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Keeping the other candidate guessing: Electoral competition when preferences are private information

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Abstract. We analyze a, model of two candidate competition in which candidate and voter preferences are private information. If candidates simultaneously commit to policy platforms the uncertainty about candidate preferences reinforces the incentive for platform divergence. After a candidate observes the other candidate's stance but before she learns about voter preferences she may face regret about her choice. This ex post irrationality suggests that a 1 period model may not capture the relevant incentives. In a multi-period proposal game in which candidates first make non-binding public proposals and then they make binding public proposals (similar to Ledyard, 1989) we find that candidates are uninformative during the first stage, as they have a disincentive to reveal their preferences to the opposing candidate. This finding offers an explanation for candidate ambiguity or inconsistency early in an election which does not involve efforts to deceive voters. Candidates may be trying to keep their opponent guessing. With a strong pre-election commitment technology, candidates can only be deterred from this type of behavior if they anticipate that a sizeable number of voters (more than a majority) will vote contrary to their preferences over policy.

1. Introduction

> "Politicians are notoriously reluctant to take clear stands on issues of the day" (Page, 1976: 742)

The spatial model of competition is the dominant paradigm for formal scholarship about two party elections The simplest version pits two office seeking parties in competition, posits that voters have symmetric single-peaked preferences over a single policy dimension and assumes that office motivated candidates can simultaneously commit to policy platforms before the election (Downs, 1957; Black, 1958). The conclusion that, in equilibrium the parties announce identical centrist policies has spurned a vast theoretical literature (e.g., Calvert, 1985; Davis and Hinich, 1970; Ledyard, 1984, McKelvey and Ordeshook, 1985, 1986a,b; Wittman, 1977, 1983). Wittman considers mixed motivation by candidates and finds that with uncertainty about the location of the median voter's ideal point (or probabilistic voting) candidate divergence can occur in equilibrium. Calvert shows that despite this finding the Downsian model is robust as small changes in candidate motivation or uncertainty lead to small deviations from candidate convergence in equilibrium play. This paper further extends the basic Downsian/Wittman model by assuming that policy motivated candidates face uncertainty about the preferences of the opposing candidate and that there is uncertainty about the location of the median voter's ideal point (or that voting is probabilistic). In a Bayesian game in which candidates simultaneously announce binding platforms and then voting occurs, the informational asymmetry about candidate preferences reinforces platform divergence.

Ledyard (1989) considers models of Downsian competition in which candidates care only about winning, have asymmetric information about the location of the median voter and candidates first make cheap talk announcements before making binding policy announcements. He finds that candidates have an incentive to hide their private information and thus first stage announcements cannot be informative in equilibrium. We follow Ledyard and augment the current model to include a preliminary cheap talk stage. If voters use weakly undominated strategies Ledyard's result extends to our setting with policy motivated candidates and private information about candidate preferences.

The intuition for the result is straightforward. If voters use weakly undominated strategies, then they care only about the policy platforms that the candidates commit to before the election. Accordingly, a candidate's statement in the first (cheap talk) period cannot effect voter behavior, but it might effect candidate behavior. Specifically, if the left candidate's first period statement is informative, revealing information about her preferences (and thus her likely second period platform) to the right candidate, then the right candidate's policy commitment will depend both on her private information and on the first period statement of the left candidate. In this case the left candidate will have an incentive to deceive the right candidate (for example by pretending to be more leftist than she actually is). Such a deception will effect the right candidate's second period platform, and thus the likelihood that the left candidate wins as well as the lottery over the final policy.

Creation of incentives for candidates to reveal their preferences early requires that voters commit to punishing candidates that unilaterally change their platforms throughout the election. While such commitment is possible in an equilibrium it requires that voter strategies are best response only because the probability of being pivotal is zero. The usual refinement of weakly undominated voting strategies excludes this possibility.

While the logic behind the result is similar to that presented by Ledyard, there is a subtle difference. When the information asymmetry is just over the location of the median voter (Ledyard), it is clear that a candidate values his opponent's private information because it allows for better forecasting of the location of the median. Here, the informational asymmetry pertains to the candidates' ideal points. The left candidate wishes to know the right candidate's private information because it allows her to better predict the right candidate's future actions. In both cases the information that a candidate can learn is useful for forecasting the expected utility of a particular action.

At a substantive level the different sources of informational asymmetry in the two models is also important. While both asymmetries might persist, the presence of repeated polling by candidates and many public third party polls suggests that the candidates probably have nearly equal assessments of voter preferences. In contrast, a candidate (especially a novice) cannot be pinned down unless they choose to. Even officials with long-histories in public life occasionally change their policy positions/platforms. Thus, this source of informational asymmetry seems at least as interesting and may be more prevalent.

Theoretically, the paper makes three advances. First pure strategy equilibria are shown to exist in a Bayesian model of electoral competition in which candidate type and strategy spaces are uncountable. Second, Ledyard's result about the lack of information revelation in cheap talk stages of the election is shown to also hold in models with no information asymmetries (between candidates) about the voter preferences but with information asymmetries about candidate preferences. Finally, the relevance of the assumption that voting strategies satisfy weak dominance is explored.

Substantively, the paper forwards a possible explanation for candidate ambiguity. Assessing the evidence (e.g., Campbell, 1983; Kroszner and Stratmann, 1999; Page, 1976; Snyder, 1990,1992) for and against campaign clarity or ambiguity is beyond the scope of this paper. In terms of theoretical work, Fishburn (1972), Shepsle (1972), and Glazer (1990) all present explanations for ambiguity that hinge on the desirability of remaining ambiguous in a one period setting. Aragonès and Postlewaite (1999), Aragonès and Neeman (2000), and Meirowitz (2001) present explanations that hinge on the desirability of remaining uncommitted in a second period of play. In stark contrast to these papers, here as in Ledyard's model the incentive to remain unclear (up until the end of the election) is driven by a candidate's desire to keep the opposing candidate from learning her private information.

The remainder of the paper is organized as follows. In Section 2 we present the basic model of elections in which candidates do not know each others preferences. We establish the existence of equilibria to this model and offer several qualitative characterizations of the equilibria. In Section 3 we present and analyze a two period version of the game. In Section 4 we conclude with a discussion. To preserve the continuity of presentation the more technical proofs are relegated to an appendix.

