximize voter welfare require mixing by the voter. A mixed ballot function is then a mapping $\lambda(\cdot, \cdot) : \beta \times P \to [0, 1]$ with $\lambda(\mathbf{x}, p)$ denoting the probability that an incumbent of party p is retained following the selection of policy $\mathbf{x}$. The

[7]In this case, one might interpret $\frac{b}{1-b}$ as the relative price of missile defense systems in terms of subsidized health insurance.

assumption of stationarity is satisfied by this description as we have required strategies to hinge only on the state variable $(\mathbf{x}, g)$.

The assumption of stationarity is tenable as voting or policy selection strategies that hinge on a long history of past elections seem peculiar when these past elections provide no payoff relevant information. Moreover, stationary equilibria exhibit retrospective voting—a feature that surfaces in the voting literature.[8] Subsequently, we discuss the consequences of relaxing the stationarity restriction. Since this is a model of incomplete information we focus on perfect Bayesian equilibria to ensure that the equilibria do not hinge on unreasonable voter beliefs. In this game the only relevant uncertainty is faced by the voter, m, when she must decide whether to retain or remove the incumbent. Accordingly, SPBE require that beliefs about b^t, c^t be consistent with the observation $\mathbf{x}^t$ and the strategy $\psi_g(\cdot)$. The beliefs are not very important to the analysis here, which distinguishes this model from many others with imperfect information. Under a fixed profile of stationary strategies $\psi_l(\cdot)$, $\psi_r(\cdot)$ the voter's preference for retaining or removing g is not dependent on b^t, c^t. Accordingly, the extent to which a ballot strategy $v(\cdot, \cdot)$ is sequentially rational does not depend on the beliefs. Intuitively, sequential rationality would constrain prospective behavior (in this case voting), but the information observable to m is of no value in predicting future play under a fixed pair of stationary policy functions. Given this observation we suppress the beliefs about b^t from subsequent statements of and arguments about equilibria. We characterize strategy profiles as *supportable as SPBE* when there exists a belief mapping for which the strategy profile and belief would constitute an SPBE. Sequential rationality imposes the following constraint on the ballot function.

CONDITION 1: *Given $\psi_l(\cdot)$, $\psi_r(\cdot)$, the mapping $v(\cdot, \cdot)$ $(\lambda(\cdot, \cdot))$ is sequentially rational iff*

$$\int u_m(\psi_g(b, c))\, dF_{bc}(b, c) > (<) \int u_m(\psi_{-g}(b, c))\, dF_{bc}(b, c)$$

implies $v(\mathbf{x}, g) = 1(0)$ $(\lambda(\mathbf{x}, g) = 1(0))$.

Given this condition in any SPBE in which a fixed party is not always in office we must have

$$\int u_m(\psi_l(b, c))\, dF_{bc}(b, c) = \int u_m(\psi_r(b, c))\, dF_{bc}(b, c). \tag{6}$$

This means that in any non-trivial equilibrium, the ballot function will not be prospective in nature. In fact, given the desire of m to create incentives for $\psi_g(b^t, c^t)$ to be close to $\mathbf{x}^*_m(b^t, c^t)$ it is natural to think about the game

[8] Our usage of the phrase retrospective voting differs from that of Fiorina (1981). By retrospective voting we mean behavior in which voting decisions over tomorrow's government are based only on the policy choice of today's government. Typical applications of the concept, involve an aspiration level.

as a mechanism design problem, where m selects a ballot function satisfying Condition 1 to maximize the left and right hand side of (6) and the policy mappings $\psi_p(\cdot, \cdot)$ are mutual best responses to the ballot function. Our analysis will serve to characterize the relevant incentive compatibility constraints on ballot functions.

Definition 1: *We say an SPBE exhibits perfect control if for every $p \in P$ $\psi_p(b, c) = \mathbf{x}^*_m(b, c)$ for almost every b, c. We say an SPBE exhibits partial control if for every $p \in P$, $\psi_p(b, c) = \mathbf{x}^*_m(b, c)$ for $b, c \in D_p$ with D_p a subset of $\mathbb{B}$ having positive measure.*

4. Results

If the constraint $B(b^t, c^t)$ were observed by the voter than the reelection rule that retains p if and only if $\psi_p(b^t, c^t) = \mathbf{x}^*_m(b^t, c^t)$ would be feasible. Moreover as long as the value of retaining office were sufficiently high either party would comply and select the voters optimum. Such an equilibrium would clearly be optimal for m. Uncertainty about the resource level c and cost b has dramatic implications for the possibility of monitoring and control.

In any SPBE with perfect control, the in-government party almost always adopts the voter's most preferred feasible policy. In an SPBE exhibiting partial control, the in-government party sometimes adopts its most preferred feasible policy that results in reelection and sometimes this party shirks adopting its most preferred feasible policy. Given a pure strategy ballot function the set of polices which will result in an incumbent victory is given by the acceptance sets $A_l = \{\mathbf{x} : v(\mathbf{x}, l) = 1\}$, $A_r = \{\mathbf{x} : v(\mathbf{x}, r) = 1\}$. The compliments of these sets result in loss of office. By choosing the acceptance sets, m influences the policies that governments will enact. If she makes A_p too small or restrictive then when party p is in office it will select a policy to maximize its current utility over policy and not retain office next period. Conversely, if A_p is too large or unrestrictive when party p is in office it will be able to retain office while selecting a policy that is far from the voter's constrained optimum. Agent m would like to select A_p so that when party p is in office she selects $\mathbf{x}^*_m(b^t, c^t)$. Perfect control by the voter is difficult because she never learns b^t, c^t and thus may not be able to discern if an observed policy is in her best interests.

4.1. Pure Strategies

We first develop intuition by considering the special case where the resource level is commonly known and characterize the incentives that preclude full control in pure strategies.

4.1.1. The Resource Level Is Commonly Known

Specifically, in this section assume that $c^t = 1$ for every t with probability one and that this is common knowledge. We now show that in pure strategies there are no SPBE in which the voter perfectly controls the candidates. Here, the institutional controls open to the public are insufficient to create the *right* incentives for the government. The problem that the voter faces is that when the price b is low (high) party $l(r)$ can choose an inefficient policy (in the sense that $x_1 b + (1 - b)x_2 < 1$) which is optimal for m under some other higher (lower) price. This inefficient policy will be more desirable to the party than m's most preferred policy given b. If m conjectures that parties are always choosing $\mathbf{x}^*_m(\cdot)$ she will not be able to discern a deviation of the form just described since she does not know b.

