Electoral Contests, Incumbency Advantages and Campaign Finance

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Abstract: Most campaigns do not revolve around policy commitments; instead, we think of campaigns as contests in which candidates spend time, energy and money to win. This paper develops models of electoral competition in which candidates select levels of effort. The analysis offers insights about which causes of the incumbency advantage are consistent with the empirical record. Marginal asymmetries in costs or technology can explain the advantage; asymmetries in voter preferences cannot. The analysis also speaks to the consequences of campaign finance reform. Reforms can be interpreted as shocks to the cost of influencing voters' perceptions; limits generally increase the likelihood that advantaged incumbents win, and even limits that target incumbents do not improve the welfare of disadvantaged challengers. Alternatively, caps on the amount of effort can either increase or decrease the probability that the disadvantaged candidate wins. Ironically, with very tight caps, the advantaged candidate wins for sure.

In the 2006 election, the average House campaign spent $596,000 and the average Senate campaign spent $3.56 million. In the 2004 presidential contest, George Bush spent approximately $345 million to John Kerry's $310 million. In sum, over $1.9 billion was spent to win the Presidency, Senate and House in 2004. Over $1.3 billion was spent in the 2006 national midterm election.[2] This may be interpreted as evidence that a fair amount of money is spent on campaigns. Empirical scholarship forwards two additional claims. First, money matters in elections for both challengers and incumbents.[3] Second, the majority of advertisements do not involve precise policy statements.[4] Given these two conclusions, it seems likely that a large portion of the $1.3 billion spent in the 2006 election was not used to announce policy commitments. Nonetheless, the candidates and parties spending this money thought that it would influence the election outcome. Downsian models of competition cannot explain how candidates choose spending/campaigning levels or what factors influence these decisions.

The argument that competition over policies on a line cannot capture the heart of elections is not novel. Shortly after the publication of Downs' (1957) work on spatial/ideological competition, Stokes (1963) observed that the spatial metaphor misses certain critical pieces of the political landscape.

> It will not do simply to exclude valence issues from the discussion of party competition. The people's choice too often depends upon them. At least in American presidential elections of the past generation it is remarkable how many valence-issues have held the center of the stage.... The failure to distinguish these types of [valence] issues, whatever they are called, is one reason why journalistic accounts of political trends so often go astray. Apparently the urge to give an ideological, position-issue interpretation of election results can be irresistible, despite the reams of copy that have been devoted to Madison Avenue technique and the art of image-building. (p. 373)

In the more than forty years since Stokes' critique, scholars have extended and refined Downsian models of electoral competition, but this work has not sought to understand the equilibrium levels of "Madison Avenue techniques" or explain variation in this behavior.[5] This paper presents a theory of electoral competition in which candidates undertake costly campaign actions to persuade impressionable voters that they should win. While this work does not probe into the "art of image-building," it seeks to understand the properties of elections when a substantial portion of campaign activity is of this form. Although the model remains ambivalent about the exact form of effort, empirical referents are readily available. Many advertisements try to create the impression that a candidate is compassionate, competent, or honest. Moreover, advertisements, that defend a candidate's previous actions or associate a candidate with the good side of an issue are often meant to persuade the voters that the candidate is deserving–not to identify her with a particular policy platform to be enacted following the election. Support for the assumption that voters respond to candidate spending surfaces in recent empirical scholarship.[6] Instead of developing rich explanations of voter behavior, I start with a fairly sparse micro-level model of voter behavior. The analysis then explores how asymmetries affect elite behavior.[7]

Several types of asymmetries may be present. First, candidates may differ in their productivity. For example, one candidate may be able to more effectively generate and spend contributions. Second, voters may have biases in favor of one candidate. A partial list of the possible sources of these asymmetries includes reputations from previous elected office, incumbency, recent policy commitments, organizational ability, and the strength or popularity of one's party. Comparing equilibrium predictions with the fact that incumbents tend to spend more–and win more–leads to the conclusion that an equilibrium incumbency advantage cannot, simply, be the result of voter preferences for the incumbent. This type of asymmetry generates an equilibrium in which the advantaged candidate is likely to win, but spends less than the disadvantaged candidate. Instead, equilibria in which the advantaged candidate tends to outspend her challenger require an asymmetry in the marginal cost or benefit of effort.

The model also generates predictions about the consequences of campaign finance reform. Policy changes that make fund-raising more difficult lower the expected level of spending. They, however, increase the equilibrium odds that the incumbent wins. The analysis generates an intuition for this *incumbency protection* claim that is stronger and more subtle than that typically forwarded by pundits. Whereas the common argument notes that reforms hurt challengers who start the race without a war chest, the model demonstrates that an increase in the cost of effort serves to emphasize the relative technological advantage that incumbents have–even if they don't start the race with war chests. The disadvantaged candidate, typically, cannot benefit from these types of reforms. Policies that make fund-raising more difficult will lower the equilibrium effort levels. Since the marginal cost of effort is defined relative to the value of winning office, another interpretation of this comparative static can be given: the incumbency advantage should be inversely related to the value of winning office. Thus, variation in the advantage may be explained by variation in the desirability of holding office.[8] If campaign finance reform takes the form of spending caps, then the consequences are more ambiguous because of equilibrium multiplicity. While the presence of binding caps can lead to equilibria in which the incumbent is less likely to win, it can also lead to equilibria in which the incumbent is more likely to win. Equilibrium effort and voter payoffs, however, unambiguously fall as a result of spending caps. With very tight caps, the advantaged candidate wins for sure.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 develops the basic game and characterizes equilibria with various types of asymmetries. The analysis highlights conclusions about campaign finance reform and the nature of the incumbency advantage. Section 4 concludes with a discussion. Since these models have a different flavor than the more common Downisan models, some of the analysis appears in the body of the paper. The remaining proofs appear in the appendix.

Related Literature

A few recent papers focus on decisions about costly levels of effort or spending. Erikson and Palfrey (2000) consider a game in which two candidates choose spending levels. Payoffs depend on candidate specific cost functions and a function that relates spending levels to probabilities of victory. Erikson and Palfrey assume that spending choices are related to payoffs in a smooth manner, and they characterize pure strategy equilibria. In the current paper, voters are explicitly modeled. I do not introduce an exogenous probability of voting incorrectly or incomplete information on the part of voters. Pure strategy equilibria typically do not exist in the model; mixed strategy equilibria are characterized. Erikson and Palfrey and this paper illustrate two distinct modeling strategies. There are models with exogenous probabilistic voting and equilibria with deterministic candidate behavior, and there are models that do not assume probabilistic voting but that exhibit equilibria with randomization by candidates. Both types of models yield non-deterministic predictions about electoral outcomes. The former describe elections as forums in which the candidates face little uncertainty about how their opponent will behave and a lot of uncertainty about how voters will respond conditional on candidate behavior. The latter offer the opposite description. Both approaches capture a plausibly relevant form of uncertainty.[9]

Sahuguet and Persico (2006) consider a model in which candidates first decide how much to spend and then decide how to distribute promises across voters.[10] Unlike the current paper, which focuses on majority rule and candidates concerned with the probability of winning, they consider a proportional representation system and candidates who care about vote share.[11] For fixed spending decisions, the redistributive game faced by the parties is equivalent to a Colonel Blotto model, and Sahuguet and Persico take advantage of similarities between all pay auctions and Blotto games. They also investigate the consequences of campaign finance reform. The results are sometimes consistent and sometimes inconsistent with the policy conclusions in this paper. I return to a comparison later in the paper.

Several papers combine spending and policy selection. Wiseman (2006) considers elections in which parties *sequentially* select the platform of their candidate and a level of endorsement for their candidate. Equilibrium play can involve policy divergence, and the focus is on the extent to which the first mover can preempt or contain the second mover. In contemporaneous work, Herrera, Levine and Martinelli (2005), Ashworth and Bueno de Mesquita (2005), Zakharov (2005) and Meirowitz (2007) consider models in which candidates first make policy selections and then simultaneously select spending levels. In Herrera et al., turnout is assumed to respond to spending, and uncertainty about voters results in a smooth mapping from candidate actions to payoffs. Comparative statics yield interesting relationships between spending and polarization. Ashworth and Bueno de Mesquita, as well as Zakharov, also assume that the mapping from candidate actions to payoffs is smooth. Neither of these papers concentrates on the nature of the incumbency advantage or the consequences of campaign finance reform. Carrillo and Castanheira (2002) analyze a model in which candidates first select policies and then exert effort to influence their quality. They find that the median voter theorem holds when quality is either perfectly or never observed, but that it does not hold when quality is partially observed. Meirowitz (2007) extends the contests in the current paper to allow candidates to also commit to policy platforms. The median voter theorem is shown to fail even when quality/effort are perfectly observed.

The models in this paper (as well as the models in several of the above-cited papers) are closely related to all pay auctions. Baye et al. (1996) characterizes n-player all-pay auctions with perfect information and considers a form of asymmetry that is similar to one of the asymmetries considered here. In Baye et al., bidders are allowed to have distinct valuations of the prize. Asymmetric valuations turn out to be nearly equivalent to asymmetric costs. On the other hand, the literature does not seem to address the role of the second form of asymmetry considered here. Accordingly, the results of this paper could be reinterpreted as providing new conclusions about complete and perfect-information all-pay auctions when the auctioneer derives some utility from awarding the prize to a particular bidder. Results for the case of binding spending caps also seem new.

A model of effort choice in campaigns

Consider an election with n (odd) voters and two candidates, labeled 1 and 2. I use c to refer to a generic candidate and i to refer to a generic voter. Sometimes the notation $-c$ is used to refer to the candidate other than c. In period 1, each candidate $c \in \{1, 2\}$ simultaneously selects a level of effort, $a_c \in [0, \infty)$. The voters observe these decisions and, in period 2, cast ballots $v_i \in \{1, 2\}$. So $v_i = c$ denotes a vote for candidate c by voter i. I let c^* denote the identity of the candidate who receives a majority of votes and, thus, wins office. The parameter $\theta_i \in \{1, 2\}$ captures the identity of the candidate who voter i prefers, if the candidates select nearly identical levels of effort. Voter utility functions are of the form

$$u(c^*, a_{c^*}; \theta_i) = \begin{cases} \alpha + a_{c^*} \text{ if } c^* = \theta_i \\ a_{c^*} \text{ if } c^* \neq \theta_i \end{cases} . \tag{1}$$

Thus, θ_i is the voter's preferred party and $\alpha \geq 0$ is the value to having one's preferred party win the election. The higher the level of effort by candidate c, a_c, the happier a voter is to elect her. If $a_c - a_{-c} < \alpha$, a voter with $\theta_i = -c$ will prefer to have $-c$ in office. So a voter, i, with $\theta_i = 1$ has a preference for candidate 1, but if candidate 2's effort is sufficiently large relative to candidate 1's, then voter i prefers to elect candidate 2. Weakly undominated voting strategies involve $v_i = \theta_i$ if $\alpha + a_{\theta_i} > a_{-\theta_i}$ and $v_i \neq \theta_i$ if $\alpha + a_{\theta_i} < a_{-\theta_i}$. When a voter is indifferent, weak dominance allows her to vote for either candidate. In characterizing equilibria, below, the behavior of indifferent voters will be addressed. Without loss of generality, assume that a majority of voters have $\theta_i = 1$.[12] This assumption implies that candidate 1 is advantaged. Under weakly undominated voting strategies, if the difference between a_1 and a_2 is less than α, candidate 1 will win.

Candidate utility functions are of the form

$$u_c(a_c, c^*) = \begin{cases} 1 - \beta_c a_c \text{ if } c^* = c \\ -\beta_c a_c \text{ if } c^* \neq c \end{cases} . \quad (2)$$

The marginal cost of effort relative to the value of winning office is, thus, $\beta_c \geq 0$ for candidate c. Allowing for different costs, $\beta_1 \neq \beta_2$, captures a second form of asymmetry. The cost of a marginal change in one's attractiveness (as perceived by voters) may be higher for one candidate than for the other. Since the value of winning is standardized at 1, the ratio $\frac{1}{\beta_c}$ may be interpreted as the value of winning relative to the cost of a unit of effort. Thus, the specification allows us to interpret differences in the cost as differences in the valuations to victory. I assume that $\beta_1 \leq \beta_2$. The case where voters prefer candidate 2 and $\beta_1 < \beta_2$ is considered in the section on cross-cutting asymmetries.