2. The basic model

We begin by describing the electoral setting. The electorate, $N = \{1, 2, \ldots, n\}$ consisting of n, odd, voters has preferences representable by symmetric single-peaked utility functions on the compact unidimensional policy space $X = [0, 1]$. By $y_m \in X$ we denote the all important median voter's ideal point. There are two candidates $C = \{l, r\}$, and each cares about the policy of the winning candidate. In the basic model candidates $c \in C$ simultaneously announce policy platforms $x_c \in X$, and voters $i \in N$ then cast simultaneous ballots $v_i \in C$. The winning candidate $w \in C$ (by simple majority rule) enacts the policy she announced, x_w. Voter and candidate preferences over final polices x can be represented by the utility function

$$u_i(x; y_i) = h(|x - y_i|) \tag{1}$$

where $h : \mathbb{R}^1 \to \mathbb{R}^1$ is strictly decreasing, concave, and twice differentiable and $y_i \in X$ is agent i's ideal point for $i \in C \cup N$.[1] We denote the vector of ideal points $(y_l, y_r, y_1, \ldots, y_n)$ by $y \in X^{n+2}$.

We introduce imperfect information by considering a Bayesian game in which the ideal points y are not publicly known. Only candidate c knows $y_c(c \in \{l, r\})$ and only voter i knows $y_i(i \in N)$. The prior distributions of the candidate ideal points are given by the distribution functions $F_l(\cdot)$ with support $[0, \frac{1}{2}]$ and $F_r(\cdot)$ with support $[\frac{1}{2}, 1]$. Unless otherwise stated we assume that voters use weakly undominated strategies. Since voters act only at the end of the game this assumption is equivalent to the assumption that voting is sincere. Under sincere voting the median voter's ideal point is a sufficient statistic, and thus we need only concern ourselves with the distribution $F_m(\cdot)$ of y_m. We assume that $F_m(y) = y$, on $y \in [0, 1]$. The assumption that the median ideal point is uniformly distributed imposes two relevant features. First, the middle of the policy space is the middle of the prior distribution of the median voter's ideal point. Thus the candidates are assured to be on opposite sides of the middle of the prior distribution of the median voter's ideal point. Second, this distribution function has the property that the probability that l(r) wins is linear in the policy platform $x_l(x_r)$ holding fixed $x_r(x_l)$.[2] We will consider either the case of $F_l(\cdot), F_r(\cdot)$ strictly increasing and continuously differentiable or concentrated at known ideal points y_l and y_r respectively. In the former case the strictly positive densities $f_l(\cdot), f_r(\cdot), f_m(\cdot)$ exist. We assume that candidates can only select policies on their side of the policy space, thus $x_l \in [0, \frac{1}{2}]$ and $x_r \in [\frac{1}{2}, 1]$. This assumption is consistent with the view that parties constrain platform selection, but there is uncertainty and flexibility about how leftist (rightist) a Democrat (Republican) will be. This assumption is maintained only for convenience of exposition. The main results (e.g., propositions 1 and 4) hold without the assumption. One substantive motivation is the finding that for most U.S. legislatures there is little overlap between the Republican and Democrat ideal point estimates (Poole and Rosenthal, 1997: Ch. 4). This finding indicates that the game does not exclude a type of behavior that occurs often.

The 1-period proposal Bayesian game is characterized by the primitives $\langle h(\cdot), F_l(\cdot), F_r(\cdot)\rangle$. The timing of a 1-period proposal Bayesian game is simple. Nature draws y. The candidates knowing only their own ideal point simultaneously announce x_c. The voters observe (x_l, x_r) and cast ballots $v_i \in \{l, r\}$. The winning candidate (by majority rule), w, enacts her announced policy x_w. Utilizing the assumption of sincere voting we let $p(x_l, x_r) := F_m\left(\frac{x_l+x_r}{2}\right) = \frac{x_l+x_r}{2}$ denote the probability that candidate l wins when the candidates use the profile of platforms x_l, x_r. Of course the probability that candidate r wins is just $1 - p(x_l, x_r)$.

2.1. *Equilibria of the 1-period proposal game*

When the candidate ideal points are known (i.e., $F_l(\cdot), F_r(\cdot)$ are both concentrated at the candidate ideal points y_l and y_r respectively) the Bayesian Nash equilibrium (BNE) to this game involves announcements solving the system

$$\begin{aligned}
\hat{x}_l(x_r) &\in \arg\max_{x\in[0,\frac{1}{2}]} \{u_l(x; y_l)p(x, x_r) + u_l(x_r; y_l)[1 - p(x, x_r)]\} \\
\hat{x}_r(x_l) &\in \arg\max_{x\in[\frac{1}{2},1]} \{u_r(x_l; y_r)p(x_l, x) + u_r(x; y_r)[1 - p(x_l, x)]\} \qquad (2)
\end{aligned}$$

Moreover, standard arguments ensure that a BNE exists (Debreu, 1952; Glicksberg, 1952; Fan, 1952).

The addition of uncertainty about y_l and y_r (i.e., $F_l(\cdot), F_r(\cdot)$ are not concentrated) results in a game where a, pure strategy for a candidate is a schedule assigning a policy platform x_c to each possible ideal point y_c in the support of $F_c(\cdot)$. Thus, the strategy space for each candidate is a space of functions. Specifically, candidate l must choose a function $x_l(y_l) : [0, \frac{1}{2}] \to [0, \frac{1}{2}]$ and candidate r must choose a function $x_r(y_r) : [\frac{1}{2}, 1] \to [\frac{1}{2}, 1]$. Using the notation A^B to denote the space of functions that map from B into A, we can represent the candidate best responses by either $\xi_l(y_l; x_r(y_r)) : [0, \frac{1}{2}] \times [\frac{1}{2}, 1]^{[\frac{1}{2},1]} \to\to [0, \frac{1}{2}]$ or $\xi_l(x_r(y_r)) : [\frac{1}{2}, 1]^{[\frac{1}{2},1]} \to\to [0, \frac{1}{2}]^{[0,\frac{1}{2}]}$.