Given any constraint $B(b, 1)$ the incumbent party $p \in P$ must decide whether to select a policy that results in reelection if such a policy is feasible (i.e., $A_p \cap B(b, 1) \neq \emptyset$). On purely policy grounds p's most preferred feasible policy is given by

$$\begin{aligned} \mathbf{x}^*_l(b, 1) &= \left(0, \frac{1}{1-b}\right) \\ \mathbf{x}^*_r(b, 1) &= \left(\frac{1}{b}, 0\right). \end{aligned} \tag{7}$$

Alternatively, given A_p and the constraint $B(b, 1)$ the optimal policy that will result in reelection is given by

$$\mathbf{x}^w_p(b, 1) = \arg \max_{\mathbf{x} \in B(b,1) \cap A_p} u_p(\mathbf{x}). \tag{8}$$

Figure 1 exhibits the intuition. In any SPBE with perfect control in which party p is in office, for almost every b it must be the case that $\mathbf{x}^w_p(b, 1) =$

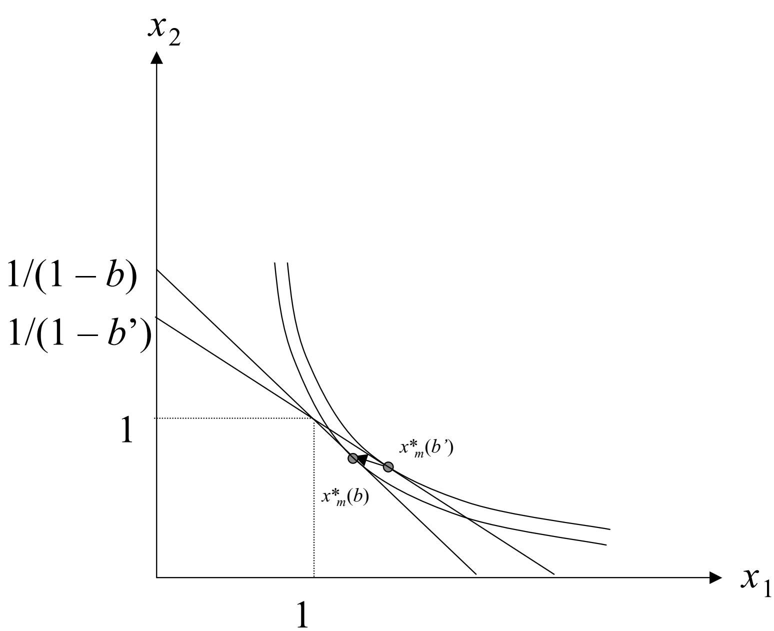

Figure 1: Monitoring problem for uncertain *b*

$\mathbf{x}^*_m(b, 1)$. By assumption 1 there exists a b^* s.t. $\mathbf{x}^*_m(b^*, 1) = (1, 1)$. We will repeatedly refer to this special slope b^*. Now consider $b^* > b > b'$. Prices b and b' both induce a constraint with a boundary that is flatter than the boundary of $B(b^*, 1)$. By assumption 1, $\mathbf{x}^*_m(b^*, 1) \nwarrow \mathbf{x}^*_m(b, 1) \nwarrow \mathbf{x}^*_m(b', 1)$. This and $b > b'$ imply that $\mathbf{x}^*_m(b, 1) \in B(b', 1)$ and $u_l(\mathbf{x}^*_m(b', 1)) < u_l(\mathbf{x}^*_m(b, 1))$. Accordingly party l facing a constraint $b' < b^*$ will strictly prefer to enact a policy $\mathbf{x}^*_m(b, 1)$ which is optimal for the voter under some other constraint and feasible under the constraint $B(b', 1)$. A similar argument holds for party r with prices $b^* < b < b'$. We have just demonstrated that the following conclusion holds.

CONCLUSION 1: *With the resource level known, in pure strategies there are no SPBE that exhibit perfect control.*

The problem that prevents us from constructing SPBE with perfect control is slightly peculiar. When m observes l, a high demander of x_2, choose a low value of x_2 she cannot rule out the possibility that the government shirked (choosing a policy that l prefers to $\mathbf{x}^*_m(b, 1)$). In contrast when l chooses high values of x_2 the voter can be certain that no shirking occurred. This is true because when $b < b^*$ a deviation from $\mathbf{x}^*_m(b, 1)$ that increases x_2 is not feasible (it is outside $B(b, 1)$). Thus, m is unable to determine if l is selecting the right policy, not when the enacted policy involves a high quantity of the government's preferred coordinate, but rather when the enacted policy involves a low quantity of the government's preferred coordinate and a very low quantity of the other coordinate. The intuition being, when the constraint favors party r, party l has an incentive to select an inefficient policy that makes it look like the constraint favors party l by a little less. There are always regions of the set $\mathbb{B}$ where party l can get away with this. It should be noted that even in non-stationary strategies perfect control is not possible. The barrier to control is not the size of the stick and carrot, but rather the inability of the voter to determine when to use the stick and when to use the carrot.

While perfect control cannot occur in a pure strategy SPBE, sometimes there are pure strategy SPBE in which the voter can exert partial control on the parties. To develop the basic intuition we first consider cases with a fair amount of symmetry making it easy to satisfy condition 1. We say *symmetry* is satisfied if the voter is indifferent between the lotteries over policy induced by the following two functions:

$$\psi_l(b) = \begin{cases} \mathbf{x}^*_m(b, 1) & \text{if } b > b^* \\ \mathbf{x}^*_l(b, 1) & \text{otherwise} \end{cases}$$
$$\psi_r(b) = \begin{cases} \mathbf{x}^*_m(b, 1) & \text{if } b < b^* \\ \mathbf{x}^*_r(b, 1) & \text{otherwise.} \end{cases} \tag{9}$$

An example satisfying symmetry involves $F_b(\cdot)$ uniform and $u_m(\mathbf{x}) = x_1^{1/2} x_2^{1/2}$. This is not the only parameterization that satisfies the condition.

Symmetry is a joint restriction on the distribution $F_b(\cdot)$ and the voter's preferences $u_m(\cdot)$. When symmetry is satisfied we can characterize strategy profiles that are supportable as SPBE in which each party selects the voter's constrained optimum when b is on its desirable side of b^* and it selects the party constrained optimum when b is on its undesirable side of b^*. The construction uses the fact that when $(1, 1) \nwarrow \mathbf{x}^t$ and $\mathbf{x}^t$solves the voter's problem for some constraint the public can trust that r has not shirked and when $\mathbf{x}^t \nwarrow (1, 1)$ and and $\mathbf{x}^t$ solves the voter's problem for some constraint the public can trust that l has not shirked. In the converse cases it is not possible to infer that the parties are not shirking. By $\mathbf{x}_m^{*-1}(\mathbf{x}) = \{b : \mathbf{x}_m^*(b, 1) = \mathbf{x}\}$ we denote the inverse of $\mathbf{x}_m^*(b, 1)$. In addition to symmetry two additional conditions are used.