A mixed strategy for candidate c is a lottery over effort levels. Such a lottery is denoted by a distribution function $F_c(\cdot)$, where $F_c(x)$ denotes the probability that $a_c \leq x$. For a given mixed strategy for candidate c, the smallest closed set of effort levels which occurs with probability one is called the support of the strategy.[13]

It is assumed that the game form (i.e., sequence of moves and payoffs) is common knowledge. The appropriate equilibrium concept is subgame perfection with weakly undominated voting strategies.[14] In an equilibrium, candidates select effort levels (a_1, a_2) that are simultaneous best responses when they anticipate that voters will vote for the candidate they prefer, and voters vote for the candidate they prefer given the effort levels. Strictly speaking, equilibria will not be unique because the way that voters resolve indifference is not pinned down by weak dominance, and many voting rules for resolving indifference (at events that occur with probability 0) can support the same equilibrium mixed strategies. This form of multiplicity is uninteresting, so I use the term *unique equilibrium* to describe the case where all equilibria involve the same candidate strategies. In the extension involving spending caps more-interesting forms of multiplicity are present.

Before turning to the analysis, a comment on how spending and effort are distinguished is in order. If candidates differ in how effectively they can generate funds, but not in how efficiently they can use money to affect voter perceptions, β_c should be viewed as a measure of fund-raising effectiveness (lower β_c corresponds to more-effective fund-raising). The quantity $\beta_c a_c$ should be interpreted as the spending level. If candidates differ primarily in how efficiently they can use money to affect perceptions, then β_c should be interpreted as a measure of campaigning effectiveness (lower β_c corresponds to more-effective marketing), and the quantity a_c should be interpreted as the spending level. If both asymmetries are present, β_c captures the aggregate technology. For example, if b_c is the marginal cost, in terms of effort of collecting a dollar of contributions, and B_c is the marginal cost, in terms of dollars of generating a unit of perceived competence (in the same scale as α_i), the marginal cost, in terms of effort of attaining a unit of perceived competence is $b_c B_c$. In this case, $\beta_c = b_c B_c$. For expositional convenience, I use the terms spending and effort choice to describe a_c and simply keep track of the term β_c.

Characterizing the equilibria

It is helpful to first determine what types of effort strategies are strictly dominated. Since the value of winning is normalized to 1, sufficiently high values of a_c cannot be rationalized. For example, regardless of the choice by candidate $-c$, effort of $a_c' = 0$ is preferable to any effort level, a_c, with $\beta_c a_c > 1$. This means that the most candidate c will accumulate is $\frac{1}{\beta_c}$. Now, since $\beta_1 \leq \beta_2$, the fact that candidate 2 will select an effort level of no more than $\frac{1}{\beta_2}$ means that an effort level more than $\frac{1}{\beta_2} - \alpha$ is wasteful for candidate 1 even though $\frac{\beta_1}{\beta_2} < 1$.

Combining these arguments, in the manner of iterative elimination of strictly dominated strategies, yields the conclusion that an effort level of $a_1 > \frac{1}{\beta_2} - \alpha$ is strictly dominated by $a_1' \in (\frac{1}{\beta_2} - \alpha, a_1)$.

A second observation is that an effort level of $a_2 \in (0, \alpha)$ will result in a loss for candidate 2 regardless of a_1. Thus, $a_2 \in (0, \alpha)$ is strictly dominated by an effort level of $a_2' = 0$. A well known result is that there are no Nash equilibria in which players use strategies that are ruled out by iterative elimination of strictly dominated strategies. This fact leads to the following lemma.

Lemma 1 *In any equilibrium (involving mixed or pure strategies),* a_1 *is in* $A_1 := [0, \frac{1}{\beta_2} - \alpha]$ *with probability one and* a_2 *is in* $A_2 := \{0\} \cup [\alpha, \frac{1}{\beta_2}]$ *with probability one.*

Since effort by 2 is valuable only if $a_2 - a_1 \geq \alpha$, the cost of winning to candidate 2 is at least $\alpha\beta_2$ (this is the minimal cost of winning if a_1 happens to be 0). Consider the case where $\alpha\beta_2 > 1$ (which is the same as $\beta_2 > \frac{1}{\alpha}$). In this case, the disadvantaged candidate, 2, is better off selecting 0 than any positive amount of effort regardless of candidate 1's effort level. Given this, the advantaged candidate, 1, will win regardless of her effort level and, thus, is best off selecting $a_1 = 0$. This brief argument establishes the following result. If $\beta_2 > \frac{1}{\alpha}$, then the unique equilibrium involves $a_1 = a_2 = 0$. In the remainder of this paper, I focus on the interesting cases in which one candidate does not have such a large advantage as to make the contest trivial.

Assumption: $\alpha < \frac{1}{\beta_2}$.

If this assumption is satisfied, pure strategy equilibria do not exist. This conclusion can be seen by considering a small number of profiles. At any profile in which both candidates select a positive amount and one wins for sure, the losing candidate would gain by a deviation to 0. At the profile $a_1 = 0, a_2 = \alpha$ if 1 wins with probability less than 1, a deviation to $a_1' = \varepsilon$ (where ε is a small but positive number) is desirable, and if 1 wins with probability

1 following this profile, a deviation to $a_2' = \alpha + \varepsilon$ is desirable. At any profile in which both candidates select a positive amount and both win with probability less than 1, a deviation by c to a slightly higher effort level will result in victory and be desirable unless the tying effort levels are $a_1 = \frac{1}{\beta_2} - \alpha$ and $a_2 = \frac{1}{\beta_2}$. But this profile cannot be an equilibrium because for candidate 2 the probability of winning is less than the cost of her effort, $\beta_2 \frac{1}{\beta_2} = 1$. So, in all of these profiles, at least one candidate has an incentive to deviate. The following result is proven in the appendix.

Proposition 1 *If $\beta_2 < \frac{1}{\alpha}$, there is no equilibrium in which candidates use pure strategies.*

In the appendix, I state and prove two lemmas (2.1 and 2.2). These results establish that: (1) in any mixed strategy equilibrium, the support of each candidate's mixed strategy is essentially the set of points that survive deletion of strictly dominated strategies (A_1 for candidate 1 and either A_2 or $[\alpha, \frac{1}{\beta_2}]$ for candidate 2); and (2) in any equilibrium, only the boundary points (0 and $\frac{1}{\beta_2} - \alpha$ for candidate 1 and $0, \alpha$ and $\frac{1}{\beta_2}$ for candidate 2) of the supports can be chosen with strictly positive probability. The proof is somewhat technical, but much of the intuition is provided by noting that if there is a point, a, which is not on the boundary of candidate c's support but which is chosen with positive probability, then the other candidate, $-c$, must not be playing points that are close to, but just below, the point that ties a. Given this, playing a is not optimal for c. This contradicts the assertion that a is in the support of c's mixed strategy in an equilibrium. The content of Lemmas 2.1 and 2.2 is restated here.

Lemma 2 *Assume that $\beta_2 < \frac{1}{\alpha}$. In any mixed strategy equilibrium, candidate 1's mixed strategy has support $[0, \frac{1}{\beta_2} - \alpha]$, and candidate 2's mixed strategy has support $\{0\} \cup [\alpha, \frac{1}{\beta_2}]$ or $[\alpha, \frac{1}{\beta_2}]$. In any equilibrium, candidate 1 does not assign strictly positive probability to points other than 0 and $\frac{1}{\beta_2} - \alpha$, and candidate 2 does not assign strictly positive probability to points other than 0, α and $\frac{1}{\beta_2}$.*

In order to characterize the equilibrium, I begin with the symmetric case. Assume that $\alpha = 0$ and $\beta_1 = \beta_2 = \beta$. Proposition 1 states that in any equilibrium at least one candidate uses a non-degenerate mixed strategy, and Lemma 2 states that in any equilibrium both candidates randomize on the support $[0, \frac{1}{\beta}]$. I first characterize an equilibrium in which at least one candidate mixes with a distribution function that is strictly increasing and continuous on the support $[0, \frac{1}{\beta}]$ (i.e., effort is drawn from a distribution function that is atomless and has no holes on the set $[0, \frac{1}{\beta}]$). I then show that no other type of equilibrium can exist. If candidate 2 randomizes in her selection of a_2 according to the strictly increasing and continuous cumulative distribution $F_2(\cdot)$ with support $[0, \frac{1}{\beta}]$, then candidate 1's expected utility is $Eu_1(a_1) = F_2(a_1) - \beta a_1$. Since selection of $a_1 = 0$ in response to candidate 2's mixed strategy results in victory for 2 with probability 1, candidate 1 receives a utility of 0 from playing $a_1 = 0$. In order for candidate 1 to be willing to randomize over the support $[0, \frac{1}{\beta}]$, the equality

$$F_2(a_1) - \beta a_1 = 0 \tag{3}$$

must be satisfied for every $a_1 \in [0, \frac{1}{\beta}]$. This means that $F_2(\cdot)$ has a constant slope and, thus, a_2 is drawn from a uniform distribution. Repeating the argument with the indices interchanged demonstrates that in any mixed strategy equilibrium, in which at least one player mixes with a strictly increasing and continuous distribution on $[0, \frac{1}{\beta}]$, both players' strategies are characterized by the uniform distribution on $[0, \frac{1}{\beta}]$,

$$F_c(a_c) = \begin{cases} 0 \text{ if } a_c < 0 \\ \beta a_c \text{ if } a_c \in [0, \frac{1}{\beta}] \\ 1 \text{ if } a_c > \frac{1}{\beta}. \end{cases} \tag{4}$$

Given Lemma 2, any other equilibria must involve at least one player assigning strictly positive probability to a point in $\{0, \frac{1}{\beta}\}$. But this is not possible since any mixture satisfying (3) and having support $[0, \frac{1}{\beta}]$, must place probability 0 on the endpoints in order to integrate to 1 on the support. Thus, the following result obtains.

Proposition 2 *Consider the symmetric case ($\alpha = 0$, $\beta_1 = \beta_2 = \beta$). In the unique equilibrium, both candidates mix according to the uniform distribution on* $[0, \frac{1}{\beta}]$.

In equilibrium, both candidates win with the same probability and have expected utility of 0. One interpretation of this last finding is that competition bids away any expected rents from holding office. I now relax the symmetry assumptions. Consider a model involving $\alpha \geq 0$ and $\beta_1 \leq \beta_2$. Lemma 2 tells us that in any equilibrium, candidate 1's mixed strategy has support $[0, \frac{1}{\beta_2} - \alpha]$, and candidate 2's mixed strategy has support $\{0\} \cup [\alpha, \frac{1}{\beta_2}]$ or $[\alpha, \frac{1}{\beta_2}]$. It turns out that these asymmetric games also possess unique equilibria. The following proposition is proven in the appendix.

Proposition 3 *Assume that $\frac{1}{\beta_2} \geq \alpha \geq 0$ and effort is less costly for candidate 1 ($\beta_2 \geq \beta_1$). The following is true in the unique equilibrium: (1) Candidate mixed strategies are*

$$F_1(a_1) = \begin{cases} 0 \textit{ if } a_1 < 0 \\ \beta_2(a_1 + \alpha) \textit{ if } a_1 \in [0, \frac{1}{\beta_2} - \alpha] \\ 1 \textit{ if } a_1 \geq \frac{1}{\beta_2} - \alpha, \end{cases} \tag{5}$$

$$F_2(a_2) = \begin{cases} 0 \textit{ if } a_2 < 0 \\ \beta_1\alpha + 1 - \frac{\beta_1}{\beta_2} \textit{ if } a_2 \in [0, \alpha) \\ \beta_1 a_2 + 1 - \frac{\beta_1}{\beta_2} \textit{ if } a_2 \in (\alpha, \frac{1}{\beta_2}) \\ 1 \textit{ if } a_2 \geq \frac{1}{\beta_2}. \end{cases} \tag{6}$$

(2) The expected efforts are $Ea_1 = \frac{1}{2\beta_2} - \alpha + \frac{\beta_2\alpha^2}{2}$ and $Ea_2 = \frac{\beta_1}{2\beta_2^2} - \frac{\beta_1\alpha^2}{2}$. (3) Candidate 2's expected utility is $Eu_2 = 0$, and candidate 1's expected utility is $Eu_1 = 1 - \frac{\beta_1}{\beta_2} + \beta_1\alpha$. (4) The

probability that candidate 1 wins is $p_1 = 1 - \frac{\beta_1}{2\beta_2} + \frac{\beta_1\beta_2\alpha^2}{2}$.