These best responses can be thought of as either specifying a policy platform x_l for each ideal point y_l and mapping for candidate r, or as specifying a mapping from $[0, \frac{1}{2}]$ into $[0, \frac{1}{2}]$ for each mapping by candidate r. Similarly the best response mapping for candidate r is a mapping $\xi_r(y_r; x_l(y_l))$: $[\frac{1}{2}, 1] \times [0, \frac{1}{2}]^{[0,\frac{1}{2}]} \rightarrow\rightarrow [\frac{1}{2}, 1]$ or $\xi_r(x_l(y_l)) : [0, \frac{1}{2}]^{[0,\frac{1}{2}]} \rightarrow\rightarrow [\frac{1}{2}, 1]^{[\frac{1}{2},1]}$. The candidate best responses must satisfy the optimality conditions

$$\begin{aligned} \xi_l(y_l; x_r(y_r)) &\in \arg\max_{x\in[0,\frac{1}{2}]} \left\{ \int_{\frac{1}{2}}^{1} \begin{pmatrix} u_l(x; y_l)p(x, x_r(y_r)) + \\ u_l(x_r(y_r); y_l)[1 - p(x, x_r(y_r))] \end{pmatrix} f_r(y_r)\, dy_r \right\} \\ \xi_r(y_r; x_l(y_l)) &\in \arg\max_{x\in[\frac{1}{2},1]} \left\{ \int_0^{\frac{1}{2}} \begin{pmatrix} u_r(x_l(y_l); y_r)p(x_l(y_l), x) + \\ u_r(x; y_r)[1 - p(x_l(y_l), x)] \end{pmatrix} f_l(y_l)\, dy_l \right\}. \end{aligned} \tag{3}$$

For every y_l candidate l's platform $x_l(y_l)$ must be optimal when candidate l faces uncertainty about the ideal points of candidate r and voter m. The former uncertainty manifests itself as uncertainty about $x_r(y_r)$ and the latter appears in the probabilistic win function $p(\cdot, \cdot)$. A BNE to this game involves a pair of mappings $(x_l^*(y_l), x_r^*(y_r))$ solving the condition $\xi_l(x_r^*(y_r)) = x_l^*(y_l)$ and $\xi_r(x_l^*(x_l^*(y_l)) = x_r^*(y_r)$. This condition characterizes a fixed point of an operator on a functional space. In the appendix we establish the existence of such an equilibrium when the function $h(\cdot)$ is strictly concave and twice differentiable.[3] Throughout we will maintain these assumptions.

Proposition 1. If $h(\cdot)$ is strictly concave and twice differentiable and the distributions of the candidate ideal points $F_l(\cdot), F_r(\cdot)$ are strictly increasing and differentiable on their supports then a BNE to the 1-period proposal Bayesian game exists.

An unsurprising feature of a BNE to the 1-period proposal Bayesian game is that the mappings $x_c^*(y_c)$ are increasing. That is, the policy platforms are responsive to ideal points in the intuitive manner. The argument hinges on using the implicit function theorem to demonstrate that any solution to (3) is a strictly increasing function.

Proposition 2. If $h(\cdot)$ is strictly concave and twice differentiable then any equilibrium platform mapping $x_c^*(y_c)$ is strictly increasing in y_c.

This fact has an important implication. After observing the candidate platforms x_l^*, x_r^* the ideal points of the candidates can be found by inverting the equilibrium mappings. The fact that candidate platforms vary with candidate types implies that the equilibrium to the 1-period proposal Bayesian game may not be robust in an interesting sense. Suppose that after the candidates make their equilibrium proposals candidate l has the opportunity to commit to a different policy. This would involve solving the problem

$$x_l^2(y_l) \in \arg\max_{x \in [0,\frac{1}{2}]} \{u_l(x; y_l)p(x, x_r) + u_l(x_r; y_l)[1 - p(x, x_r)]\} \tag{4}$$

The solution to this problem satisfies the first order condition

$$\frac{\partial u_l(x; y_l)}{\partial x} p(x, x_r) + \frac{\partial p(x, x_r)}{\partial x}[u_l(x; y_l) - u_l(x_r; y_l)] = 0 \tag{5}$$

While this problem is similar to the one that $x_l^*(y_l)$ solves, (5) is contingent on x_r and the first order condition from (3)

$$O = \mathbb{E}_{y_r}\left[\frac{\partial U_l(x, y_l)}{\partial x}\right] = \int_{\frac{1}{2}}^{1} \left(\begin{array}{c} \frac{\partial p(x, x_r(y_r))}{\partial x}[u_l(x, y_l) - u_l(x_r(y_r); y_l)] \\ +p(x, x_r(y_r))\frac{\partial u_l(x; y_l}{\partial x} \end{array} \right) f_r(y_r) dy_r. \tag{6}$$

involves integration over $x_r(y_r)$.[4] Accordingly, for almost every x_r that can be played in a BNE to the 1-period proposal Bayesian game, the solution to (5) does not correspond to the platform $x_l^*(y_l)$ that solves (6). This implies that the following result holds.

Proposition 3. If the candidate ideal point distributions, $F_l(\cdot), F_r(\cdot)$, are strictly increasing and differentiable on their supports then with probability 1 a BNE to the 1-period proposal Bayesian game is not ex post rational.

In other words candidate l will have an incentive to select a different platform after observing x_r if she alone is given the chance to change her platform. Of course the same is true of candidate r. She has an incentive to offer a different proposal after learning x_l. This non-robustness (or ex post irrationality) does not surface in pure strategy equilibria to the spatial model of electoral competition when candidates do not face uncertainty about each other's preferences. In those models the candidates announce platforms that are simultaneous best responses to each other's platforms and thus have no regret about their choice after observing the platform of their opponent. Here candidates are announcing platforms that are only optimal given beliefs about the type of the other player, and when information about this type is revealed, a deviation in platforms becomes desirable. Given the empirical unreasonableness of assuming that candidates can only make platform announcements once, this finding motivates consideration of a multi-period proposal game.

3. The 2-period proposal game

The 2-period proposal Bayesian game differs from the 1 period version only in that it features two periods in which candidates make platform announcements. In the first period each candidate simultaneously announces a platform x_c^1 and then after observing the profile (x_l^1, x_r^1) each candidate simultaneously announces a platform x_c^2. Voters then cast their ballots knowing all of the platforms $x_l^1, x_r^1, x_l^2, x_r^2)$. We maintain the assumption that candidates can commit to policies before voters cast their ballots. The simplest nontrivial assumption is to assume that the platforms x_c^2 are binding. That is, if a candidate has offered two different platforms $x_c^1 \neq x_c^2$ and she is elected, she will enact the policy x_c^2. In principal voters may choose to punish candidates that behave in this manner $(x_c^1 \neq x_c^2)$ but the extent to which this phenomena occurs is an equilibrium question. As long as voters use weakly undominated strategies they will vote for whichever candidate selects a better second period platform ($|x_c^2 - y_i|$ is smallest). A 2-period proposal Bayesian game is characterized by the same data as a 1-period proposal Bayesian game, $\langle h(\cdot), F_l(\cdot), F_r(\cdot)\rangle$. A strategy for player 1 is a pair of mappings $x_l^1(y_l) : [0, \frac{1}{2}] \rightarrow [0, \frac{1}{2}]$ and $x_l^2(x_r^1; y_l) : [\frac{1}{2}, 1] \times [0, \frac{1}{2}] \rightarrow [0, \frac{1}{2}]$. Similarly, a strategy for player r is a pair of mappings $x_r^1(y_r) : [\frac{1}{2}, 1] \rightarrow [\frac{1}{2}, 1]$ and $x_r^2(x_l^1; y_r) : [0, \frac{1}{2}] \times [\frac{1}{2}, 1] \rightarrow [\frac{1}{2}, 1]$. As before we assume that voters use weakly undominated strategies, and so the candidate with a x_c^2 that is closest to y_m wins the election and implements the policy x_c^2. At the end of this section we will relax this selection/assumption.