CONCLUSION 2: *If symmetry is satisfied the following profile is supportable as an SPBE with partial control*

$$A_l = \left\{\mathbf{x} : \mathbf{x}_m^{*-1}(\mathbf{x}) > b^*\right\}$$
$$A_r = \left\{\mathbf{x} : \mathbf{x}_m^{*-1}(\mathbf{x}) < b^*\right\}$$
$$\psi_l(b) = \begin{cases} \mathbf{x}_m^*(b, 1) & \text{if } b > b^* \\ \mathbf{x}_l^*(b, 1) & \text{otherwise} \end{cases}$$
$$\psi_r(b) = \begin{cases} \mathbf{x}_m^*(b, 1) & \text{if } b < b^* \\ \mathbf{x}_r^*(b, 1) & \text{otherwise} \end{cases}$$

if the following conditions are satisfied

$$\frac{\max_{b\in[b^*,1-\gamma]} \left[u_l\left(\mathbf{x}_l^*(b, 1)\right) - u_l\left(\mathbf{x}_m^*(b, 1)\right)\right]}{\eta_l + \int [u_l(\psi_l(b', 1)) - u_l(\psi_r(b', 1))]\, dF_b(b')} \leq \delta \tag{C1}$$

$$\frac{\max_{b\in[\gamma,b^*]} \left[u_r\left(\mathbf{x}_r^*(b, 1)\right) - u_r\left(\mathbf{x}_m^*(b, 1)\right)\right]}{\eta_r + \int [u_r(\psi_r(b', 1)) - u_r(\psi_l(b', 1))]\, dF_b(b')} \leq \delta. \tag{C2}$$

Proof: Given symmetry is satisfied each party's policy function induces a lottery over policy with the same expected utility for m and thus Condition 1 is satisfied, so the ballot function is sequentially rational. It remains only to verify that the policy mappings are mutual best responses.

-Consider party l: Assume that $\psi_r(b)$ and A_l, A_r are as defined. It is sufficient to show that no unilateral single-period deviation from $\psi_l(b)$ is desirable. If $b > b^*$ then selection of $\mathbf{x}_m^*(b, 1)$ involves reelection and selection of any other feasible policy involves either loss of office or less of x_2. We let $v_l^l(b)$ denote the continuation value to l from being in office with constraint parameter b and $v_l^r(b)$ be the continuation value to l for having r in office with constraint parameter b. We define

$$Ev_l = \left[F_b(1-\gamma) - F_b(b^*)\right]\int v_l^l(b')\, dF_b(b') + \left[F_b(b^*) - F_b(\gamma)\right]\int v_l^r(b')\, dF_b(b'). \quad (10)$$

The continuation value to l from selecting $\mathbf{x}_m^*(b, 1)$ (with $b > b^*$) and staying in office is

$$v_l^l(b) = u_l\big(\mathbf{x}_m^*(b)\big) + (1+\delta)\eta_l + \delta\int u_l(\psi_l(b'))\, dF_b(b') + \delta^2 Ev_l. \quad (11)$$

The continuation value to l from selecting $\mathbf{x}_l^*(b, 1)$ and losing office is

$$v_l^r(b) = u_l\big(\mathbf{x}_l^*(b)\big) + \eta_l + \delta\int u_l(\psi_r(b'))\, dF_b(b') + \delta^2 Ev_l. \quad (12)$$

Subtracting and rearranging demonstrates the deviation is not desirable for any $b > b^*$ if (C1) is satisfied. Now if $b < b^*$ the strategy profile $\psi_l(b)$ is clearly optimal as no policy that would attain reelection is in $B(b, 1)$ and thus selection of $\mathbf{x}_l^*(b, 1)$ is a best response. Interchanging l and r and the appropriate ranges of b in the argument yields the result for party r. ■

This SPBE involves successful control over governments that receive constraints which they find relatively desirable, and no control over governments that receive constraints, which are not desirable. In the latter case the government shirks, giving itself as desirable a policy as possible and then leaves office. In the event of a desirable constraint the government forgoes the opportunity to shirk because it values the prospect of retaining office. The value to office consists of the exogenous term η_p and the endogenous term

$$\int \left[u_p(\psi_p(b')) - u_p(\psi_{-p}(b'))\right] dF_b(b'). \quad (13)$$

When symmetry is satisfied but C1 or C2 fails, it may be possible to attain SPBE with control for a smaller subset of the possible (b, g) pairs. We do not consider this extension as no additional intuition is gained, and C1 and C2 are satisfied as long as (η_l, η_r, δ) are big enough. Symmetry on the other hand involves a knife edged condition and we want to understand what happens when the condition does not hold. If symmetry is violated and the parties use the policy functions defined in proposition 2, punishment of one of the parties is no longer the best response for the voter as condition 1 would require that the one party is always reelected and the other party is never reelected. The voter strictly prefers having one party in office and that party will not find the threat of punishment credible following a single period deviation. In this case the selection problem seems to make credible solution of the control problem impossible. We can, however, modify this SPBE to accommodate cases where symmetry is not satisfied. A trick used throughout the paper involves making the strategy of the advantaged party less attractive to the voter. One, such, modification involves reducing the values of b for which the advantaged party selects $\mathbf{x}_m^*(b, 1)$. The Appendix considers the case where symmetry is violated and contains the derivation of a pure strategy equilibrium that involves partial control.

4.1.2. The Resource Level Is Private Information

We now return to the more general case where both b^t and c^t are only observed by the party in office and extend conclusion 1 into the following benchmark result.

PROPOSITION 1: *There are no pure strategy SPBE in which partial control occurs.*

Proof: Suppose there is a pure strategy SPBE in which partial control occurs. This means that there exist (b, c) in the interior of $\mathbb{B}$ for which partial control occurs on a neighborhood of (b, c). By assumption 1, on an open subset, O, of this neighborhood, $\mathbf{x}_m(\cdot, \cdot)$ has strictly positive coordinates. This means that selection of $\psi_p(b, c) = \mathbf{x}_m^*(b, c)$ is not optimal for l (in terms of policy preferences) and thus must result in reelection. Now consider a pair $(b', c') \in O$ with $c' > c$ and $b' > b$. Since control occurs on O in equilibrium $\psi_p(b', c') = \mathbf{x}_m^*(b', c')$ and $\psi_p(b, c) = \mathbf{x}_m^*(b, c)$. By assumption 1 the second dimension of the former policy (under b', c') is strictly higher than the first dimension of the latter policy (under b, c). This implies that government l strictly prefers $\psi_p(b', c')$ to $\psi_p(b, c)$. Since $\mathbf{x}_m(\cdot, \cdot)$ is continuous as long as b', c' is chosen so that $\mathbf{x}_m^*(b, c) \notin B(b', c')$ we can select b', c' s.t. $\mathbf{x}_m^*(b', c') \in B(b, c)$. (In this construction, b', c' are chosen so that the boundary of $B(b', c')$ intersects the boundary of $B(b, c)$ at a point with more of the second coordinate and less of the first than $\mathbf{x}_m^*(b, c)$). This means that there is a feasible deviation from $\mathbf{x}_m^*(b, c)$ that l prefers on policy grounds which will result in reelection (since $\mathbf{x}_m^*(b', c')$ results in reelection). Accordingly $\psi_p(b, c) = \mathbf{x}_m^*(b, c)$ cannot be part of an equilibrium. ■

Figure 2 illustrates this point by plotting two constraints $B(b, c)$ and $B(b', c')$ and two possible points $\mathbf{x}_m^*(b, c)$ and $\mathbf{x}_m^*(b', c')$. Upon observing $\mathbf{x}_m^*(b', c')$ the voter cannot determine if b, c have attained and l has shirked or if b', c' have attained and l has not shirked.