Point 2 illustrates that the advantaged candidate's expected effort level can be either higher or lower than the disadvantaged candidate's expected effort level. Point 3 of the proposition, however, indicates that the advantaged candidate unambiguously does better since

$$Eu_1 = 1 - \frac{\beta_1}{\beta_2} + \beta_1\alpha = \frac{\beta_2 - \beta_1}{\beta_2} + \beta_1\alpha \tag{7}$$

and $\beta_2 \geq \beta_1$ implies that $Eu_1 \geq 0 = Eu_2$. In fact, either form of asymmetry is sufficient for the advantaged candidate to have a strictly higher expected utility. Additionally, point 4 demonstrates that the advantaged candidate is more likely to win, as

$$p_1 = 1 - \frac{\beta_1}{2\beta_2} + \frac{\beta_1\beta_2\alpha^2}{2} \geq \frac{2\beta_2 - \beta_2}{2\beta_2} = \frac{1}{2}. \tag{8}$$

In fact, if either form of asymmetry is present, $p_1 > \frac{1}{2}$.

I consider only the case of linear costs to keep the number of parameters low. A model with increasing marginal costs has equilibria with similar features. If the costs were an increasing differentiable and strictly convex function, $\zeta(a_c)$ with $\zeta(0) = 0$, then dominance arguments would ensure that candidates never spend more than $\zeta^{-1}(1)$ and, thus, an equilibrium involves mixing on the set $[0, \zeta^{-1}(1)]$. In particular, the equilibrium to the symmetric game with cost $\zeta(\cdot)$ involves candidates using the distribution function $F_c(a_c) = \zeta(a_c)$ on support $[0, \zeta^{-1}(1)]$. Analysis of assymetric models with increasing marginal costs is beyond the scope of this paper.

The following sections provide insights about the relationship between asymmetries and equilibrium behavior by extracting corollaries of Proposition 3. The approach is to consider only one form of asymmetry at a time.

Asymmetric voter preferences

To model the asymmetric case where a majority of voters have a preference for candidate 1, but the costs are identical, assume that $\beta_2 = \beta_1 = \beta$ and $\alpha \in (0, \frac{1}{\beta})$. Recall that with $\alpha > \frac{1}{\beta}$, there is a unique pure strategy equilibrium ($a_1 = a_2 = 0$). To see why the equilibrium in Proposition 3 is correct in this case, consider the relevant indifference conditions. In order for candidate 1 to be indifferent between levels of a_1 in $(0, \frac{1}{\beta} - \alpha)$, it must be the case that for any distinct points $a_1', a_1'' \in (0, \frac{1}{\beta} - \alpha)$,

$$F_2(a_1' + \alpha) - \beta a_1' = F_2(a_1'' + \alpha) - \beta a_1''. \tag{9}$$

This implies that $F_2(\cdot)$ has a constant density for values in $(\alpha, \frac{1}{\beta})$. Similarly, in order for candidate 2 to be indifferent between different levels of a_2 in $(\alpha, \frac{1}{\beta})$, $F_1(\cdot)$ needs to have a constant density for values in $(0, \frac{1}{\beta} - \alpha)$. The remaining question is whether these distributions have atoms on their boundaries. In the unique equilibrium, it turns out, the disadvantaged candidate assigns strictly positive probability only to $a_2 = 0$, and the advantaged candidate assigns strictly positive probability only to $a_1 = 0$.

[Insert Figure 1 here]

Figure 1 depicts the distribution functions for the equilibrium. In order for candidate 2 to be willing to play $a_2 = \alpha$, she must expect to win with positive probability when she selects this effort level; otherwise, she would have a payoff of $-\beta\alpha < 0$ and prefer to just select $a_2' = 0$. This means that candidate 1 must select $a_1 = 0$ with positive probability. This is the only level of $a_1$that does not beat $a_2 = \alpha$. On the other hand, the advantaged candidate must be indifferent between playing $a_1 = 0$ and any other choice in $[0, \frac{1}{\beta} - \alpha]$. This requires that she win with positive probability when $a_1 = 0$. Since the only point in candidate 2's support that loses to $a_1 = 0$ is $a_2 = 0$, it must be the case that $a_2 = 0$ with positive probability.

Interestingly, in this equilibrium, candidate 2's effort first-order stochastically dominates candidate 1's. That is, the disadvantaged candidate tends to exert more effort than the advantaged candidate. With only this form of asymmetry, the model predicts that the advantaged candidate spends less than the disadvantaged candidate. (This is true if we let either a_c or $\beta_c a_c$ denote candidate c's campaign spending.) A second conclusion can be drawn by analysis of the probability that candidate 1 wins, p_1. Differentiation yields, $\frac{\partial p_1}{\partial \beta} = \alpha^2 \beta$ and $\frac{\partial p_1}{\partial \alpha} = \beta^2 \alpha$, which are both positive; the probability that the advantaged candidate wins is increasing in the cost and the preference advantage. Examining the equilibrium utilities leads to the conclusion that the expected utility of the disadvantaged candidate is unaffected by small changes in the cost of effort, β, and the preference asymmetry, α. The advantaged candidate's expected utility is increasing in her advantage, α, and the cost, β.

This result offers a novel perspective on the incumbency advantage. A candidate that is advantaged by voter preferences will tend to campaign less aggressively. While this conclusion is at odds with the established literature on the incumbency advantage, it is inappropriate to dismiss the model as uninteresting. Combining this analysis and the fact that incumbents tend to win and outspend their challengers leads to the following conclusion: The incumbency advantage cannot be explained by the simple claim that candidates that have already won an election tend to be more attractive to voters. If incumbents were simply more attractive than challengers, spending data would look different.

The model does offer a sharp prediction about the consequences of policies that increase the cost of fund-raising. Such policies have no effect on the disadvantaged candidate's payoffs, but they actually increase the payoffs of the advantaged candidate. By making effort more costly, such policies increase the importance of the preference advantage. Moreover, the larger α is, the larger the gain to candidate 1 from a slight increase in β is. More striking is the finding that policies that make effort more costly will increase (not decrease) the likelihood that the advantaged incumbent wins. As it becomes more challenging to generate contributions, the relationship between a preference advantage and an electoral advantage becomes more direct.

Asymmetric costs

Again, assume that $\alpha = 0$. Now consider the case where effort is costlier for candidate 2, $\beta_2 > \beta_1$. The unique equilibrium involves candidate 1 selecting her effort from a uniform distribution on $[0, \frac{1}{\beta_2}]$ and candidate 2 placing probability $1 - \frac{\beta_1}{\beta_2}$ on $a_2 = 0$; with probability $\frac{\beta_1}{\beta_2}$, she selects a_2 from a uniform distribution on $[0, \frac{1}{\beta_2}]$.

[Figure 2 here]

Figure 2 depicts the equilibrium lotteries. The logic behind this finding can be seen by noting that the most that candidate 1 is going to select is $\frac{1}{\beta_2}$. Since the cost of this level of effort is $\frac{\beta_1}{\beta_2} < 1$, candidate 1 will realize a positive expected utility (in equilibrium). This means that the expected utility from any choice $a_1 \in [0, \frac{1}{\beta_1}]$ must be positive. In order for this to be true when she accumulates $a_1 = 0$, she must win with positive probability. This requires that candidate 2 select $a_2 = 0$ with strictly positive probability. Here, the equilibrium lottery over effort levels by the advantaged candidate first-order stochastically dominates the equilibrium lottery over effort levels by the disadvantaged candidate. This conclusion reverses the finding from the preference advantage case. Under the interpretation that a_c is the spending by candidate c, the model predicts that advantaged candidates tend to outspend disadvantaged candidates.

Recall that the cost β_c might be interpreted as capturing both the cost of translating effort into contributions, b_c, and the cost of translating contributions into voter support, B_c, through the identity $\beta_c = b_c B_c$. If incumbency results in asymmetries in the first component ($b_1 \neq b_2$), then it is best to interpret $b_c a_c$ as the measure of spending. In this case, I can express the difference in expected spending levels as

$$b_1 E a_1 - b_2 E a_2 = \frac{b_1}{2 b_2 B_2}(1 - \frac{B_1}{B_2}). \tag{10}$$

This quantity is positive as long as $B_1 < B_2$ (i.e., the advantaged candidate can more efficiently translate contributions into support). The probability that the advantaged candidate wins exceeds $\frac{1}{2}$ and varies with changes in the costs. Differentiation yields $\frac{\partial p_1}{\partial \beta_1} = -\frac{1}{2\beta_2} < 0$ and $\frac{\partial p_1}{\partial \beta_2} = \frac{\beta_1}{2\beta_2^2} > 0$. In addition, the expected utility to the disadvantaged candidate is unaffected by small changes in the costs of effort; the expected utility of the advantaged candidate is increasing in her cost advantage ($\frac{\beta_2}{\beta_1}$).

In 2006, the average incumbent Senator up for reelection spent \$10,634,000, while the average challenger spent \$1,615,000. The average open-seat campaign spent \$2,722,000. House elections exhibit a similar pattern: \$1,155,000 for incumbents, \$244,000 for challengers and \$514, for campaigns for open seats.[15] In light of this pattern, the more plausible model involves asymmetric costs. Influencing voter perceptions is easier (has a lower cost) for incumbents than for challengers.

Cross-cutting asymmetries

Thus far, I have assumed that the same candidate has a cost and preference advantage ($\beta_1 \leq \beta_2, \alpha \geq 0$ respectively). In some settings, however, one candidate has a cost advantage, while the other has a preference advantage. One such example might involve partisan realignments or electoral tides that swing against the incumbent party. The incumbent might possess a cost advantage and a preference disadvantage. In this section, I assume that $\beta_1 < \beta_2$ and that the preference advantage is for candidate 2. A majority of the voters prefer candidate 2 to candidate 1 unless, $a_1 - \alpha \geq a_2$. So, in order for candidate 1 to be strictly preferred to candidate 2 (by a majority), she must accumulate $a_1 > a_2 + \alpha$. As before, this game is trivial if the preference advantage is too large. In particular, if $\alpha > \frac{1}{\beta_1}$, candidate 1 is not willing to exert enough effort to overcome her preference disadvantage; in this case, the equilibrium involves $a_1 = a_2 = 0$. For the remainder of this section I assume that $\beta_1 < \frac{1}{\alpha}$.

The analysis proceeds as above, first determining what strategies are dominated. Since victory results in a payoff of 1, the greatest amount of effort that candidate 2 is willing to exert is $a_2 = \frac{1}{\beta_2}$. Given this, any effort a_1' by candidate 1 that is greater than an effort of $a_1 = \frac{1}{\beta_2} + \alpha$ is dominated by an effort level that is between a_1 and a_1'. Moreover, since (i) candidate 1 loses to candidate 2 if $a_1 < \alpha$, regardless of a_2, and (ii) the effort $a_1 > 0$ has cost $\beta_1 a_1$ to candidate 1, any effort $a_1' \in (0, \alpha)$ is dominated by the effort $a_1 = 0$.