The 2-period proposal Bayesian game is dramatically different in one important sense-candidates might learn about the opposing candidate's preferences from first period platforms and candidates may have an incentive to manipulate their opponent's beliefs through first period proposals. The Bayesian Nash equilibrium concept is inappropriate as it does not constrain candidates to utilize information in a reasonable manner. More precisely, if $x_{-c}^1(\cdot)$ is responsive to y_{-c} then a reasonable notion of rationality would require that candidate c use updated posterior beliefs about y_{-c} to forecast x_{-c}^2 and this information should be used in determining the optimal second period platform. Accordingly, we require that candidate c have beliefs about y_{-c} conditional on the observed information x_{-c}^1. A conditional belief for c is a probability distribution over y_{-c} that is conditioned on the first period platform x_{-c}^1. We denote such a belief as $\mu_l(\cdot|x)$ and $\mu_r(\cdot|x)$. By $\Delta[\frac{1}{2}, 1]$ and $\Delta[0, \frac{1}{2}]$ we denote the sets of probability distributions over $[\frac{1}{2}, 1]$ and $[0, \frac{1}{2}]$ respectively.

Definition 1. A pair of second period beliefs is a double $\langle \mu_l(\cdot|x), \mu_r(\cdot|x)\rangle$

of distribution functions $\mu_l(\cdot|x) : [\frac{1}{2}, 1] \rightarrow \Delta[\frac{1}{2}, 1]$ and $\mu_r(\cdot|x)$: $[0, \frac{1}{2}] \rightarrow \Delta[0, \frac{1}{2}]$. These beliefs are consistent relative to strategies $(x_l^1(\cdot), x_r^1(\cdot))$ if for any $a \in [0, 1]$.

$$
\begin{aligned}
\mu_l(a|x) &= \frac{\int_{\{y_r:x_r^1(y_r)=x\}} 1_a(x_r^1(y_r))dF_r(y_r)}{\int_{[\frac{1}{2},1]} 1_a(x_r^1(y_r))dF_r(y_r)} \\
\text{if } x_r^1(y_r) &= x \text{ for some } y_r \in [\frac{1}{2}, 1] \qquad (7) \\
\mu_r(a|x) &= \frac{\int_{\{y_l:x_l^1(y_l)=x\}} 1_a(x_l^1(y_l))dF_l(y_l)}{\int_{[0,\frac{1}{2}]} 1_a(x_l^1(x_l^1(y_l))dF_l(y_l)} \\
\text{if } x_l^1(y_l) &= x \text{ for some } y_l \in [0, \frac{1}{2}].
\end{aligned}
$$

The function $1_a(x_c^1(y_c))$ is an indicator function taking value 1 if $x_c^1(y_c) \leq a$ and value 0 otherwise. Consistency of beliefs requires that Bayes' rule is satisfied when it is well defined. Beliefs that are consistent place positive probability only on sets of ideal points containing a point y_{-c} that solves $x_{-c}^1(y_{-c}) = x$. When the strategies $(x_l^1(\cdot), x_r^1(\cdot))$ are strictly monotone this is equivalent to inverting the mapping $x_{-c}^1(\cdot)$ and believing that y_{-c} satisfies $x_{-c}^1(y_{-c}) = x$ (with probability 1) where x is the observed stance of $-c$ in period 1. We will require that candidates use strategies that are mutual best responses given their beliefs and that these beliefs satisfy the consistency condition of definition 1.

Definition 2. A perfect Bayesian equilibrium in weakly undominated voting strategies to the 2-period proposal Bayesian game is a pair of second period beliefs $\langle\mu_l(\cdot|x), \mu_r(\cdot|x)\rangle$, candidate strategies $\langle x_l^1(\cdot), x_l^2(\cdot;\cdot), x_r^1(\cdot), x_r^2(\cdot;\cdot)\rangle$ and voting functions satisfying the conditions:

(1) The beliefs are consistent relative to the strategies $(x_l^1(\cdot), x_r^1(\cdot))$.
(2) The second period strategies $\langle x_l^2(\cdot;\cdot), x_r^2(\cdot;\cdot)\rangle$ are best responses for every history x_l^1, x_r^1 given the beliefs.
(3) The first period strategies $(x_l^1(\cdot), x_r^1(\cdot))$ are optimal holding fixed the second period strategies.
(4) $v_i = c$ if $|x_c^2 - y_i| < |x_{-c}^2 - y_i|$.

Condition 4 captures sincere voting which is implied by the requirement of weakly undominated voting strategies. Condition 2 requires that the second period strategies form a BNE to the 1 period Bayesian game in which candidate beliefs are given by $\mu_l(\cdot|x_r^1)$ and $\mu_r(\cdot|x_l^1)$. Condition 3 requires that holding fixed $x_{-c}^1(\cdot)$, $x_{-c}^2(\cdot;\cdot)$ and $x_c^2(\cdot;\cdot)$ the first period strategy $x_c^1(\cdot)$ solves

$$x_l^1(y_l) \in \arg\max_{x\in[0,\frac{1}{2}]}$$

$$\left\{\int_{\frac{1}{2}}^{1}\begin{pmatrix} u_l(x_l^2(x_r^1(y_r);y_l);y_l)p(x_l^2(x_r^1(y_r);y_l),x_r^2(x;y_r))+ \\ u_l(x_r^2(x;y_r);y_l)[1-p(x_l^2(x_r^1(r_r);y_l);y_l),x_r^2(x;y_r))] \end{pmatrix} f_r(y_r)dy_r\right\}$$

$$x_r^1(y_r) \in \arg\max_{x\in[\frac{1}{2},1]} \tag{8}$$

$$\left\{\int_{0}^{\frac{1}{2}}\begin{pmatrix} u_r(x_l^2(x;y_l);y_r)p(x_l^2(x;y_l),x_r^2(x_l^1(y_l);y_r))+ \\ u_r(x_r^2(x_l^1(y_l);y_r);y_r)[1-p(x_l^2(x;y_l),x_r^2(x_l^1(y_l);y_r))] \end{pmatrix} f_l(y_l)dy_l\right\}$$

These two problems involve the selection of x_c^1 to maximize expected utility when the second period platform x_{-c}^2 is contingent on x_c^1 and the uncertain ideal point y_{-c} and the second period platform x_c^2 is contingent on x_{-c}^1 which is itself contingent on the uncertain y_{-c}.