4.2. The Possibility of Control with Mixed Strategies

We now consider mixed strategies and characterize optimal equilibria. Our main result is that if the slope of $h_p(\cdot)$ is not too steep there are mixed strategy SPBE in which perfect control occurs.

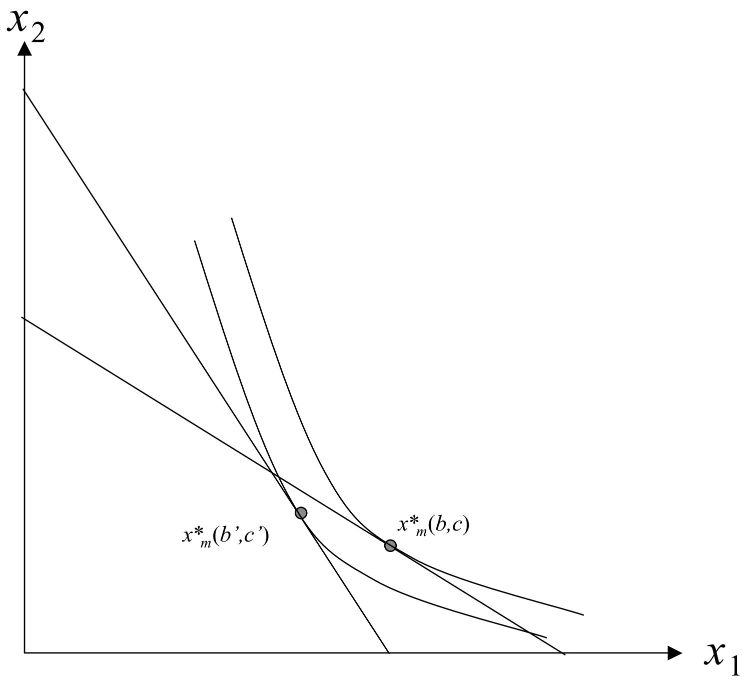

Figure 2: Monitoring problem for uncertain *b,c*

PROPOSITION 2: *With uncertainty about b and c (or just b), if*

$$h_l\left(\frac{\bar{\varsigma}}{\gamma}\right) - h_l(0) \leq \delta\eta_l \qquad (*)$$

$$h_r\left(\frac{\bar{\varsigma}}{\gamma}\right) - h_r(0) \leq \delta\eta_r. \qquad (**)$$

there exist constants q_l and q_r such that perfect control is supportable in a mixed strategy SPBE with the following ballot functions:

$$\lambda(\mathbf{x}, l) = \frac{1}{\delta\eta_l}(u_m(\mathbf{x}) - h_l(x_2)) + q_l$$

$$\lambda(\mathbf{x}, r) = \frac{1}{\delta\eta_r}(u_m(\mathbf{x}) - h_r(x_1)) + q_r.$$

Proof: The construction hinges on creating mixed ballot functions that induce each party to choose $\mathbf{x}^*_m(b, c)$. Recall that $\lambda(\mathbf{x}, p)$ denotes the probability that p is retained if she selects policy $\mathbf{x}$. Using the single deviation principle we can derive the incentive compatibility condition that a mixed ballot function must satisfy. Suppose both parties will select $\mathbf{x}^*_m(b, c)$ whenever they are in office (except possibly for party l this period). Given this, a ballot function $\lambda(\cdot, l)$ and a constraint $B(b, c)$, party l must solve the problem

$$\arg \max_{\mathbf{x} \in B(b,c)} h_l(x_2) + \lambda(\mathbf{x}, l)\delta\eta_l + k, \tag{14}$$

where k is a constant with respect to the choice variable $\mathbf{x}$. By definition we have

$$\mathbf{x}_m^*(b, c) = \arg \max_{\mathbf{x} \in B(b,c)} u_m(\mathbf{x}). \tag{15}$$

Since (14) and (15) involve the same constraint, if $\lambda(\cdot, \cdot)$ is chosen so that the first order conditions from problem (14) are equated with the first order conditions from from (15), party l will have an incentive to choose $\mathbf{x}_m^*(b, c)$. This requires

$$\begin{aligned} \frac{\partial h_l(x_2)}{\partial x_2} + \delta\eta_l \frac{\partial \lambda(\mathbf{x}, l)}{\partial x_2} &= \frac{\partial u_m(\mathbf{x})}{\partial x_2} \\ \delta\eta_l \frac{\partial \lambda(\mathbf{x}, l)}{\partial x_1} &= \frac{\partial u_m(\mathbf{x})}{\partial x_1}. \end{aligned} \tag{16}$$

Rearranging yields

$$\begin{aligned} \frac{\partial \lambda(\mathbf{x}, l)}{\partial x_2} &= \frac{1}{\delta\eta_l}\left(\frac{\partial u_m(\mathbf{x})}{\partial x_2} - \frac{\partial h_l(x_2)}{\partial x_2}\right) \\ \frac{\partial \lambda(\mathbf{x}, l)}{\partial x_1} &= \frac{1}{\delta\eta_l}\left(\frac{\partial u_m(\mathbf{x})}{\partial x_1}\right). \end{aligned} \tag{17}$$

A function that satisfies this condition is

$$\lambda(\mathbf{x}, l) = \frac{1}{\delta\eta_l}(u_m(\mathbf{x}) - h_l(x_2)) + q, \tag{18}$$

where q is a constant. It remains only to verify that it is possible to construct a mapping $\lambda(\cdot, l)$ with image $[0, 1]$ that satisfies these conditions. This requires that

$$\max_{\mathbf{x} \in \beta} \lambda(\mathbf{x}, l) - \min_{\mathbf{x} \in \beta} \lambda(\mathbf{x}, l) < 1. \tag{19}$$

This difference is bounded by

$$\frac{1}{\delta\eta_l}\left(h_l\left(\frac{\bar{\varsigma}}{\gamma}\right) - h_l(0)\right). \tag{20}$$

Thus, if (*) is satisfied an incentive compatible mixed ballot function for party l can be constructed. Similar logic yields the first order conditions

$$\begin{aligned} \frac{\partial \lambda(\mathbf{x}, r)}{\partial x_1} &= \frac{1}{\delta\eta_r}\left(\frac{\partial u_m(\mathbf{x})}{\partial x_1} - \frac{\partial h_r(x_1)}{\partial x_1}\right) \\ \frac{\partial \lambda(\mathbf{x}, r)}{\partial x_2} &= \frac{1}{\delta\eta_r}\left(\frac{\partial u_m(\mathbf{x})}{\partial x_2}\right) \end{aligned} \tag{21}$$

and if (**) is satisfied an incentive compatible mixed ballot function for party r can be constructed. We are then left with the result.[9] ■

Since the stationarity assumption only limits the sizes of the carrot and stick, when (*) and (**) are satisfied (and thus the carrot and stick are big enough), the restriction to stationary strategies does not limit the public's ability to control the government in mixed strategies, as there are no equilibria that do better than these stationary mixed strategy equilibria. When conditions (*) and (**) are not satisfied allowing punishment to last for multiple periods can make full control in mixed strategies possible.