If $\frac{1}{\beta_1} \geq \frac{1}{\beta_2} + \alpha$, this argument characterizes the strategies that survive iterative deletion of strictly dominated strategies. If, however, $\frac{1}{\beta_1} < \frac{1}{\beta_2} + \alpha$, then $a_1 \in (\frac{1}{\beta_1}, \frac{1}{\beta_2} + \alpha)$. This results in a negative payoff for 1. In this case, the set of effort levels for candidate 1 that survive iterative deletion of dominated strategies is $\{0\} \cup [\alpha, \frac{1}{\beta_1}]$. Given this, a strategy $a_2 \in (\frac{1}{\beta_1} - \alpha, \frac{1}{\beta_2})$ is dominated by a strategy a_2' that is slightly smaller. The strategies for candidate 2 that survive are $[0, \frac{1}{\beta_1} - \alpha]$. The following analogue to Lemma 2 characterizes the supports.

Lemma 3 *Assume that the asymmetries are cross-cutting ($\beta_1 < \beta_2$ and 1's advantage is $-\alpha < 0$). If $\frac{1}{\beta_1} - \frac{1}{\beta_2} \geq \alpha$, then the strategy sets that survive iterative deletion are $\{0\} \cup [\alpha, \frac{1}{\beta_2} + \alpha]$ for candidate 1 and $[0, \frac{1}{\beta_2}]$ for candidate 2. If $\frac{1}{\beta_1} - \frac{1}{\beta_2} < \alpha$, then the strategy sets that survive iterative deletion are $\{0\} \cup [\alpha, \frac{1}{\beta_1}]$ for candidate 1 and $[0, \frac{1}{\beta_1} - \alpha]$ for candidate 2.*

The following proposition characterizes the unique equilibrium when the asymmetries are cross-cutting. The proof appears in the appendix.

Proposition 4 *Assume that the asymmetries are cross-cutting. (a) If* $\alpha < \frac{1}{\beta_1} - \frac{1}{\beta_2}$*, the unique equilibrium has the following properties: (1) The equilibrium mixtures are given by*

$$F_1(a_1) = \begin{cases} 0 \text{ if } a_1 < \alpha \\ \beta_2(a_1 - \alpha) \text{ if } a_1 \in [\alpha, \frac{1}{\beta_2} + \alpha) \\ 1 \text{ if } a_1 \geq \frac{1}{\beta_2} + \alpha, \end{cases} \tag{11}$$

$$F_2(a_2) = \begin{cases} 0 \text{ if } a_2 < 0 \\ 1 - \frac{\beta_1}{\beta_2} + \beta_1 a_2 \text{ if } a_2 \in [0, \frac{1}{\beta_2}] \\ 1 \text{ if } a_2 \geq \frac{1}{\beta_2}. \end{cases} \tag{12}$$

(2) The expected efforts are $Ea_1 = \alpha + \frac{1}{2\beta_2}$ *and* $Ea_2 = \frac{\beta_1}{2\beta_2^2}$*. (3) Candidate 2's expected utility is* $Eu_2 = 0$*, and candidate 1's expected utility is* $Eu_1 = 1 - \frac{\beta_1}{\beta_2} - \beta_1\alpha$*. (4) The probability that candidate 1 wins is* $p_1 = 1 - \frac{\beta_1}{2\beta_2} \geq \frac{1}{2}$*.*

(b) If $\alpha \in (\frac{1}{\beta_1} - \frac{1}{\beta_2}, \frac{1}{\beta_1})$*, the unique equilibrium has the following properties: (1) The equilibrium mixtures are given by*

$$F_1(a_1) = \begin{cases} 0 \text{ if } a_1 < 0 \\ 1 - \beta_2(\frac{1}{\beta_1} - \alpha) \text{ if } a_1 \in [0, \alpha) \\ 1 - \beta_2(\frac{1}{\beta_1} - \alpha) + \beta_2(a_1 - \alpha) \text{ if } a_1 \in [\alpha, \frac{1}{\beta_1}) \\ 1 \text{ if } a_1 \geq \frac{1}{\beta_1}, \end{cases} . \tag{13}$$

$$F_2(a_2) = \begin{cases} 0 \text{ if } a_2 < 0 \\ \beta_1(a_2 + \alpha) \text{ if } a_2 \in [0, \frac{1}{\beta_1} - \alpha] \\ 1 \text{ if } a_2 \geq \frac{1}{\beta_1} - \alpha. \end{cases} \tag{14}$$

(2) The expected efforts are $Ea_1 = \alpha(1 - \frac{\beta_1}{\beta_2}) + \frac{\beta_2\alpha^2}{2} + \frac{\beta_2}{2\beta_1^2}$ *and* $Ea_2 = \beta_1(\frac{1}{\beta_1} - \alpha)\left(\frac{1}{2\beta_1} - \frac{\alpha}{2}\right)$*. (3) Candidate 2's expected utility is* $Eu_2 = 1 - \beta_2(\frac{1}{\beta_1} - \alpha)$*, and candidate 1's expected utility*

is $Eu_1 = 0$. *(4) The probability that candidate 1 wins is* $p_1 = \frac{\beta_2}{2\beta_1} - \frac{\beta_2\beta_1}{2}\alpha^2$.

Propositions 3 and 4 yield the following assessment of the relationship between asymmetries and candidate payoffs. Candidate 1 (lower cost) obtains a positive payoff (while candidate 2 obtains 0 payoff) as long as candidate 2 does not have too large a preference advantage. If 1 has a preference advantage or 2 has a small preference advantage, candidate 1 obtains a higher payoff. However, if 2's preference advantage is large enough, this asymmetry swamps candidate 1's cost advantage, and candidate 1 obtains a payoff of 0, while candidate 2 obtains a positive payoff. The probability that candidate 1 wins, p_1, is equal to $1 - \frac{\beta_1}{2\beta_2} \geq \frac{1}{2}$, when the preference advantage is small. When the preference advantage for candidate 2 is large, $p_1 = \frac{\beta_2}{2\beta_1} - \frac{\beta_2\beta_1}{2}\alpha^2$. As α varies from $\frac{1}{\beta_1} - \frac{1}{\beta_2}$ up to $\frac{1}{\beta_1}$, the value p_1 decreases from $1 - \frac{\beta_1}{2\beta_2}$ and approaches 0 as $\alpha \to \frac{1}{\beta_1}$. Note that at $\alpha = \frac{1}{\beta_1}$ the equilibrium involves $a_1 = a_2 = 0$.

Campaign finance

While the model is fairly sparse, it does possess enough moving parts to facilitate a few policy experiments. In the remainder of this section, I investigate how policy shocks affect the equilibria. Following the conclusion that cost asymmetries are needed to yield predictions consistent with the observed incumbency advantage, the analysis focuses on the case of $\beta_1 < \beta_2$ and $\alpha = 0$.

Policies that change costs

How do policies that alter the costs affect behavior and payoffs? Motivating examples include a campaign finance reform that alters the tax deductibility of contributions, or regulations that limit the amount any individual (or firm) can contribute. In the first case, the cost of transferring a dollar to a campaign is raised (and so a fixed level of effort is likely to yield fewer contributions), and in the second case, more contributors must be contacted to generate a fixed amount in contributions. In both of these cases, the amount of effort required to generate contributions increases with the magnitude of the policy intervention.

First, consider a policy that affects both candidates symmetrically, by changing each cost by the common multiple $k > 0$. The expected effort levels, probabilities of victory and payoffs can then be treated as functions of k. By inspection, $Ea_1(k) = \frac{1}{2k\beta_2}$ and $Ea_2(k) = \frac{k\beta_1}{2k^2\beta_2^2}$ are both decreasing in k.[16] The probability that the advantaged candidate wins, $p_1(k) = 1 - \frac{k\beta_1}{2k\beta_2}$, is unaffected by changes in k. Moreover, the expected utilities of the candidates are unaffected by changes in k. Since this change does not affect the likelihood that 1 wins, but it does reduce the expected effort of the winning candidate, voter utility unambiguously falls.[17]

It is also possible for a policy change to affect the candidates differentially. Suppose that candidate 1 is from a party that typically raises most of its funds from corporate contributions, while candidate 2's party typically raises most of its funds from private donors. In this case, a reform that limits individual contributions without limiting corporate contributions would increase both candidates' cost terms but it would raise candidate 2's cost more. While the comparative statics on expected effort levels and the probability that the advantaged candidate wins are ambiguous (and, thus, the effect on the payoffs are ambiguous), the effects on candidate payoffs are unambiguous. The payoff to the disadvantaged candidate is unaffected, while the payoff to the advantaged candidate, $Eu_1 = 1 - \frac{\beta_1}{\beta_2}$, and her probability of victory, $p_1 = 1 - \frac{\beta_1}{2\beta_2}$, rise (fall) if candidate 2's cost increases more (less) than candidate 1's.

This result suggests that both advantaged and disadvantaged candidates may not oppose regulatory policies that make fund-raising and spending difficult. Moreover, small changes that disproportionately affect the candidates can effect only the payoffs of the advantaged candidate. Finally, if the goal of campaign finance reform is to weaken the incumbency advantage, success requires that the advantaged candidate's marginal cost is increased more than the disadvantaged candidate's.

Spending caps

A second possible type of campaign finance reform involves spending caps. To model this policy intervention, I consider a cap $k < \frac{1}{\beta_2}$ and require that candidates select a_1 and a_2 in $[0, k]$. If k is small enough, this game has pure strategy equilibria. In particular, if there exists some value $z \in (0, 1)$ such that $z - \beta_1 k \geq 0$ and $(1 - z) - \beta_2 k \geq 0$, then $a_1 = a_2 = k$ and voters resolving the tie in favor of candidate 1 with probability z is an equilibrium. Given this likelihood of winning at $a_i = k$, each candidate prefers this effort level to any other feasible effort level, which results in victory with probability 0. Such a z exists as long as $k \leq \frac{1}{\beta_1+\beta_2}$. When this inequality holds strictly, there are a continuum of equilibrium probabilities that 1 wins, and a continuum of equilibrium payoffs.

Proposition 5 *Assume that candidates are constrained to select $a_c \leq k$ with $k \leq \frac{1}{\beta_1+\beta_2}$ and that $\beta_1 < \beta_2$ and $\alpha = 0$. There exist pure strategy equilibrium with $a_1 = a_2 = k$.*

I now focus on the case where the constraint is not so tight, $k \in (\frac{1}{\beta_1+\beta_2}, \frac{1}{\beta_2})$. If candidate 1 is playing both a_1 and a_1' (both in $(0, k)$) in an equilibrium, then it must be the case that $F_2(a_1) - \beta_1 a_1 = F_2(a_1') - \beta_1 a_1'$; if this equality did not hold, then candidate 1 would not be indifferent between a_1 and a_1'. This means that if $(0, k)$ is in the support of candidate 2's mixed strategy, $F_2(a_2)$ must have density β_1 on $(0, k)$. A fairly straighforward application of the arguments used to prove lemmas 2.1 and 2.2 rules out holes and atoms in the interior of $[0, k]$. Thus, $(0, k)$ is in the support of candidate 2's mixed strategy, and we find that candidate 2 assigns probability $k\beta_1 < 1$ to the interval $(0, 1)$. Candidate 2, thus, assigns strictly positive probability to $0, k$ or both. Similarly, in order for candidate 2 to be willing to play the strategies in the interior of her support, $F_1(a_1)$ must have density β_2 on $(0, k)$; candidate

1 must assign strictly positive probability to 0,k or both. Given these conclusions, characterizing the equilibria requires determining which combinations of allocations of probability to 0 and k can occur in a pair of mutual best responses. In the appendix, I state and prove Lemma 4, which establishes that in any equilibrium to this game, two candidates *cannot* assign strictly positive probability to the same endpoint. Thus, in equilibrium, one candidate must put positive probability only on k and the other must put positive probability only on 0. There are, thus, two possible strategy profiles. One has the advantaged candidate assign probability $1-\beta_2 k$ to $a_1 = k$ and place density β_2 on $[0,k)$, while the disadvantaged candidate assigns probability $1-\beta_1 k$ to $a_2 = 0$ and places density β_1 on $(0,k]$. In the second strategy profile, the advantaged candidate assigns probability $1-\beta_2 k$ to $a_1 = 0$ and places density β_2 on $(0,k]$, while the disadvantaged candidate assigns probability $1-\beta_1 k$ to $a_2 = k$ and places density β_1 on $[0,k)$. Both of these strategy profiles are, in fact, equilibria.