A natural conjecture is that first period strategies will be strictly monotone. This means that candidates reveal their preferences through their first period stances. A special case of this involves first period strategies that are *truthful*, $x_c^1(y_c) = y_c$. In fact the revelation principal implies that if an equilibrium with strictly monotone first period strategies exists then one with truthful first period strategies must exist. If first period strategies are strictly monotone then consistent beliefs will be concentrated at the true ideal points. This means that a deviation by c to a different x_c^1 will fool $-c$ into believing that c has a different type, y_c. Thus, if first period strategies are strictly monotone in a PBE then it must be the case that candidates have no incentive to mislead each other. In the appendix we show that almost every type y_c will have an incentive to mislead $-c$. The argument hinges on showing that for all but knife edged parameterizations of $F_l(\cdot)$, $F_r(\cdot)$, for almost any value of y_c candidate c is better off (in expectation) facing an opponent $-c$ that has a particular incorrect assessment of the location of y_c.

Some interpretation of the claim is needed. We do not show that there are never instances where BNE exist in which candidates reveal their preferences through x_c^1. Rather, these instances are rare and unstable. From any parameterization of the game in which this type of PBE exists we can make an arbitrarily small perturbation to $F_l(\cdot)$ or $F_r(\cdot)$ or both, and it will no longer be the case that the candidates use strictly monotone first period strategies in a PBE with weakly undominated voting. It is not however true that every arbitrary small perturbation will eradicate the existence of strictly monotone equilibria. It is possible to show that for a fixed magnitude of perturbation almost every perturbation of that magnitude will eradicate the strictly monotone equilibrium. Proposition 3 states that in the generic or regular case of 2-period proposal Bayesian games, candidates will not reveal their ideal points in the first period. In other words, the model predicts that candidates do not provide a lot of information early in the campaign. The possibility that some information is conveyed in the first period is not, ruled out.

Proposition 4. For any strictly decreasing and strictly concave loss function $h(\cdot)$, on a generic subset of candidate ideal point distributions $F_l(\cdot), F_r(\cdot)$ there is not a BNE in weakly undominated voting strategies to the 2-period proposal Bayesian game in which candidates use strictly monotone strategies $x_c^1(y_c)$ in the first period.

3.1. *Truthful equilibria*

If we relax the assumption that voting involves weakly undominated strategies, condition 4 of definition 2 is replaced by

(4′) Voting strategies form a Nash Equilibria of the simultaneous voting game over platforms x_l^2, x_r^2.

With this modification there are PBE in which first period proposals are truthful.

Proposition 5. If voters use weakly dominated strategies there is a PBE to the 2-period proposal Bayesian game in which candidates are truthful $x_c^1 = x_c^2$):

Proof. Consider the following voting strategies:

$$v_i = \begin{cases} l \text{ if } x_l^1 = x_l^2 \text{ and } x_r^1 = x_r^2 \text{ and } |x_l^1 - y_i| \leq |x_r^1 - y_i| \\ l \text{ if } x_r^1 \neq x_r^2 \text{ and } x_l^1 = x_l^2 \\ r \text{ if } x_l^1 = x_l^2 \text{ and } x_r^1 = x_r^2 \text{ and } |x_l^1 - y_i| > |x_r^1 - y_i| \\ r \text{ otherwise} \end{cases}$$

Given these strategies if c is not going to change her platform from one period to the next then $-c$'s best response is to not change her platform, since any pair (x_l^2, x_r^2) induces a, non-degenerate lottery over the winning platform, and a unilateral switch in policy results in a certainty of loss. Given this, a pair of simultaneous best responses is: $x_l^1(y_l), x_r^1(y_r)$ solving (8) and $x_c^2(x_{-c}^1, y_c) = x_c^1$ for $c \in C$. The above voting strategies satisfy (4′) because they are either sincere in the event that $x_c^1 = x_{-c}^2$ for $c \in C$ or every voter is voting the same way and thus no voter is pivotal. □

The construction hinges on having voters vote sincerely if both candidates have offered consistent or truthful platforms, and having all voters vote against a candidate if she is the only one that was inconsistent ($x_c^1 \neq x_c^2$). This punishment is sufficient to induce candidates to play strategies that are BNE in the 1 period game, and then to not change their platforms in the second period. However, the punishment requires voters to vote against their interests just because they are not pivotal (at off the path histories). Not surprisingly these equilibria involve strategies that are not coalition proof at histories following inconsistency by one candidate. If voters actually care about platform consistency then it is possible for truthful equilibria to survive the weak dominance refinement. The analysis of such models is outside the scope of this paper.

4. Discussion

Inclusion of uncertainty about candidate preferences results in minimal changes to the qualitative predictions of single period commitment models. However, the analysis suggests that with this form of uncertainty a model with a single period in which platforms are announced may be unreasonable. Unlike Nash equilibria, Bayesian Nash equilibria, are not ex post rational. The equilibrium predictions of the two period model counter the expectation that voters will sanction candidates that are inconsistent throughout the campaign. This finding hinges on the fact that as long as candidates commit before the vote, previous inconsistencies are irrelevant. In the current model, this form of punishment must involve use of weakly dominated voting strategies. A more plausible explanation of punishment might hinge on voters trying to infer candidate quality or trust-worthiness. In this case they might face more uncertainty from a candidate that was not consistent. In the current model the strong commitment assumption removes this problem as voters face no payoff relevant uncertainty at the time of voting. Accordingly, in any Perfect Bayesian equilibrium with weakly undominated voting strategies, voters must simply compare the platforms of the available candidates, and candidates cannot expect punishment or reward for the degree of platform consistency they exhibit in the campaign.