5. Valence Advantages

While the parties differ in terms of their policy preferences, we have not considered the case where one party is more attractive to the voters independent of policy. One possibility is that the l party has a valence advantage so that when the l party is in office and enacts policy $\mathbf{x}$ the voter's period payoff is $u_m(\mathbf{x}) + \varepsilon$ where $\varepsilon > 0$ is the net valence advantage of party l. When party r is in office and enacts policy $\mathbf{x}$ the voter's period payoff is $u_m(\mathbf{x})$. Since the mixed strategy equilibrium that supports proposition 2 involves perfect control, $\varepsilon > 0$ means that m is no longer indifferent between each party. As a consequence if the parties are controlled then mixing is not a best response for m. If ε is not too big it is possible to characterize a mixed strategy equilibrium in which party r always selects $x^*_m(b, c)$ and party l (the advantaged party) shirks a bit. Qualitatively the equilibrium is quite similar to the one characterized above with $\varepsilon = 0$.

In this spirit, we consider an extension in which there is a time varying random valence shock. Assume that ε^t is independently drawn from a distribution $H(\cdot)$ on $[\underline{k}, \bar{k}]$. If $0 \in [\underline{k}, \bar{k}]$ then the identity of the advantaged party is also random. We assume that the realization of ε^t is revealed prior to the choice of which party will govern in period t. Once realized, the value is public information. The shock, ε^t, combines with policy utility, $u_m(\mathbf{x}_t)$, to give the period t payoff to the voter. One plausible interpretation is that the incumbent receives accurate polling data measuring partisan or personality-based preferences (independent of policy). Intuitively, if l is in office for period $t-1$ and a positive value of ε^t is realized then the voter will be predisposed to keep l in office for period t. As a consequence, the shock ε^t results in variation in the identity of the advantaged candidate and the magnitude of the advantage. If the parties are using strategies that yield the same expected policy utility then the shock ε^t will result in a strict preference over the parties in period t. This strict preference has important implications for control. Given the logic behind condition 1, some degree of control requires that the

[9]The mixed strategy equilibria can also be constructed in the simpler model without uncertainty about c.

new shock dependent strategies $\psi_p(b^t, c^t, \varepsilon^t)$ and $\lambda(\mathbf{x}^t, p, \varepsilon^t)$ involve shock compensation. The natural extension of condition 1 is then

CONDITION 2: *Given the mappings* $\psi_l(\cdot,\cdot,\cdot)$, $\psi_r(\cdot,\cdot,\cdot)$, *the mapping* $\upsilon(\cdot,\cdot,\cdot)$ *is sequentially rational iff*

$$\int u_m(\psi_l(b, c, \varepsilon))\, dF_{bc}(b, c) + \varepsilon > (<) \int u_m(\psi_r(b, c, \varepsilon))\, dF_{bc}(b, c)$$

implies $\lambda(\mathbf{x}, l, \varepsilon) = 1(0)$ *and* $\lambda(\mathbf{x}, r, \varepsilon) = 0(1)$.

We now characterize an optimal (for the voter) mixed strategy equilibrium. Optimallity requires that the less desirable party enact the voter's most preferred policy. Suppose when $\varepsilon < 0$, l selects

$$\bar{\psi}_l(b, c, \varepsilon) = \mathbf{x}_m^*(b, c). \tag{22}$$

Given this and Condition 2, indifference by the voter requires that when $\varepsilon < 0$ party r's strategy satisfies the condition

$$\int u_m\big(\mathbf{x}_m^*(b, c)\big)\, dF_{bc}(b, c) + \varepsilon = \int u_m(\psi_r(b, c, \varepsilon))\, dF_{bc}(b, c). \tag{23}$$

For a fixed $\varepsilon < 0$ let $C_r(\varepsilon)$ denote the set of functions $\psi_r(b, c, \varepsilon)$ that satisfy this constraint. This set is non-empty as long as the lower bound $\underline{k}$ on the support of ε is greater than

$$k^- := \int u_m\left(\left(\frac{c}{b}, 0\right)\right) dF_{bc}(b, c) - \int u_m\big(\mathbf{x}_m^*(b, c)\big)\, dF_{bc}(b, c). \tag{24}$$

Assume that $\underline{k} \geq k^-$ and let $\bar{\psi}_r(b, c, \varepsilon)$ be a selection from the set

$$\arg\max\{u_r(x(b, c)) \text{ s.t. } x(b, c) \in C_r(\varepsilon)\}. \tag{25}$$

Suppose when $\varepsilon > 0$, r selects

$$\bar{\psi}_r(b, c, \varepsilon) = \mathbf{x}_m^*(b, c). \tag{26}$$

Given this and condition 2 indifference by the voter requires that when $\varepsilon > 0$ party l's strategy satisfies the condition

$$\int u_m\big(\mathbf{x}_m^*(b, c)\big)\, dF_{bc}(b, c) = \int u_m(\psi_l(b, c, \varepsilon))\, dF_{bc}(b, c) + \varepsilon. \tag{27}$$

For a fixed $\varepsilon > 0$ let $C_l(\varepsilon)$ denote the set of functions $\psi_l(b, c, \varepsilon)$ that satisfy this constraint. This set is non-empty as long as the upper bound $\bar{k}$ on the support of ε is less than

$$k^+ := \int u_m\big(\mathbf{x}_m^*(b, c)\big)\, dF_{bc}(b, c) - \int u_m\left(\left(0, \frac{c}{1-b}\right)\right) dF_{bc}(b, c). \tag{28}$$

Assume that $\bar{k} \leq k^+$ and let $\bar{\psi}_l(b, c, \varepsilon)$ be a selection from the set

$$\arg\max\{u_l(x(b, c)) \text{ s.t. } x(b, c) \in C_l(\varepsilon)\}. \tag{29}$$