Proposition 6 *Assume that candidates are constrained to select* $a_c \leq k$ *with* $k \in (\frac{1}{\beta_1+\beta_2}, \frac{1}{\beta_2})$ *and that* $\beta_1 < \beta_2$ *and* $\alpha = 0$. *(i) There are two equilibria:*

$$F_1^1(a_1) = \begin{cases} 0 \text{ } \textit{if } a_1 < 0 \\ \beta_2 a_1 \text{ } \textit{if } a_1 \in [0,k) \\ 1 \text{ } \textit{if } a_1 \geq k, \end{cases} \tag{15}$$

$$F_2^1(a_2) = \begin{cases} 0 \text{ } \textit{if } a_2 < 0 \\ 1-\beta_1 k \text{ } \textit{if } a_2 = 0 \\ 1-\beta_1 k + \beta_1 a_2 \text{ } \textit{if } a_2 \in (0,k] \\ 1 \text{ } \textit{if } a_2 \geq k \end{cases} \tag{16}$$

and

$$F_1^2(a_1) = \begin{cases} 0 \ \textit{if} \ a_1 < 0 \\ 1 - \beta_2 k \ \textit{if} \ a_1 = 0 \\ 1 - \beta_2 k + \beta_2 a_1 \ \textit{if} \ a_1 \in (0, k] \\ 1 \ \textit{if} \ a_1 \geq k, \end{cases} \tag{17}$$

$$F_2^2(a_2) = \begin{cases} 0 \ \textit{if} \ a_2 < 0 \\ \beta_1 a_2 \ \textit{if} \ a_2 \in [0, k) \\ 1 \ \textit{if} \ a_2 \geq k. \end{cases} \tag{18}$$

(ii) In the first equilibrium, the expected utilities are $Eu_1^1 = 1 - \beta_1 k$ *and* $Eu_2^1 = 0$*; the probability that 1 wins is* $p_1^1 = 1 - \frac{\beta_2 \beta_1 k^2}{2}$*. In the second equilibrium, the expected utilities are* $Eu_1^2 = 0$ *and* $Eu_2^2 = 1 - \beta_2 k$*; the probability that 2 wins is* $p_1^2 = \frac{\beta_2 \beta_1 k^2}{2}$*. (iii) In either equilibrium, the candidate payoffs are weakly increasing in the spending cap* k*. (iv) The distribution of the voters' payoff (winning candidate's effort level) is higher in the second equilibrium than in the first equilibrium and higher without caps than in either equilibrium to the game with caps.*

This game, thus, has multiple equilibria that are not payoff-equivalent. Since either candidate would attain a strictly positive payoff if it exerted k units of effort and won for sure, it is possible to construct equilibria in which either player attains a strictly positive payoff. In the other games considered in this paper, the upper bound of one candidate's support is the effort level that just makes the candidate indifferent between exerting this level of effort and winning and exerting an effort level of 0 and not winning.[18]

In contrast to the case of campaign finance reforms that change the costs, a policy that imposes a binding cap makes it possible for the disadvantaged candidate to be more likely to win. This finding must be tempered because, with binding caps, there are two equilibria; in one the advantaged candidate is more likely to win (and receive a strictly positive payoff), but in the second equilibrium the disadvantaged candidate is more likely to win (and receive a strictly positive payoff). Compared to the case of no caps, this reform can either increase or decrease p_1. In the first equilibrium, p_1^1 is higher than $1 - \frac{\beta_1}{2\beta_2}$, the value of p_1 with no policy intervention. In the second equilibrium, p_1^2 is lower than $1 - \frac{\beta_1}{2\beta_2}$.[19]

Sahuguet and Persico (2006) analyze a model in which candidates choose how much to spend and then how to allocate the spending across voters. Their model involves asymmetric initial levels of valance but symmetric cost functions. The candidates in their model are vote maximizing. This paper considers candidates who maximize the probability of victory. Despite these distinctions, interesting comparisons can be made. In the case of linear costs (this example is closest to the models in the current paper), Sahuguet and Persico find that when there is no finance intervention, the disadvantaged candidate "catches up." In this case, each party attains a 50-percent vote share, and campaign finance reform does not affect competitiveness. In this paper, whether the disadvantaged candidate spends more or less (in expectation) depends on the type of asymmetry, but regardless of the asymmetry the advantaged candidate remains electorally advantaged (is more likely to win in equilibrium). With non-linear costs, Sahuguet and Persico find that spending limits (caps) have anti-competitive effects, while interventions that lower the marginal cost of spending can have pro- or anti-competetive effects (depending on the shape of the cost function). In this paper, I find that with cost asymmetries, interventions that lower the marginal cost of spending do not affect the equilibrium levels of competitiveness. Spending caps, however, can either increase or decrease competitiveness. The ambiguity stems from equilibrium multiplicity. So, the two models yield different conclusions about how spending limits or interventions on cost affect competitiveness.

A final observation about caps is worth making.[20] Return to to the case in which candidate 1 has a preference advantage ($\alpha > 0$). Under a policy of imposing tight caps on effort/spending, the unique equilibrium has the advantaged candidate win with probability one. Neither candidate exerts any effort. This is true because candidate 1 will win following any feasible profile of candidate actions and, thus, the unique pair of best responses is $a_1 = a_2 = 0$.

Proposition 7 *In the model with assymetric (or symmetric) costs and a preference advantage for candidate 1 ($\alpha > 0$), if $k < \alpha$ the unique equilibrium has $a_1 = a_2 = 0$ and candidate 1 wins with probabiliy 1.*

The equilibrium characterized in Proposition 7 maximizes the expected sum of candidate utilities. It also maximizes the probability that the preferred candidate wins.

Discussion

This paper develops a model of campaigns in which candidates determine how much effort to exert in the contest to persuade voters to support them. Equilibrium analysis offers insights into what types of asymmetries are likely, and unlikely, to cause the incumbency advantage observed in American elections. Marginal advantages result in equilibria in which the incumbent outspends the challenger. Preference advantages generate equilibria in which the challenger outspends the incumbent. In both cases, the advantaged candidate is more likely to win and receives a positive expected payoff. For the disadvantaged candidate, any rents to office are bid away (in expectation) by the costs of campaigning. When the asymmetries are cross-cutting, whether or not the candidate with a marginal advantage or preference advantage is favored depends on the relative magnitudes of the asymmetries.

Comparative statics analysis offers several insights about the consequences of campaign finance reform. Symmetric or neutral changes to the cost of generating contributions will generally increase the likelihood that incumbents win. Thus, the model supports the idea that campaign finance reform protects incumbents. This is true, even if reform is symmetric and not biased in favor of the incumbent. This is a stronger argument than the one typically presented in the literature. Most pundits implicitly assume that reform helps incumbents because it disproportionately hurts challengers, who have less access to money. The analysis suggests that neutral changes can reinforce the asymmetry present in the electoral contest and, thus, help the incumbent. Surprisingly, even small asymmetric changes, aimed at hurting incumbents more than challengers, cannot improve the welfare of disadvantaged challengers. Despite this last point, it is possible to increase the likelihood that the challenger wins by disproportionately raising the cost to the incumbent. This change does not actually improve the challenger's welfare because equilibrium behavior results in more campaigning. Since neither candidate benefits, campaign finance reform that increases the cost of raising contributions is not likely to be championed by challengers or incumbents (unless it becomes a relevant policy issue). Limits on spending that are binding for both candidates result in a game with multiple equilibria. Depending on equilibrium selection, the incumbent (advantaged candidate) can be more or less likely to win; the incumbent's payoff can be either decreasing or constant in the level of the cap.

Appendix A: Proofs

Proof of Proposition 1: Recall that a majority of voters prefer candidate 1, so that with close to equal levels of a candidate 1 is favored. Candidate 1 has the following objective function (assuming weakly undominated voting).

$$Eu_1(a_1; a_2) = \begin{cases} 1 - \beta_1 a_1 \text{ if } a_1 + \alpha > a_2 \\ -\beta_1 a_1 \text{ if } a_1 + \alpha < a_2 \\ q - \beta_1 a_1 \text{ otherwise} \end{cases} \tag{19}$$

Candidate 2 has a similar objective function

$$Eu_2(a_2; a_1) = \begin{cases} 1 - \beta_2 a_2 \text{ if } a_1 + \alpha < a_2 \\ -\beta_2 a_2 \text{ if } a_1 + \alpha > a_2 \\ 1 - q - \beta_2 a_2 \text{ otherwise.} \end{cases} \tag{20}$$

The best response correspondences are not everywhere defined. For instance, 1's optimal level of effort in response to an effort level of $a_2 \in (\alpha, \frac{1}{\beta_2})$ does not exist. An effort level of $a_1 = a_2 - \alpha$ results in a tie and let the probability of winning be q, but selection of $a_1' = a_2 - \alpha + \varepsilon$ results in victory for certain. As long as $\varepsilon < \frac{1-q}{\beta_2}$, the deviation is desirable. Thus, as long as $q < 1$ no best response for agent 1 exists if $a_2 \in (\alpha, \frac{1}{\beta_2})$. However, if $q = 1$, then no best response for candidate 2 will exist if $a_1 \in [0, \frac{1}{\beta_2} - \alpha)$. For any such a_1, selection of $a_2 = a_1 + \alpha + \varepsilon$ results in victory. For ε small enough this is preferred to losing. Accordingly, it is not possible to select q (even as a function of a_1, a_2) to render both best response mappings well defined at $a_2 = a_1 + \alpha$. Thus, there cannot be an equilibrium with $a_2 = a_1 + \alpha$. If $a_2 > a_1 + \alpha$, then a slight reduction in a_2 is a profitable deviation. Finally, if $a_2 < a_1 + \alpha$, then either $a_1 = 0$ or a slight decrease in a_1

is desirable, and if $a_1 = 0$, then since $\alpha < \frac{1}{\beta_2}$ a deviation to a value of a_2 slightly greater than α is desirable.

The result does not hinge on the asymmetry imposed by θ or the cost difference. To see this, assume that $\alpha = 0$. For $\beta_2 = \beta_1 = \beta > \frac{1}{2}$ the candidates are not willing to accumulate $a = 1$ in order to tie (with a fair randomization). In this instance $a_1 = a_2 = 1$ is not an equilibrium as one candidate would prefer to deviate to $a_c = 0$ since $\beta > \frac{1}{2}$ and one candidate must win with probability of no more than $\frac{1}{2}$ in a tie. On the other hand, $a_1 = a_2 = 0$ cannot be an equilibrium as $\alpha = 0$ implies that any deviation to $a_c > 0$ would result in victory. Finally, for $a_c \in (0, \frac{1}{\beta})$ a best response to a_c does not exist as any effort that is slightly greater than a_c results in victory.■

I state and prove lemma 2 as two distinct results. One additional definition is needed. A point x is in the interior of a set X if there exists some open set $U \subset X$ s.t $x \in U$.

Lemma 2.1: If $\beta_2 < \frac{1}{\alpha}$, there cannot be an equilibrium in which candidate 1 assigns strictly positive probability to a point in $(0, \frac{1}{\beta_2} - \alpha)$ and/or candidate 2 assigns strictly positive probability to a point in $(\alpha, \frac{1}{\beta_2})$.