It is not difficult to see that in more than 2 periods a similar result would obtain. Candidates will not reveal their preferences until the period where their opponent commits. This logic offers an explanation for policy inconsistency or ambiguity. With the exception of Ledyard's analysis which is not directly interpreted in terms of candidate ambiguity, existing theoretical explanations of ambiguity are quite distinct from the present one. Shepsle offered a simple partial equilibrium model in which ambiguity was desirable when voters were risk acceptant. Glazer was one of the first to argue that ambiguity could be a strategic response to uncertainty about the voter preferences. Alternatively, Alesina and Cukierman, and Aragonès and Neeman analyze models of elections in which ambiguity is motivated by the desire to remain unconstrained in the future. In these models, candidates exogenously prefer to be less constrained in office, whereas voters prefer precise stances. Meirowitz offers an explanation for ambiguity that hinges on the option value of waiting until information about voter preferences is revealed. In contrast the current explanation of early ambiguity does not hinge on risk aversion of voters, or a second period value to being unconstrained. Rather ambiguity early in the campaigning is explained by the desire to not reveal information about one's preferences to the opposing candidate.

Even though it is quite sparse the 2-period Bayesian platform game offers insight into the incentives faced by candidates. As all models in the Downsian tradition demonstrate, there is clearly an incentive to select platforms that are desirable to the voters. A second incentive only surfaces when the informational assumptions are relaxed (as in this model). There is also a strong incentive to not provide the competing candidate with information. An open question is whether there are equilibria (with weakly undominated voting) in which candidate first period platforms provide any information about their preferences. The possibility of equilibria, (on a generic set of games) in which first period platforms are partially revealing is not ruled out by proposition 4. However, even if such equilibria exist, it is clear that first and second period platforms will riot be equivalent and thus candidate inconsistency will still be present.

Some readers may find unsatisfactory the prediction that candidates hide information from the voters early in the campaign only to reveal it on the eve of the election, The prediction, that voters do not punish candidates for being unclear or inconsistent demonstrates that models which assume pre-election commitment (like those analyzed by Downs, Wittman and Calvert), may not be appropriate for the study of campaign dynamics. There is a large collection of dynamic theories of elections that do not assume commitment (Banks and Duggan, 2002; Banks and Sundaram, 1993, 1998; Austen-Smith and Banks, 1989; Ferejohn, 1986). Future work on campaigns might be well served by building on these frameworks. What the current paper does show is that within the class of models that assume commitment, there may be incentives for candidates to not provide information. In contrast to the view of pundits that inconsistency is an attempt to satisfy competing interests by "playing both sides of the fence" the analysis suggests that the position oscillation is driven by the candidates' strategic desire to "keep their opponent guessing".

Notes

1. Consideration of distinct loss functions $h_i(\cdot)$ for each player complicates the notation, but does not effect the results in any deep sense.
2. The assumption that the prior on the median voter's ideal point is uniform is stronger than necessary for some of the results. However the assumption is used in the proofs that are provided. See Davis and Hinich (1970) for more on this issue.
3. This result is a special case of the theorem found in Meirowitz (2003).
4. The technical assumptions needed to interchange the order of integration and differentiation are satisfied given (1) the continuity assumptions of the model, and (2) the boundedness of the supports of y_c.

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Appendix

Proof of Proposition 1

We show that the game satisfies the sufficient conditions of proposition 1 in Meirowitz (2003). This result pertains to a Bayesian game in which each agent $i \in N$ has type and action spaces Θ_i and A_i and utility function

$$u_i(a;\theta) : \prod_{i\in N} A_i \times \prod_{i\in N} \Theta_i \to \mathbb{R}^1$$

with conditional beliefs $\eta_i(\theta_{-i}|\theta_i)$ about the other player's types. In such a game the strategy space for player i is the set of measurable functions from Θ_i into A_i. We denote this set as $\Theta_i^{A_i}$. By Θ_{-i}, A_{-i} and $\Theta_{-i}^{A_{-i}}$ we denote the analogous product sets for all players other than i. We now state the relevant result.

Proposition. A Bayesian game has a pure strategy BNE if for each $i \in N$
(1) Θ_i and A_i are nonempty, convex and compact subsets of Euclidean space.
(2) $u_i(a;\theta)$ is continuous.
(3) for every θ and measurable function $x_{-i}(\theta_{-i})$

$$U_i(a_i,\theta_i) = \int u_i(a_i, x_{-i}(\theta_{-i}); \theta_i, \theta_{-i}) d\eta_i(\theta_{-i}|\theta_i)$$

is strictly quasiconcave in a_i.
(4) for every $\varepsilon_i > 0$ there exists some constant δ_i s.t. if

$$x_i(\theta_i) \in \arg\max_{a_i,\in A_i} \left\{ \int u_i(a_i, x_{-i}(\theta_{-i}); \theta_i, \theta_{-i}) d\eta_i(\theta_{-i}|\theta_i) \right\}$$

for some $x_{-i}(\theta_{-i}) \in \Theta_{-i}^{A_{-i}}$ then $\sup_{\{(\theta_i',\theta_i'')\in\Theta_i : ||\theta_i'-\theta_i''||<\delta_i\}} ||x_i(\theta_i') - x_i(\theta_i'')|| < \varepsilon_i$.
(5) for a.e. measurable set $A \subset \Theta_{-i}$, $\eta_i(A|\theta_i)$ is continuous in θ_i.

Since Θ_i and A_i are $[0,\frac{1}{2}]$ for $i = l$ and $[\frac{1}{2},1]$ for $i = r$ condition 1 is satisfied. The utility functions are just

$$u_l(x_l,x_r;y_l,y_r) = h(|x_l - y_l|\left(\frac{x_l + x_r)}{2}\right) + h(|x_r - y_l|)\left(1 - \frac{x_l + x_r)}{2}\right)$$
$$u_r(x_l,x_r;y_l,y_r) = h(|x_l - y_r|\left(\frac{x_l + x_r)}{2}\right) + h(|x_r - y_r|)\left(1 - \frac{x_l + x_r)}{2}\right)$$

which are both continuous (satisfying condition 2). Interchanging the order of integration and differentiation (allowable given the continuity and compactness assumptions on $F_r(\cdot)$ and its support) yields the second derivative (for $x_l > y_l$)

$$\mathbb{E}_{y_r}\left[\frac{\partial^2 U_l(x, y_l)}{\partial x^2}\right] = \int_{\frac{1}{2}}^{1} \left(\begin{array}{c} \left(\frac{1}{2}\right)\left(\frac{\partial u_i(x;y_l)}{\partial x}\right) + \\ \left(\frac{x+x_r(y_r)}{2}\right)\frac{\partial^2 u_l(x;y_l)}{\partial x^2} + \left(\frac{1}{2}\right)\left(\frac{\partial u_l(x;y_l)}{\partial x}\right) \end{array} \right) f_r(y_r)dy_r. \quad (9)$$

When $h(\cdot)$ is strictly concave, arguments in the body of the paper imply that for every y_r the integrand is strictly negative.