Thus, given ε the voter is indifferent between the parties if they use the following ε dependent strategies

$$\psi_l(b, c, \varepsilon) = \begin{cases} \mathbf{x}_m^*(b, c) & \text{if } \varepsilon \leq 0 \\ \bar{\psi}_l(b, c, \varepsilon) & \text{otherwise} \end{cases} \tag{30}$$

$$\psi_r(b, c, \varepsilon) = \begin{cases} \mathbf{x}_m^*(b, c) & \text{if } \varepsilon \geq 0 \\ \bar{\psi}_r(b, c, \varepsilon) & \text{otherwise.} \end{cases} \tag{31}$$

It remains only to characterize party and ε dependent reelection mixtures that make these strategies best responses for the parties. Conditional on $\varepsilon \leq (\geq) 0$ the function $\lambda(\cdot, l)$ $(\lambda(\cdot, r))$ characterized above works. For the remaining cases it is sufficient for the voter to use functions $\bar{\lambda}(\cdot, l, \varepsilon)$, $\bar{\lambda}(\cdot, r, \varepsilon)$ that satisfy the condition: for every $\varepsilon > 0$

$$\bar{\psi}_l(b, c, \varepsilon) \in \arg \max_{\mathbf{x} \in B(b,c)} h_l(x_2) + \bar{\lambda}(\mathbf{x}, l, \varepsilon)\delta\eta_l \tag{32}$$

and for every $\varepsilon < 0$

$$\bar{\psi}_r(b, c, \varepsilon) \in \arg \max_{\mathbf{x} \in B(b,c)} h_r(x_2) + \bar{\lambda}(\mathbf{x}, r, \varepsilon)\delta\eta_r. \tag{33}$$

Since $u_p(\bar{\psi}_p(b, c, \varepsilon)) \geq u_p(\mathbf{x}_m^*(b, c))$ the bounds (*) and (**) are sufficient to ensure that functions $\bar{\lambda}(\cdot, l, \cdot)$, $\bar{\lambda}(\cdot, r, \cdot)$ satisfying the above conditions exist.

It remains only to show that there cannot be an SPBE in which the per period payoff to the voter exceeds the payoff from selection of $\mathbf{x}_m^*(b, c)$ by the disadvantaged candidate. By way of a contradiction assume this were true. Specifically, assume that in some equilibrium for some $\varepsilon' < 0$ party r's policy function $\psi'_r(b, c, \varepsilon)$ is such that $\int u_m(\psi'_r(b, c, \varepsilon'))\, dF_{bc}(b, c) > \int u_m\big(\mathbf{x}_m^*(b, c)\big)\, dF_{bc}(b, c) + \varepsilon'$. This would require that in period t if $\varepsilon^t < \varepsilon'$, selection of party r is strictly preferred to selection of party l. But condition 2 implies that in this case m would need to select r for office in period t following ε^t regardless of $\mathbf{x}^{t-1}$. Since $\psi'_r(b, c, \varepsilon)$ is not optimal for r (for at least some b, c) this contradicts the fact that $\psi'_r(b, c, \varepsilon)$ is an equilibrium policy function.

We are thus left with the following result.

PROPOSITION 3: *With uncertainty about (b^t, c^t) and ε^t drawn from the support $[k^-, k^+]$ if conditions (*), (**), $\bar{k} \leq k^+$ and $\underline{k} \geq k^-$ are satisfied the following strategies are supportable in a mixed SPBE:*

$$\psi_l(b, c, \varepsilon) = \begin{cases} \mathbf{x}_m^*(b, c) & \textit{if } \varepsilon \leq 0 \\ \bar{\psi}_l(b, c, \varepsilon) & \textit{otherwise} \end{cases}$$

$$\psi_r(b, c, \varepsilon) = \begin{cases} \mathbf{x}_m^*(b, c) & \textit{if } \varepsilon \geq 0 \\ \bar{\psi}_r(b, c, \varepsilon) & \textit{otherwise.} \end{cases}$$

$$\lambda(\mathbf{x}, l, \varepsilon) = \begin{cases} \bar{\lambda}(\mathbf{x}, l, \varepsilon) & \textit{if } \varepsilon > 0 \\ \lambda(\mathbf{x}, l) & \textit{otherwise} \end{cases}$$

$$\lambda(\mathbf{x}, r, \varepsilon) = \begin{cases} \bar{\lambda}(\mathbf{x}, r, \varepsilon) & \textit{if } \varepsilon < 0 \\ \lambda(\mathbf{x}, r) & \textit{otherwise.} \end{cases}$$

Furthermore, no other SPBE yields a higher payoff to the voter.

The optimallity of this SPBE compared to one in which the advantaged party is simply put in office offers a stark contradiction of Fearon's claim that in settings in which the voters have limited information and candidates are distinguishable selection is the appropriate perspective. Proposition 3 shows that control is possible and in fact preferred to simple selection.

Party preferences can be interpreted as rents that the advantaged party gets to extract by selecting policies that it prefers to $\mathbf{x}_m^*(b, c)$.

COROLLARY 1: *The extent to which party p will shirk in period t, $s_p(\varepsilon^t) := \int \|\psi_p(b^t, c^t, \varepsilon^t) - \mathbf{x}_m^*(b^t, c^t)\| dF_{bc}(b^t, c^t)$ has the following relationship with ε^t:*

(*1*) *If $\varepsilon^t > 0$ then $s_l(\varepsilon^t)$ is increasing ($s_r(\varepsilon^t)$ is constant) in $|\varepsilon^t|$*

(*2*) *If $\varepsilon^t < 0$ then $s_r(\varepsilon^t)$ is increasing ($s_l(\varepsilon^t)$ is constant) in $|\varepsilon^t|$.*

Non-policy preferences distort the control relationship by causing the valence advantaged party to shirk-selecting policies closer to her ideal. In the one shot setting with commitment, Groseclose (2001) finds that the opposite holds in equilibrium. When candidates have policy and office motivation and face uncertainty about the location of the median voter the candidate with a valence advantage will select policies closer to the center of the policy space (and thus further from her ideal policy).

6. Extensions

In this section we discuss how the results are affected by a few extensions. It is possible to extend the analysis to the case of multiple voters in several ways. Since voter decisions involve choices over lotteries on $\mathbb{R}_+^2$ there will be a representative voter, if the voter preferences over lotteries on $\mathbb{R}_+^2$ are representative. Banks and Duggan (2006) have shown that if preferences are quadratic and an interior core exists (in an arbitrary dimensional space) then the voter with the ideal point corresponding to the core is decisive over lotteries. Following this result we can construct a model with an odd population of voters having quadratic preferences: (1) with ideal points that are in the first quadrant, (2) with ideal points that have a large enough magnitude so that preferences are monotone on the set β, and (3) such that the agent ideal points satisfy the Plott (1967) conditions (an example would be ideal points that are colinear) so that the core in $\mathbb{R}_+^2$ is non-empty. In this problem Banks and Duggan's result implies that there is a voter who's preferences over lotteries in β are decisive for majority rule. This voter would be called m.