Proof: The proof involves showing that if one candidate (say 1) plays a point (say b) in $(0, \frac{1}{\beta_2} - \alpha)$ with positive probability, then the other candidate's best response will be such that selecting b itself is not a best response for candidate 1. Suppose candidate 1 plays $a_1 = b$ with strictly positive probability. Let $p_b > 0$ denote the probability that $a_1 = b$ in this equilibrium. Since a choice $a_2 < b + \alpha$ loses to $a_1 = b,$ a selection of $a_2 = b + \alpha - \varepsilon$ yields a strictly lower payoff than a selection of $a_2' = b + \alpha + \varepsilon$ if $0 < p_b - 2\varepsilon\beta_2$. Thus, for $\varepsilon < \frac{p_b}{2\beta_2}$ candidate 2 must select $a_2 \in (b+\alpha-\varepsilon, b+\alpha)$ with probability 0 in this equilibrium. Now consider 2 cases. Either (i) $a_2 = b+\alpha$ is in the support of 2's strategy or (ii) it is not. If (i) then 2 must win (with probability one) in the event of a tie of the form $a_1 = b$ and $a_2 = b+\alpha$; otherwise, if 2 won with probability $p' < 1$ in this kind of tie, a deviation to $a_2 = b+\alpha+\varepsilon'$ (for $\varepsilon' < \frac{p_b(1-p')}{2\beta_2}$) would be attractive. In this case, 1 is strictly better off playing $a_1' \in (b-\varepsilon, b)$ than she is playing $a_1 = b$ as a_1' wins with the same probability as the policy $a_1 = b$ but it has a strictly lower cost. If (ii) then 1 is also strictly better off playing $a_1' \in (b-\varepsilon, b)$ then she is playing $a_1 = b$ as a_1' wins with the same probability as the policy $a_1 = b$ but it has a strictly lower cost. Thus, there cannot be an equilibrium in which 1 selects $a_1 = b$ with strictly positive probability. A nearly identical argument establishes the result for candidate 2.■

Lemma 2.2: Assume $\beta_2 < \frac{1}{\alpha}$ and consider any mixed strategy equilibrium If $(b,d) \subset [0, \frac{1}{\beta_2} - \alpha]$, the probability that $a_1 \in (b,d)$ is strictly positive in the equilibrium. If $(b,d) \subset [\alpha, \frac{1}{\beta_2}]$, the probability that $a_2 \in (b,d)$ is strictly positive in the equilibrium.

Proof: Consider candidate 1's strategy. Suppose that for some interval $(b,d) \subset [0, \frac{1}{\beta_2} - \alpha]$ (with $b < d$) there is an equilibrium in which the probability that $a_1 \in (b,d)$ is 0. Let (b', d') denote the largest interval containing (b,d) s.t. $a_1 \in (b', d')$ with probability 0 in this equilibrium. It must be the case that 2's mixed strategy assigns probability 0 to the interval $(b'+\alpha, d'+\alpha]$ (as $a_2 \in (b'+\alpha, d'+\alpha]$ wins with the same probability as $a_2' = \frac{b'+\alpha+a_2}{2}$ but costs strictly more than a_2'). Observe that for $\varepsilon > 0$, $a_1 \in [d', d'+\varepsilon]$ wins with probability of at most $F_2(d'+\alpha+\varepsilon)$. From lemma 2.1, candidate 2 does not play $d'+\alpha$ with positive probability unless $d' = \frac{1}{\beta_2} - \alpha$. This means that if $d' \neq \frac{1}{\beta_2} - \alpha$, then $F_2(d'+\alpha+\varepsilon)$ is continuous in ε (and thus for any $\delta > 0$ there exists an $\varepsilon > 0$ s.t. $F_2(d'+\alpha+\varepsilon) - F_2(b'+\alpha+\varepsilon) < \delta$)

and thus since $\beta_1 b' < \beta_1 d'$ for some ε, a policy like $a_1' = \frac{b'+d'}{2}$ yields a strictly higher expected payoff to 1 than a policy in $[d', d' + \varepsilon]$. Accordingly, unless $d' = \frac{1}{\beta_2} - \alpha$ the set (b', d') cannot be the largest interval containing (b, d) s.t. $a_1 \in (b', d')$ with probability 0 in this equilibrium. Thus I have shown that if the probability that $a_1 \in (b, d)$ is 0 in an equilibrium then the probability that $a_1 \in (b, \frac{1}{\beta_2} - \alpha)$ is 0 and the probability that $a_2 \in (b + \alpha, \frac{1}{\beta_2})$ is 0 in that equilibrium. If $\frac{1}{\beta_2}$ is in candidate 2's support, then she must win with probability 1 if she plays this policy. Alternatively, candidate 1 must win with probability one when she selects $a_1 = \frac{1}{\beta_2} - \alpha$. This means that both of these endpoints cannot be played with positive probability in the same equilibrium. There cannot be an equilibrium (with the holes isolated above) where exactly one of these endpoints is chosen with positive probability since a deviation to a slightly lower effort level would still win but cost less than the endpoint. Thus, in equilibrium $F_2(b' + \alpha) = 1$. This means that candidate 1 must win with probability 1 when she selects $a_1 = b'$ (otherwise for small $\delta > 0$ selections of $a_1 \in (b' - \delta, b')$ would yield a lower expected payoff than $a_1' = b' + \delta$). But this logic also leads to the conclusion that 2 must win with probability 1 when she selects $a_2 = b' + \alpha$. Thus, we have a contradiction as candidates 1 and 2 cannot both win with probability 1 in the event of a tie.■

Proof of Proposition 3: 1) By Lemma 2, candidate 1's mixed strategy has support $[0, \frac{1}{\beta_2} - \alpha]$ and candidate 2's mixed strategy has support $\{0\} \cup [\alpha, \frac{1}{\beta_2}]$ or $[\alpha, \frac{1}{\beta_2}]$. To characterize candidate 1's mixed strategy consider candidate 2's incentives. The expected utility to candidate 2 from choosing $a_2 = \frac{1}{\beta_2}$ is

$$Eu_2(\frac{1}{\beta_2}) = (1 - F_1^+(\frac{1}{\beta_2} - \alpha)q) - 1 \tag{21}$$

where $F_1^+(\frac{1}{\beta_2} - \alpha)$ is the probability that candidate 1 chooses $a_1 = \frac{1}{\beta_2} - \alpha$ and q is the probability that a tie goes to candidate 1. By inspection this value is at most 0. The fact that $\frac{1}{\beta_2}$ is the upper bound of 2's support (Lemma 2) does not imply that 2 is indifferent between this strategy and the strategies in the interior of her support. However, since the probability that 2 wins with $a_2 = \frac{1}{\beta_2} - \varepsilon$ is less than the probability that she wins with $a_2 = \frac{1}{\beta_2}$ and the latter costs only $\varepsilon\beta_2$ less, $Eu_2(\frac{1}{\beta_2})$ must be at least as large as the limit of 2's expected utility for $a_2 = \frac{1}{\beta_2} - \varepsilon$ as $\varepsilon \downarrow 0$. Since this limit is 0 $Eu_2(\frac{1}{\beta_2})$ must be non-negative. In order for $Eu_2(\frac{1}{\beta_2})$ to be non-negative either $q = 0$ or $F_1^+(\frac{1}{\beta_2} - \alpha) = 0$ must hold. With at least one of these conditions satisfied, it is clear that player 2's expected utility is 0. Accordingly,

$$F_1(a_2 - \alpha) = \beta_2 a_2 \tag{22}$$

for any value of $a_2 \in (0, \frac{1}{\beta} - \alpha)$. This implies that

$$F_1(a_1) = \beta_2(a_1 + \alpha) \tag{23}$$

for any value of $a_1 \in (0, \frac{1}{\beta_2} - \alpha)$. Since distribution functions are right continuous taking the limit as $a_1 \downarrow 0$ yields $F_1(0) = \beta_2\alpha$. This distribution has an atom at 0 of $\beta_2\alpha$. Since $F_1(\frac{1}{\beta_2} - \alpha) = 1$ and $\lim_{a_1 \uparrow \frac{1}{\beta_2} - \alpha} \beta_2(a_1 + \alpha) = 1$ this distribution is continuous at the upper bound of its support. Thus, one of the two conditions stated above, $F_1^+(\frac{1}{\beta_2} - \alpha) = 0$, is satisfied, so candidate 2' expected utility is 0. Thus, the unique distribution for candidate 1's strategy that satisfies candidate 2's indifference condition is

$$F_1(a_1) = \begin{cases} 0 \text{ if } a_1 < 0 \\ \beta_2(a_1 + \alpha) \text{ if } a_1 \in [0, \frac{1}{\beta_2} - \alpha] \\ 1 \text{ if } a_1 \geq \frac{1}{\beta_2} - \alpha. \end{cases} \tag{24}$$

In order for candidate 1 to be willing to randomize, an indifference condition needs to be satisfied. Indifference between $a_1 \in (0, \frac{1}{\beta_2} - \alpha)$ and $a_1' = \frac{1}{\beta_2} - \alpha$ requires

$$F_2(a_1 + \alpha) - \beta_1 a_1 = 1 - F_2^+(\frac{1}{\beta_2})(1-q) - \frac{\beta_1}{\beta_2} + \beta_1 \alpha \tag{25}$$

where q is the probability that 1 wins in a tie involving $a_1 = \frac{1}{\beta_2} - \alpha$. Thus

$$F_2(a_1 + \alpha) = \beta_1(a_1 + \alpha) - F_2^+(\frac{1}{\beta_2})(1-q) + 1 - \frac{\beta_1}{\beta_2} \tag{26}$$

for $a_1 \in (0, \frac{1}{\beta_2} - \alpha)$. Rearranging yields

$$F_2(a_2) = \beta_1(a_2) - F_2^+(\frac{1}{\beta_2})(1-q) + 1 - \frac{\beta_1}{\beta_2} \tag{27}$$

for $a_2 \in (\alpha, \frac{1}{\beta_2})$. Since distribution functions are right continuous, $F_2(\alpha) = \beta_1\alpha + 1 - \frac{\beta_1}{\beta_2} - F_2^+(\frac{1}{\beta_2})(1-q)$. Note that if $F_2^+(\frac{1}{\beta_2})(1-q) > 0$ then for some $\varepsilon > 0$, candidate 1 will strictly prefer $a_1' = \frac{1}{\beta_2} + \varepsilon$ to $a_1 = \frac{1}{\beta_2}$ (the former wins with strictly higher probability and costs only $\beta_1\varepsilon$ more). So this means that $F_2(\alpha) = \beta_1\alpha + 1 - \frac{\beta_1}{\beta_2}$. It remains to establish whether this distribution puts any mass on $a_2 = 0$. Since candidate 1 wins at $a_1 = \frac{1}{\beta_2} - \alpha$ she has expected utility of $Eu_1 = 1 - \frac{\beta_1}{\beta_2} + \beta_1\alpha$. This means that the expected utility of $a_1 = 0$ must be $\beta_1\alpha + 1 - \frac{\beta_1}{\beta_2}$. This requires that candidate 1 win with probability $\beta_1\alpha + 1 - \frac{\beta_1}{\beta_2}$ at $a_1 = 0$. This means that $F_2(0) = \beta_1\alpha + 1 - \frac{\beta_1}{\beta_2}$, and there is no atom at $a_2 = \alpha$. By definition $F_2(\frac{1}{\beta_2}) = \lim_{a_2 \uparrow \frac{1}{\beta_2}} F_2(a_2) + F_2^+(\frac{1}{\beta_2})$. Since the limit term is 1, the fact that $F_2(\frac{1}{\beta_2}) = 1$ implies that $F_2^+(\frac{1}{\beta_2}) = 0$. Thus

$$F_2(a_2) = \begin{cases} 0 \text{ if } a_2 < 0 \\ \beta_1\alpha + 1 - \frac{\beta_1}{\beta_2} \text{ if } a_2 \in [0, \alpha) \\ \beta_1 a_2 + 1 - \frac{\beta_1}{\beta_2} \text{ if } a_2 \in (\alpha, \frac{1}{\beta_2}) \\ 1 \text{ if } a_2 \geq \frac{1}{\beta_2}. \end{cases} \tag{28}$$

(2) Computation of the expected values of a_1 and a_2 is simplified by a decomposition of the distribution functions into weighted averages of point masses and uniform distributions. Taking expectations yields,

$$\begin{aligned} Ea_1 &= \frac{1-\beta_2\alpha}{2}(\frac{1}{\beta_2} - \alpha) \\ &= \frac{1-\beta_2\alpha}{2\beta_2} - \frac{\alpha - \beta_2\alpha^2}{2} \\ &= \frac{1}{2\beta_2} - \alpha + \frac{\beta_2\alpha^2}{2} \end{aligned} \tag{29}$$

$$\begin{aligned} Ea_2 &= \left(\frac{\beta_1}{\beta_2} - \beta_1\alpha\right)\left(\frac{\alpha}{2} + \frac{1}{2\beta_2}\right) \\ &= \frac{\beta_1}{2\beta_2^2} - \frac{\beta_1\alpha^2}{2}. \end{aligned} \tag{30}$$

(3) The expected utilities are derived above (in the proof of part (1)).