This implies that the integral is itself negative. Accordingly for every y_l the objective function is strictly concave on the relevant choice set $[y_l, \frac{1}{2}]$. A similar argument applies to r's objective function. Thus condition 3 is satisfied. Condition 4 is satisfied if the set of strategies $x_c(\cdot)$ that are best responses for some strategy $x_{-c}(\cdot)$ have a bounded slope. A best response $x_l^*(y_l)$ solves

$$0 = \mathbb{E}_{y_r}\left[\frac{\partial U_l(x, y_l)}{\partial x}\right]$$

The implicit function theorem implies that

$$\frac{\partial x_l^*(y_l)}{\partial y_l} = \frac{-\mathbb{E}_{y_r}\left[\frac{\partial U_l(x,y_l)}{\partial x \partial y_l}\right]}{\mathbb{E}_{y_r}\left[\frac{\partial^2 U_l(x,y_l)}{\partial x^2}\right]}. \quad (10)$$

The denominator is bounded away from 0 for all x_l and functions $y_r(\cdot)$ and the numerator is bounded since $h(\cdot)$ is strictly concave and the choice sets for l and r are bounded. A similar argument holds for r. Thus condition 4 is satisfied. Condition 5 is satisfied as the types are independent. □

Proof of Proposition 2

We prove the result for $c = l$. An analogous argument applies for $c = r$. We start with equation (10). We have shown in the proof of proposition 1 that the denominator is strictly negative. The numerator is given by

$$-\mathbb{E}_{y_r}\left[\frac{\partial U_l(x, y_l)}{\partial x \partial y_l}\right] = -\int_{\frac{1}{2}}^{1} \left(\begin{array}{c} \frac{\partial p(x,x_r(y_r))}{\partial x}\left[\frac{\partial u_i(x,y_l)}{\partial y_l} - \frac{\partial u_l(x_r(y_r):y_l)}{\partial y_l}\right] \\ +p(x, x_r(y_r))\frac{\partial^2 u_l(x:y_l)}{\partial x \partial y_l} \end{array} \right) f_r(y_r)dy_r. \quad (11)$$

The first term in the integral is positive as $y_l < x_l < x_r$ occurs in equilibrium, and $h(\cdot)$ is strictly concave. The second term in the integral is equivalent to $p(x, x_r(y_r))[-h''(x - y_l)]$ which is positive since $h''(\cdot) < 0$ as $h(\cdot)$ is strictly concave. Thus the integral is positive and thus $\frac{\partial x_l^*(y_l)}{\partial y_l}$ is the ratio of a negative number over a negative number and is thus positive. □

Proof of Proposition 4

It is not difficult to see that a revelation principal holds: A PBE in weakly undominated voting strategies with strictly monotone first period strategies exists iff a PBE in weakly undominated voting strategies with truthful first period strategies exists. Given this, the argument begins by assuming that we have a PBE in which the first period strategies are truthful, $x_c^1(y_c) = y_c$. We have already argued that in this case consistency of posterior beliefs requires that the beliefs are concentrated at the correct ideal points. Accordingly, in such a PBE the second period strategies must solve the problem

$$x_c^2(x_{-c}^1, y_c) \in \arg\max_x \left\{ \begin{array}{c} u_c(x; y_c)p(x, x_{-c}^2(y_c; x_{-c}^1))+ \\ u_c(x_{-c}^2(y_c; y_{-c}); y_c)[1 - p(x, x_{-c}^2(y_c; x_{-c}^1))] \end{array} \right\} \tag{12}$$

Recalling that $p(x_l, x_r) := \frac{x_l + x_r}{2}$, and differentiating yields the first order condition

$$\frac{u_c(x; y_c)}{2} + \left(\frac{x + x_{-c}^2(y_c; x_{-c}^1)}{2} \right) \frac{\partial u_c(x; y_c)}{\partial x} - \frac{u_c(x_{-c}^2(y_c; x_{-c}^1); y_c)}{2} = 0 \tag{13}$$

Applying the implicit function theorem yields the derivative

$$\frac{\partial x_c^2(x_{-c}^1, y_c)}{\partial x_{-c}^1} = \frac{-\left[\frac{\partial x_{-c}^2(y_c i x_{-c}^1)}{\partial x_{-c}^1} \left(\frac{\partial u_c(x; y_c)}{\partial x_c^2} - \frac{\partial u_c(z); y_c)}{\partial z} | z = x_{-c}^2(y_c; x_{-c}^1) \right) \right]}{\left[\frac{\partial u_c(x; y_c)}{\partial x} + (x + x_{-2}^2(y_c; x_{[}^1 - c])) \frac{\partial^2 u_c(x; y_c)}{\partial x \partial x} + \frac{\partial u_c(x; y_c)}{\partial x} \right]}. \tag{14}$$

Note that the argument that the optima $x_c^2(x_{-c}, y_c)$ solving (12) is differentiable hinges only on the implicit function theorem and the differentiability of $u_c(\cdot, \cdot)$ and $p(\cdot, \cdot)$. In proposition 2 we demonstrated that in the 1 period Bayesian game the optimal platform is strictly increasing in the ideal point. Since the second period platforms solve a 1 period Bayesian game in which the agents beliefs are given by consistent beliefs it must be the case that c anticipates that $\frac{\partial x_{-c}^2(y_c; x_{-c}^1)}{\partial x_{-c}^1} > 0$. For $c = l$, we know that $\frac{\partial u_c(x; y_c)}{\partial x} < 0$ near an equilibrium and since $x_c^2 < \frac{1}{2} < x_{-c}^2$ the strict concavity of $h(\cdot)$ implies that $\frac{\partial u_c(x; y_c)}{\partial x} - \frac{\partial u_c(z); y_c)}{\partial z}|_{z = x_{-c}^2(y_c; x_{-c}^1)} > 0$. Thus the numerator is negative. Again the fact that $\frac{\partial u_c(x; y_c)}{\partial x} < 0$ and $\frac{\partial^2 u_c(x; y_c)}{\partial x^2} < 0$ implies that the denominator is negative. Thus