A more satisfying direction is to consider a model with n (odd) voters each having Von Neumann-Morgenstern utility functions on $\mathbb{R}_+^2$ of the log-Cobb Douglass form,

$$u_i(x_1, x_2) = \alpha_i \ln x_1 + (1 - \alpha_i) \ln x_2.$$

The expected utility extension of these preferences to lotteries on $\mathbb{R}_+^2$ yields the representation of the expected utility of a lottery μ on $\mathbb{R}_+^2$

$$\int u_i(x_1, x_2)\, d\mu(x_1, x_2) = \alpha_i \int \ln x_1\, d\mu_1(x_1) + (1 - \alpha_i) \int \ln x_2\, d\mu_2(x_2),$$

where $\mu_d(x_d)$ is the marginal of $\mu(x_1, x_2)$ with respect to x_d and the right-hand side is attained from the linearity of the expectation operator. Accordingly, lottery μ is weakly preferred to lottery φ iff

$$\alpha_i \int \ln x_1\, d\mu_1(x_1) + (1 - \alpha_i) \int \ln x_2\, d\mu_2(x_2)$$
$$\geq \alpha_i \int \ln x_1\, d\varphi_1(x_1) + (1 - \alpha_i) \int \ln x_2\, d\varphi_2(x_2).$$

This holds iff

$$\frac{\alpha_i}{1 - \alpha_i} \geq \frac{\int \ln x_2\, d\varphi_2(x_2) - \int \ln x_2\, d\mu_2(x_2)}{\int \ln x_1\, d\mu_1(x_1) - \int \ln x_1\, d\varphi_1(x_1)}.$$

Thus, we have shown that n agents have order restricted preferences over the set of lotteries on $\mathbb{R}_+^2$ and they are ordered by α_i. Since order restricted preferences are representative, the agent with the median value of α_i is decisive and we can call this voter m.

When parties care about both dimensions analogues to propositions 2 and 3 can be readily attained. Moreover, relaxing the assumption that the feasibility constraint has a linear boundary is quite simple. As long as the public constrained optima and the parties' constrained optima are well defined, the construction in proposition 2 can be extended to establish the existence of mixed strategy equilibria with full control when agents value office enough.[10] Finally, the assumption that policy is two-dimensional can be relaxed. Specifically, the following result generalizes proposition 2.

PROPOSITION 4: *Let $\mathbb{B}$ denote a set of constraint sets with each $B \in \mathbb{B}$ being a compact subset of $\mathbb{R}_+^k$. Further assume that $u_m(\cdot) : \mathbb{R}_+^k \to \mathbb{R}$ is a strictly increasing and differentiable function and that for each $p \in \{l, r\}$ $u_p(\cdot) : \mathbb{R}_+^k \to \mathbb{R}$ is a weakly*

[10]See Meirowitz (2004) for social choice theoretic results on collective choice problems of this form.

increasing and differentiable function. Let δ, η_l, and η_r denote the discount rate and respective values to office. If for each $p \in \{l, r\}$

$$\sup_{B \in \mathbb{B}} \left\{ \max_{\mathbf{x} \in B} u_p(\mathbf{x}) - \max_{\mathbf{x} \in B} u_m(\mathbf{x}) \right\} \leq \delta \eta_p \qquad (^{***})$$

there exist constants q_l, q_r such that perfect control is supportable in a mixed strategy SPBE with the following ballot functions:

$$\lambda(\mathbf{x}, p) = \frac{1}{\delta \eta_p} (u_m(\mathbf{x}) - u_p(\mathbf{x})) + q_p. \qquad (34)$$

Proof: First, continuity of the utility functions and compactness of the constraint sets implies that constrained optima exist and the values $\max_{\mathbf{x} \in B} u_i(\mathbf{x})$ are well defined for $i \in \{m, l, r\}$. Following the structure of the proof of proposition 2, we construct $\lambda(\cdot, \cdot)$ so that the following two objective functions have the same first order conditions

$$\begin{gathered} \max_{\mathbf{x} \in B} \{u_m(\mathbf{x})\} \\ \max_{\mathbf{x} \in B} \{u_p(\mathbf{x}) + \lambda(\mathbf{x}, p) \delta \eta_p + k_p\}. \end{gathered} \qquad (35)$$

This requires that for each dimension d

$$\frac{\partial \lambda(\mathbf{x}, \mathbf{p})}{\partial x_d} = \frac{1}{\delta \eta_p} \left(\frac{\partial u_m(\mathbf{x})}{\partial x_d} - \frac{\partial u_p(\mathbf{x})}{\partial x_d} \right). \qquad (36)$$

Equation (34) gives a function satisfying these conditions. This mapping has image [0, 1] if

$$\max_{B \in \mathbb{B}} \left\{ \max_{\mathbf{x} \in B} \lambda(\mathbf{x}, p) \right\} - \min_{B \in \mathbb{B}} \left\{ \min_{\mathbf{x} \in \beta} \lambda(\mathbf{x}, p) \right\} < 1. \qquad (37)$$

This condition is satisfied if (***) is satisfied. ■

7. Discussion

We consider an agency model of repeated elections with a novel conception of the monitoring problem. The government faces privately observed feasibility constraints and the public only observes the chosen policy. This monitoring problem is sharp and in pure strategies the voters cannot create incentives for selection of their constrained optimum. In mixed strategies full control is possible. In addition the model generates some reasonable predictions about representation: policy differs over time, governments are sometimes retained and sometimes thrown out. Both of these findings are in contrast to the repeated election models and cannot be compared with the static Downsian election models.

In the optimal mixed strategy equilibria voters treat incumbents from different parties differently. This prediction does not surface in either the quality-based moral hazard and adverse selection models (Barro, Ferejohn, Austen-Smith and Banks, Banks and Sundaram, Ashworth) or the spatial representation models (Duggan, Banks, and Duggan). In their analysis of state elections Lowry, Alt, and Ferree find evidence that governors are disproportionately rewarded for increases in scale and balance. While drawing definitive conclusions from their specification is difficult, this work is suggestive that parties are treated differently.

In terms of the principal agent perspective the theory is primarily one of control. An extension in which one party has a valence advantaged poses a non-trivial selection problem. We find that the voter's preferred equilibria focus on controlling the parties and not selecting the better party. The better party receives a policy rent in that she gets to shade policy towards her own optimum. The model points out that the presence of rents to holding office and sufficient patience can be utilized to construct time consistent incentives for policy selection even when parties have distinct policy preferences and one party is clearly better than the other. This point seems particularly relevant to debates about term limits and office compensation.