(4) Candidate 1 wins if $a_2 = 0$ (this occurs with probability $\beta_1\alpha + 1 - \frac{\beta_1}{\beta_2}$). Conditional on $a_1 > 0$ and $a_2 > \alpha$ (which happens with probability $(\frac{\beta_1}{\beta_2} - \beta_1\alpha)(1 - \beta_2\alpha)$ candidate 1 wins

with probability $\frac{1}{2}$. Thus

$$p_1 = (\beta_1\alpha + 1 - \frac{\beta_1}{\beta_2}) + \frac{1}{2}(\frac{\beta_1}{\beta_2} - \beta_1\alpha)(1 - \beta_2\alpha) \tag{31}$$
$$= 1 - \frac{\beta_1}{2\beta_2} + \frac{\beta_1\beta_2\alpha^2}{2}.$$

■

Proof of Proposition 4: a. In order to characterize the equilibrium I begin with the requisite indifference conditions. In order for candidate 1 to weakly prefer $a_1 = \alpha$ which has a cost of $\beta_1\alpha$ over $a_1' = 0$ her probability of winning when $a_1 = \alpha$ must be at least $\beta_1\alpha$. This means that candidate 2 must select 0 (the only policy in her support that does not definitely beat $a_1 = \alpha$) with probability no less than $\beta_1\alpha$. In order for candidate 1 to be indifferent between two strategies a_1 and a_1' both in $(\alpha, \frac{1}{\beta_2} + \alpha]$ it must be the case that $F_2(a_1 - \alpha) - \beta_1 a_1 = F_2(a_1' - \alpha) - \beta_1 a_1'$. This condition is satisfied if and only if $F_2(\cdot)$ has density β_1 on $(0, \frac{1}{\beta_2})$. Integrating means that there is $1 - \frac{\beta_1}{\beta_2}$ probability to assign to $a_2 \in \{0, \frac{1}{\beta_2}\}$. So if $\beta_1\alpha < 1 - \frac{\beta_1}{\beta_2}$ then this strategy for player 2 will satisfy the indifference condition for player 1. Player 2 assigns probability $1 - \frac{\beta_1}{\beta_2}$ to $a_2 = 0$ (player 1 wins for sure in the event that she selects $a_1 = \alpha$ and candidate 2 selects $a_2 = 0$) and player 1 receives payoff $1 - \frac{\beta_1}{\beta_2} - \beta_1\alpha$ in equilibrium. The relevant condition simplifies to $\alpha < \frac{1}{\beta_1} - \frac{1}{\beta_2}$. If this holds then the cost asymmetry (advantage for 1) swamps the preference asymmetry (advantage for 2) and 1 gets a positive expected payoff while 2 gets 0 expected payoff. Assuming that this condition is satisfied, candidate 1's mixture must assign probability 0 to $a_1 = 0$ (or else candidate 2 would attain a strictly positive payoff from $a_2 = 0$). Moreover, $F_1(\cdot)$ must have density β_2 on $(\alpha, \frac{1}{\beta_2} + \alpha)$ in order for candidate 2 to be indifferent over $(0, \frac{1}{\beta_2})$. This means that candidate 1 must assign probability $1 - \beta_2(\frac{1}{\beta_2}) = 0$ to the points $\{\alpha, \frac{1}{\beta_2} + \alpha\}$.

Part a1 thus exhibits the unique pair of distributions that satisfy these conditions. Part a3 follows from the conclusions about candidate payoffs above, and parts a2 and a4 involve direct calculations.

b. It remains to characterize the equilibrium if $\alpha \in (\frac{1}{\beta_1} - \frac{1}{\beta_2}, \frac{1}{\beta_1})$. Following the lemma, I investigate what must be true in order for candidates to mix over the support characterized in the second part of the lemma. I begin with the upper bounds of the supports. Consider $a_1 = \frac{1}{\beta_1}$. If candidate 1 does not win when this effort level is chosen, then the strategy $a_1 = 0$, would be strictly preferred. This means that candidate 2 cannot play $a_2 = \frac{1}{\beta_1} - \alpha$ with positive probability. Since candidate 2's mixed strategy must have a density of β_1 on $(0, \frac{1}{\beta_1} - \alpha)$ and $\beta_1(\frac{1}{\beta_1} - \alpha) < 1$ we have just determined that candidate 2 plays $a_2 = 0$ with probability $1 - \beta_1(\frac{1}{\beta_1} - \alpha) = \beta_1\alpha$. So given this strategy for candidate 2, candidate 1 attains a payoff of $F_1(a_1 - \alpha) - \beta_1 a_1 = \beta_1\alpha + \beta_1(a_1 - \alpha) - \beta_1 a_1 = 0$ from any $a_1 \in (\alpha, \frac{1}{\beta_1}]$. In order for candidate 1 to attain a payoff of 0 from $a_1 = \alpha$ she must win with probability $\beta_1\alpha$ following $a_1 = \alpha$. Since candidate 2 plays 0 with this probability, candidate 1 must win in the event of this type of tie. Since $a_1 = 0$ is strictly less attractive to the voter than any effort by candidate 2, the expected payoff to candidate 1 from $a_1 = 0$ is 0. I now consider the incentives for candidate 2. Now since $\beta_2(\frac{1}{\beta_1} - \alpha) < 1$ candidate 2 must win with probability 1 when she plays $a_2 = \frac{1}{\beta_1} - \alpha$ (otherwise a slightly higher effort would be strictly preferred). Since candidate 1 selects $a_1 = \frac{1}{\beta_1}$ with probability 0, candidate 2 wins with probability 1 if $a_2 = \frac{1}{\beta_1} - \alpha$. This means that candidate 2 attains an expected utility of $1 - \beta_2(\frac{1}{\beta_1} - \alpha)$. In order for candidate 2 to attain this expected utility when she plays $a_2 = 0$ candidate 1 must select $a_1 = 0$ with probability $1 - \beta_2(\frac{1}{\beta_1} - \alpha)$. In order for candidate 2 to be indifferent over strategies in $(0, \frac{1}{\beta_1} - \alpha)$, candidate 1's mixed strategy must have density β_2 on $(\alpha, \frac{1}{\beta_1})$. Since $1 - \beta_2(\frac{1}{\beta_1} - \alpha) + \beta_2(\frac{1}{\beta_1} - \alpha) = 1$, candidate 1 can play 0 with the appropriate probability and then mix on $[\alpha, \frac{1}{\beta_1}]$ with the appropriate density. Part b1 thus exhibits the distributions that satisfy these conditions. Part b3 follows from the conclusions about payoffs above, and parts b2 and b4 involve direct calculations.■

The proof of proposition 6 relies on the following lemma.

Lemma 4: There cannot be an equilibrium with supports $A_1 = A_2 = [0, k]$ in which both candidates select 0 with strictly positive probability and there cannot be an equilibrium with supports A_1 and A_2 in which both candidates select k with strictly positive probability.

Proof: The argument proceeds in two parts. -Suppose both candidates play 0 with strictly positive probability, z_1, z_2 respectively. Let s denote the probability that 1 wins in a tie at $a_1 = a_2 = 0$. In order for both 0 and $a_c \in (0, k)$ to be in c's support it must be the case that $sz_2 = F_2(a_1) - \beta_1 a_1$ and $(1-s)z_1 = F_1(a_2) - \beta_2 a_2$. Since I have concluded that $F_c(\cdot)$ has density β_{-c} on $(0, k)$, these conclusion imply that we have $sz_2 = z_2 + \beta_1 a_1 - \beta_1 a_1$ and $(1-s)z_1 = z_1 + \beta_2 a_2 - \beta_2 a_2$. Note that the right hand sides simplify to z_2 and z_1 respectively. Thus we have

$$sz_2 = z_2$$

$$(1-s)z_1 = z_1$$

The first equation can only be solved if $s = 1$ and the second can only be solved if $s = 0$. Thus at most 1 player can select 0 with positive probability. Moreover if c selects 0 with positive probability then $-c$ must win a tie at $a_1 = a_2 = 0$ with probability 1. This implies that if candidate c selects $a_c = 0$ with probability 1 then she has an expected payoff of 0.

-Suppose that both candidates play k with strictly positive probability, t_1, t_2 respectively. Moreover, suppose that candidate 1 selects 0 with probability z_1.

Let y denote the probability that 1 wins in a tie at $a_1 = a_2 = k$. So

$$z_1 + t_1 + \beta_2 k = 1.$$

In order for 2 to play this strategy we must have

$$\begin{aligned} Eu_2(k) &= 1 - t_1 + t_1(1-y) - \beta_2 k \\ &= z_1 + \beta_2 a_2 - \beta_2 a_2 = Eu_2(a_2) \end{aligned}$$

for $a_2 \in [0, k)$. This equality implies that $1-t_1+t_1(1-y)-\beta_2 k = z_1$. Substituting $1 - t_1 = z_1 + \beta_2 k$ (from above) yields,

$$z_1 + t_1(1-y) = z_1.$$

Thus $t_1(1-y) = 0$. Suppose $t_1 > 0$, so $1 - y = 0$. This means that 1 wins a tie at k with probability 1, and we have $Eu_1(k) = 1 - \beta_1 k$ and since 2 plays $a_2 = 0$ with probability 0, $Eu_1 = 0$. But this means that 1 is not indifferent over the support of her mixed strategy–a contradiction. A similar argument obtains when player 2 is assumed to play both 0 and k with positive probability.■

Proof of Proposition 6: In the first strategy profile, candidate 1 wins if she selects $a_1 = k$ and thus her expected utility from $a_1 = k$ is $1-\beta_1 k$. To verify that this is her expected utility from every $a_1 \in [0, k]$, I consider two cases. First suppose that $a_1 \in (0, k)$. In this case 1 wins with probability $F_2(a_1) = 1 - \beta_1 k + \beta_1 a_1$ and the cost is $\beta_1 a_1$ resulting in an expected utility of $1 - \beta_1 k$. At $a_1 = 0$ candidate 1 wins with probability $1 - \beta_1 k$ (recall that I argued above in an equilibrium in which 2 selects $a_2 = 0$ with positive probability, 1 wins in a tie at $a_1 = a_2 = 0$). To see that candidate 2's strategy is a best response, it is sufficient to show that her expected utility is 0 at any $a_2 \in [0, k]$. We have already concluded that candidate 2 wins with probability 0 if $a_2 = 0$. Thus, the expected utility to $a_2 = 0$ is 0. The probability that candidate 2 wins at $a_2 = k$ is $1 - (1 - \beta_2 k)$, and the cost is $\beta_2 k$. Thus, the expected payoff is 0. Finally for $a_2 \in (0, k)$ $F_1(a_2) = \beta_2 a_2$ and the cost to 2 is $\beta_2 a_2$.

In the second strategy profile, candidate 2 wins if she selects $a_2 = k$ and thus her expected utility from $a_2 = k$ is $1 - \beta_2 k$. To verify that this is her expected utility from every $a_2 \in [0, k]$ I consider two cases. First suppose that $a_2 \in (0, k)$. In this case 2 wins with probability $F_1(a_2) = 1 - \beta_2 k + \beta_2 a_2$ and the cost is $\beta_2 a_2$. This results in an expected utility of $1 - \beta_2 k$. At $a_2 = 0$ candidate 2 wins with probability $1 - \beta_2 k$ (recall that I argued above in an equilibrium in which 1 selects $a_1 = 0$ with positive probability, 2 wins in a tie at $a_1 = a_2 = 0$). To see that candidate 1's strategy is a best response, it is sufficient to show that her expected utility is 0 at any $a_1 \in [0, k]$. I have already concluded that candidate 1 wins with probability 0 if $a_1 = 0$. Thus, the expected utility to $a_1 = 0$ is 0. The probability that candidate 1 wins at $a_1 = k$ is $1 - (1 - \beta_1 k)$ and the cost is $\beta_1 k$ thus the expected payoff is 0. Finally for $a_1 \in (0, k)$ $F_2(a_1) = \beta_1 a_1$ and the cost to 1 is $\beta_1 a_1$.