$$\frac{\partial x_l^2(x_r^1, y_l)}{\partial x_r^1} > 0. \tag{15}$$

Similar arguments establish that

$$\frac{\partial x_r^2(x_l^1, y_r)}{\partial x_l^1} > 0. \tag{16}$$

Accordingly, candidate l can increase x_r^2 by announcing a higher x_l^1 and candidate r can decrease x_l^2 by announcing a higher x_r^1. When candidate c announces x_c^1 she does not yet know y_{-c} and thus cannot simply manipulate the deterministic function $x_{-c}^2(x_c^1, y_{-c})$. Nonetheless, candidate c in a truthful PBE will anticipate knowing y_{-c} prior to selecting x_c^2 and thus the question is whether prior to learning y_{-c} candidate c will have an incentive to shift up or down the distribution over $x_{-c}^2(x_c^1, y_{-c})$ which is induced by the uncertainty over y_{-c}. We have just shown that she can shift this distribution with her selection of x_c^1. It remains only to show that for almost every pair of densities $f_t(\cdot)$, $f_r(\cdot)$ there are realizations of ideal points, y_c, for which candidate c without knowing y_{-c} would rather mislead $-c$ to increase or decrease the resulting lottery over x_{-c}^2. Differentiation of the objective function for x_l^1 in (8) yields the first, order condition

$$\begin{aligned}&\int_{\frac{1}{2}}^{1}\left([u_l(x_l^2(y_r;y_l);y_l)-u_l(s_r^2(x;y_r);y_l)]\frac{\partial x_r^2(x;y_r)}{\partial x}\right)f_r(y_r)dy_r=\\&\int_{\frac{1}{2}}^{1}\left([x_l^2(y_r;y_l)+x_r^2(x;y_r))-2]\frac{\partial u_l(x_r^2(x;y_r);y_l)}{\partial x_r^2(x;y_r)}\frac{\partial x_r^2(x;y_r)}{\partial x}\right)f_r(y_r)dy_r\end{aligned} \tag{17}$$

To simplify subsequent expressions let this equation be denoted

$$\int_{\frac{1}{2}}^{1}\gamma(x,y_l,y_r)f_r(y_r)dy_r=\int_{\frac{1}{2}}^{1}\zeta(x,y_l,y_r)f_r(y_r)dy_r. \tag{18}$$

Since $y_l < y_r$ and $x_l < x_r$ we know that $\gamma(x, y_l, y_r)$ is strictly positive for every y_r. Letting $\frac{\partial u_l(z;y_c)}{\partial z} = \frac{\partial u_l(a;y_c)}{\partial a}|_{a=z}$ the term $\frac{\partial}{\partial y_r}[u_l(x_l^2(y_r;y_l);y_l) - u_l(x_r^2(x;y_r);y_l)]$ is given by

$$\frac{\partial u_l(x_l^2(y_r;y_l)}{\partial x_l^2(y_r;y_l)}\frac{\partial x_l^2(y_r,y_l)}{\partial y_r}-\frac{\partial u_l(x_r^2(x;y_r);y_l)}{\partial x_r^2(x;y_r)}\frac{\partial x_r^2(x;y_r)}{\partial y_r}. \tag{19}$$

Since $h(\cdot)$ is strictly concave and $x_l^2 < x_r^2$ this value is positive for every y_r. Since $\frac{\partial x_r^2(x;y_r)}{\partial x} > 0$ this implies by the product rule that $\gamma(x, y_l, y_r)$ is strictly increasing (decreasing) in y_r whenever $\frac{\partial x_r^2(x;y_r)}{\partial x}$ is weakly increasing (decreasing) in y_r. Conversely since $\frac{\partial u_l(x_r^2(x;y_r);y_l)}{\partial x_r^2(x;y_r)}$ is strictly decreasing in y_r and $[x_l^2(y_r;y_l) + x_r^2(x;y_r)) - 2]$ is strictly increasing in y_r, $\zeta(x, y_l, y_r)$ is strictly decreasing (increasing) in y_r whenever $\frac{\partial x_r^2(x;y_r)}{\partial x}$ is weakly increasing (decreasing) in y_r. Assume that at $(h(\cdot), f_l(\cdot), f_r(\cdot))$ equation (18) is satisfied for every $y_l \in [0, \frac{1}{2}]$ by the mapping $x = y_l$. Since $\gamma(x, y_l, y_r)$ and $\zeta(x, y_l, y_r)$ have opposite sign slopes on some interval and when $x - y_l$ they integrate to the same number under $f_r(\cdot)$ there must be some y_r^* where the graphs cross when $x = y_l$. That is $\gamma(y_l, y_l, y_r^*) = \zeta(y_l, y_l, y_r^*)$. Moreover on a neighborhood $(y_r^* - \delta, y_r^* + \delta)$ of y_r^* they are only equal at y_r^*. For a small $\varepsilon > 0(\varepsilon < F_f(y_r^* + \delta) - F_r(y_r^* - \delta))$ let λ_ε be a continuous function $\lambda_\varepsilon : [y_r^* - \delta, y_r^* + \delta] \to [-\varepsilon, \varepsilon]$ with $\lambda_\varepsilon(y_r^*) = 0$ and $\lambda_\varepsilon(y) < 0$ for $y \in [y_r^* - \delta, y_r^*]$ and $\lambda_\varepsilon(y) > 0$ for $y \in [y_r^* + \delta, y_t^*]$. Let Λ denote a sequence of such functions with

$\varepsilon \downarrow 0$. Clearly λ_ε converges to the constant function 0 on the domain $[y_r^* - \delta, y_r^* + \delta]$. Now consider the perturbed density

$$f_r^\varepsilon(y_r) = \begin{cases} f_r(y_r) + \lambda_\varepsilon(y_r) \text{ if } y_r \in (y_r^* - \delta, y_r^* + \delta) \\ f_r(y_r) \text{ otherwise} \end{cases} . \tag{20}$$

For any $\varepsilon > 0$ the condition (18) involving $f_r^\varepsilon(y_r)$ instead of $f_r(y_r)$ will not be satisfied at $x = y_l$. Since any open set of $f_r(\cdot)$ in the max-topology contains some function $f_r^\varepsilon(\cdot)$ (for some $\varepsilon > 0$) the solution $x = y_l$ does not exist for some perturbed density in any open set of $f_r(\cdot)$ (in the max-topology). Thus generically, the strategy $x_l^1(y_l) = y_l$ is not a best response. Similar arguments establish the fact that $x_r^1(y_r) = y_r$ is not optimal on a generic set of densities $f_l(\cdot)$. Thus we have proven the result. □