Appendix

We say that party l is advantaged if given the two policy mappings:

$$\psi_l(b) = \begin{cases} \mathbf{x}_m^*(b,1) & \text{if } b > b^* \\ \mathbf{x}_l^*(b,1) & \text{otherwise} \end{cases}$$
$$\psi_r(b) = \begin{cases} \mathbf{x}_m^*(b,1) & \text{if } b < b^* \\ \mathbf{x}_r^*(b,1) & \text{otherwise,} \end{cases} \tag{A1}$$

we have $\int u_m(\psi_l(b))\, dF_b(b) > \int u_m(\psi_r(b))\, dF_b(b)$.

Specifically, if l is the advantaged party then for some $b^\# > b^*$ we will construct an equilibrium which implements the policy function that whenever $b > b^\#$, l selects $\mathbf{x}_l^*(b,1)$. This will be a best response for an l with $b > b^\#$ as long as m does not punish l for selecting policies that are only feasible if $b > b^\#$. This modification makes the value of having l in office decrease. Accordingly, in constructing an SPBE with partial control when symmetry fails we will use a $b^\#$ which is chosen to equate the expected utility to m of having each party in office. The advantaged party will then shirk for some values of b on the desirable side of b^* (namely, the extreme ones with $b > b^\#$). It is obvious that analysis with r advantaged would be completely analogous. We now formalize this extension.

We first define the critical point $b^\#$ by the equation

$$\int_\gamma^{b^*} u_m\big(\mathbf{x}_l^*(b,1)\big)\, dF(b) + \int_{b^*}^{b^\#} u_m\big(\mathbf{x}_m^*(b,1)\big)\, dF(b) + \int_{b^\#}^{1-\gamma} u_m\big(\mathbf{x}_l^*(b,1)\big)\, dF(b) = \int u_m(\psi_r(b))\, dF_b(b), \tag{A2}$$

where the mapping $\psi_r(\cdot)$ is identical to that in (A1). Using the intermediate value theorem we can establish the existence and uniqueness of the point $b^{\#}$ when l is advantaged.

LEMMA 1: *If l is advantaged and*

$$\int u_m(\psi_r(b))\, dF_b(b) > \int u_m\big(\mathbf{x}_l^*(b)\big)\, dF_b(b), \tag{A3}$$

then exactly one point $b^{\#} \in (b^*, 1-\gamma)$ exists that solves (A1).

Proof: Let the left-hand side of (A2) be denoted by the function $\phi(\cdot) : (b^*, 1-\gamma) \to \mathbb{R}^1$. Note that by continuity

$$\lim_{b^{\#} \to b^*} \phi(b^{\#}) = \int u_m\big(\mathbf{x}_l^*(b, 1)\big)\, dF_b(b). \tag{A4}$$

By (A3) the right-hand side of the above is less than $\int u_m(\psi_r(b))\, dF_b(b)$, thus

$$\lim_{b^{\#} \to b^*} \phi(b^{\#}) < \int u_m(\psi_r(b))\, dF_b(b). \tag{A5}$$

On the other hand

$$\lim_{b^{\#} \to 1-\gamma} \phi(b^{\#}) = \int_{\gamma}^{b^*} u_m\big(\mathbf{x}_l^*(b, 1)\big)\, dF_b(b) + \int_{b^*}^{1-\gamma} u_m\big(\mathbf{x}_m^*(b, 1)\big)\, dF_b(b). \tag{A6}$$

The right-hand side of this is greater than $\int u_m(\psi_r(b))\, dF_b(b)$ because l is advantaged. Since $\phi(\cdot)$ is continuous there is a value $b^{\#}$ solving (A2) by the intermediate value theorem. Since $\phi(\cdot)$ is strictly monotone this value is unique. ■

Condition (A3) states that l is not so advantaged that the voter would prefer having l select her optimum in every state to having r use the partial control strategy. The analogue to conclusion 2 when l is advantaged (but not by too much) can now be stated and proven.

CONCLUSION 3: *If l is advantaged and condition (A3) holds then the following profile is supportable as an SPBE with partial control*

$$A_l = \left\{\mathbf{x} : b^{\#} > \mathbf{x}_m^{*-1}(\mathbf{x}) > b^*\right\} \cup \left\{\mathbf{x} \in \beta : x_2 \geq \frac{1}{1-b^{\#}}\right\}$$

$$A_r = \left\{\mathbf{x} : \mathbf{x}_m^{*-1}(\mathbf{x}) < b^*\right\}$$

$$\psi_l'(b) = \begin{cases} \mathbf{x}_m^*(b,1) & \textit{if } b \in (b^*, b^\#) \\ \mathbf{x}_l^*(b,1) & \textit{otherwise} \end{cases}$$

$$\psi_r'(b) = \psi_r(b) = \begin{cases} \mathbf{x}_m^*(b,1) & \textit{if } b < b^* \\ \mathbf{x}_r^*(b,1) & \textit{otherwise} \end{cases}$$

if the following conditions are satisfied

$$\frac{\max_{b\in[b^*,b^\#]} \left[u_l\big(\mathbf{x}_l^*(b)\big) - u_l\big(\mathbf{x}_m^*(b)\big)\right]}{\eta_l + \int \left[u_l\big(\psi_l'(b')\big) - u_l\big(\psi_r'(b')\big)\right] dF_b(b')} \le \delta \tag{C1'}$$

$$\frac{\max_{b\in[\gamma,b^*]} \left[u_r(\mathbf{x}_r^*(b)) - u_r\big(\mathbf{x}_m^*(b)\big)\right]}{\eta_r + \int \left[u_r\big(\psi_r'(b')\big) - u_r\big(\psi_l'(b')\big)\right] dF_b(b')} \le \delta. \tag{C2'}$$

Proof: By construction $b^\#$ is chosen so that Condition 1 is satisfied by $\psi'_l(b)$ and $\psi'_r(b)$. It remains only to show that the policy functions are mutual best responses.

-Consider party l: Assume that $\psi'_r(b)$ and A_l, A_r are given by the proposition. It is sufficient to show that no unilateral single-period deviation from $\psi_l'(b)$ is desirable. If $b > b^\#$ then selection of $\mathbf{x}_l^*(b, 1)$ is clearly optimal as it results in reelection and is the optimal feasible policy for l. Thus no deviation from $\psi'_l(b)$ is desirable in this case. If $b \in (b^*, b^\#)$, no policy in $\{\mathbf{x} \in \beta : x_2 \geq \frac{1}{b^\#}\}$ is feasible and thus l faces exactly the choice she did under the ballot function in conclusion 2. Thus the proof of the optimallity of $\psi'_l(b)$ for $b \in (b^*, b^\#)$ is the same as that for the optimallity of $\psi_l(b)$ for $b \in [b^*, 1-\gamma]$ in the proof of conclusion 2 and the associated condition C1$'$ attains. Similarly the optimallity of $\psi'_{rl}(b)$ follows from a similar argument. ■

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