The expected utilities are pinned down in the above derivations. The values of p_1 are found by noting that in the first equilibrium 1 wins if $a_1 = k$ or $a_2 = 0$ and when neither of these events occur 1 wins with probability $\frac{1}{2}$. Thus in the first equilibrium $p_1 = 1 - \beta_2 k(\beta_1 k)\frac{1}{2}$. In the second equilibrium candidate 2 wins if $a_2 = k$ or $a_1 = 0$, and when neither of these events occur 2 wins with probability $\frac{1}{2}$. Thus in the second equilibrium $p_1 = \beta_2 k(\beta_1 k)\frac{1}{2}$.

Expected levels of effort and voter payoffs differ in the two equilibria. In the first equilibrium the payoff to the voter is equivalent to the expected effort level of the candidate with the highest level. The probability that the winning effort level is less than x, is is given by $F_1^t(x)F_2^t(x)$ for $t = 1, 2$. The voters expected payoff is given by $\int x dF_1^t(x)F_2^t(x)$. Since the distribution function of 1's strategy in the second equilibrium first-order stochastically dominates the distribution function of 2's strategy in the first equilibrium and the distribution function of 2's strategy in the second equilibrium first-order stochastically dominates the distribution function of 1's strategy in the first equilibrium, $F_1^2(x)F_2^2(x)$ first order stochastically dominates $F_1^2(x)F_2^2(x)$. This means that the voters' payoff is higher in the second equilibrium than the first equilibrium. Comparisons between the voter payoff (or effort level) to this (second) equilibrium and the equilibrium in the game without caps are facilitated by a similar dominance argument. Without caps the probability that $a_2 = 0$ is $1 - \beta_1/\beta_2$ and with the remaining probability a_2 is uniform on $[0, \frac{1}{\beta_2}]$; conversely without caps a_1 is uniform on $[0, \frac{1}{\beta_2}]$. With caps in the second equilibrium the probability that 0 is played is $1-\beta_2 k$ (by candidate 1) and thus, the distribution function from candidate 2 in the no caps equilibrium first-order dominates the distribution function from candidate 1 in the equilibrium with caps as long as $1-\frac{\beta_1}{\beta_2} < 1-\beta_2 k$. Since the caps bind only if $k < \frac{1}{\beta_2}$, we have $\beta_2 k < \beta_2 \frac{1}{\beta_2}$. The right hand side is less than $\frac{\beta_1}{\beta_2}$ so $\beta_2 k < \frac{\beta_1}{\beta_2}$ implying that $1 - \frac{\beta_1}{\beta_2} < 1 - \beta_2 k$. It remains to compare the distribution function of 1' strategy when there are no caps with the distribution function for candidate 2 in the second equilibrium to the game with caps. Since $\beta_2 < \beta_1$ the former dominates the latter and thus the equilibrium voter payoff/effort level in the game without caps first-order-stochastically dominates the voter payoff/effort level in either equilibrium to the game with caps.■

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Notes

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[2] These values are based on FEC data as of November 7, 2006 for the 2006 election and Feb. 18, 2005 for the 2004 election. Data reported on www.opensecrets.org.

[3] A consensus that challenger spending is positively related to vote share has been reached (Jacobson 1978). While estimation of this relationship for incumbents is more challenging, specifications that control for the quality of the challenger find strong relationships for incumbents, as well (Green and Krasno, 1988; Erikson and Palfrey, 2000).

[4] The evidence from political advertisements is convincingly similar despite sometimes significant variation in how advertisements are categorized. In their sample of advertisements for Senate campaigns between 1988 and 1992, Kahn and Kenney (1999; 61) find that while 80 percent of the ads "stressed issues in some way only 36 percent made an issue the major focus." In studying over 1000 ads from 1998, Spillotes and Vavreck (2002) find that 32 percent of the candidates made at least one ad that was classified as committing to a specific policy position. Similarly, Abranjano and Morton (2004) consider 2000 Congressional elections and find that over 69 percent of the ads in their sample did not involve substance.

[5] Common directions include: models with exogenous valence differences (Londregan and Romer, 1993; Groseclose, 2001; Aragonnes and Palfrey, 2002; Dix and Santore, 2002); mod-

els in which a small number of informed special interests signal candidate quality through contributions (Gerber, 1996; Prat 2002; Coate, 2004; Ashworth, 2004); and models in which funding decisions drive policy selection (Austen-Smith, 1987; Baron, 1989, 1994; Morton and Myerson, 2000; Meirowitz and Wiseman 2005). In the first type of scholarship, only policy is endogenous. In the latter two categories of research, spending decisions are made only by contributors, whereas candidates make decisions about policies or service provision.

[6]Clinton and Owen (2006) find evidence of persuasion in experimental work. Huber and Arceneaux (2005) find "little evidence that voters learn from presidential advertisements, but strong evidence that they are persuaded by them." (p.1) in the 2000 Presidential election.

[7]See Wittman (2005) for an argument that, regardless of how advertising influences voting behavior, contributions and spending must improve the welfare of the median voter if voters have rational expectations or use sensible rules of thumb.

[8]While there is some evidence of strong incumbency advantages for small offices (Snyder and Ansolabehere, 2005), a test of this comparative static requires controlling for changes in the cost of campaigning.

[9]Readers uncomfortable with mixed strategy equilibria should recognize that standard purification arguments can be applied. For every mixed strategy equilibria to every game in this paper, there is a different game in which candidates have private types that possesses a pure strategy equilibrium that induces almost the same lottery over the effort levels (Milgrom and Weber, 1985).

[10]I thank a very helpful reviewer for pointing out this paper.

[11]When candidates seek to maximize vote share, their payoff is continuous in strategies. In contrast, when candidates seek to maximize the probability of winning, the payoff function

is discontinuous.

[12]A very natural extension of the compelete and perfect information model is to think of a model in which the identity of the candidate with majority support is randomly chosen. A related extension is to think of each voter having a randomly generated α_i. It is, however, important to understand the simplest version of these contests first, and, thus, this paper focuses on the complete-and-perfect information version.

[13]For example, a strategy that plays $a_c = 0$ with probability $\frac{1}{4}$ and $a_c = \frac{1}{\beta_2}$ with probability $\frac{3}{4}$ has as its support $\{0, \frac{1}{\beta_2}\}$. Similarly, a strategy in which a_c is drawn from the uniform distribution on the interval (a, b) has as its support $[a, b]$.

[14]Subgame perfection requires that candidate strategies are based on reasonable expectations about how voters will choose between candidates. Weak dominance requires that voters do not support a candidate that they like strictly less than the other. This latter requirement is standard in voting games and rules out Nash equilibria in which unpopular alternatives receive a super-majority of votes simply because no voter is pivotal. For convenience, I simply use the terms *equilibrium* and *equilibria* to describe profiles that satisfy these requirements.

[15]Data from opensecrets.org, based on FEC data released on Nov 7, 2006.

[16]The derivative is $\frac{\partial E a_2(k)}{\partial k} = -\frac{1}{2k^2}\frac{\beta_1}{\beta_2^2} < 0$.

[17]An anonymous reviewer has pointed out that it is possible to interpret voter utilities less literally. Under the interpretation that effort influences voting behavior but not voter welfare, changes in k have no effect on aggregate welfare.

[18]A class of similar models involve candidates facing feasibility constraints, k_1 and k_2, as well as costs β_1 and β_2. For some combinations of the parameters, these games also exhibit

multiple equilibria with distinct payoffs.

[19]Of course, as k converges to $\frac{1}{\beta_2}$, the value of p_1 in the first equilibrium converges to $1 - \frac{\beta_1}{2\beta_2}$.

[20]I thank an anonymous reviewer for suggesting this line of reasoning.

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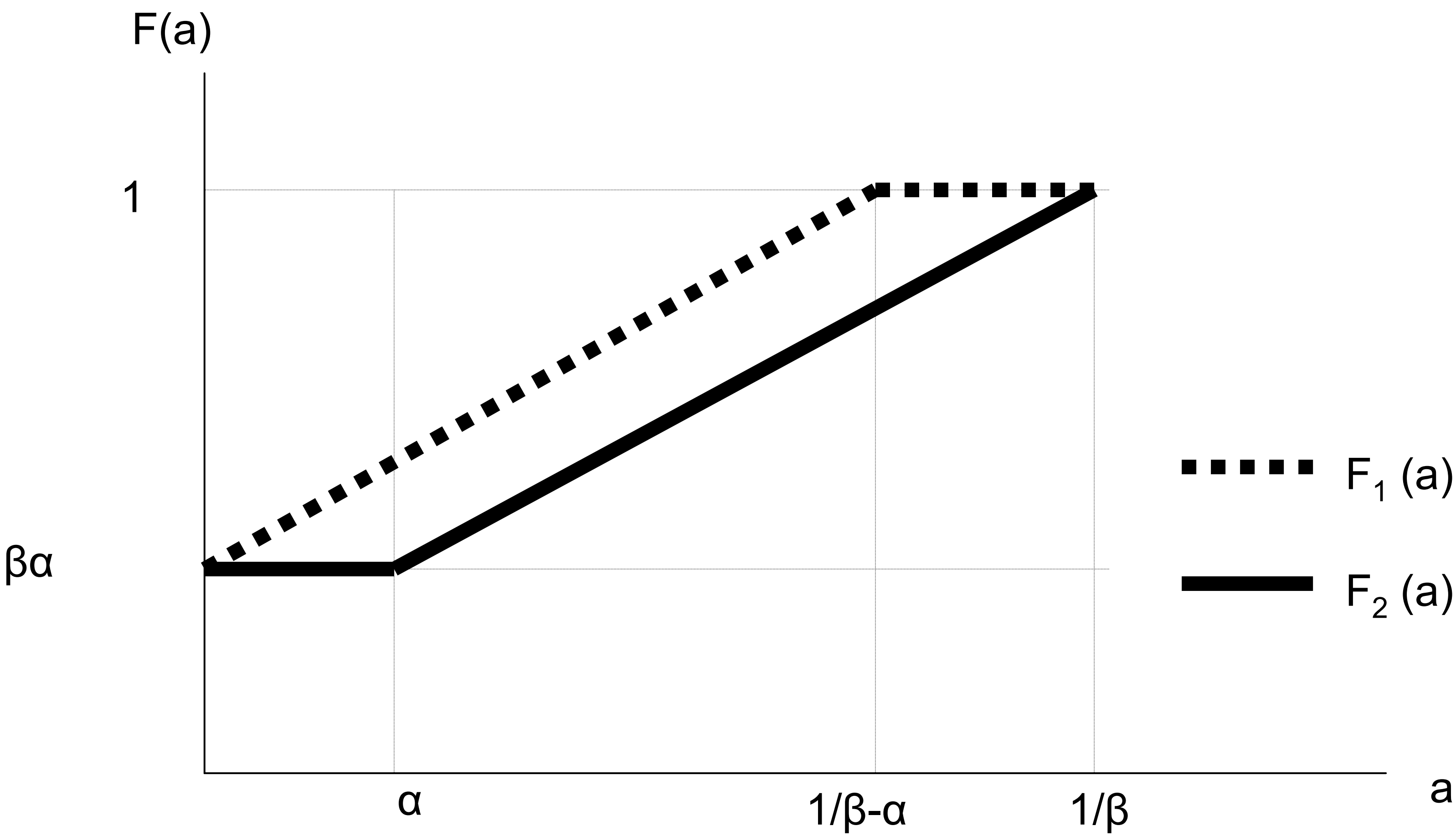

Fig 1.Equilibrium mixtures -asymmetric preference

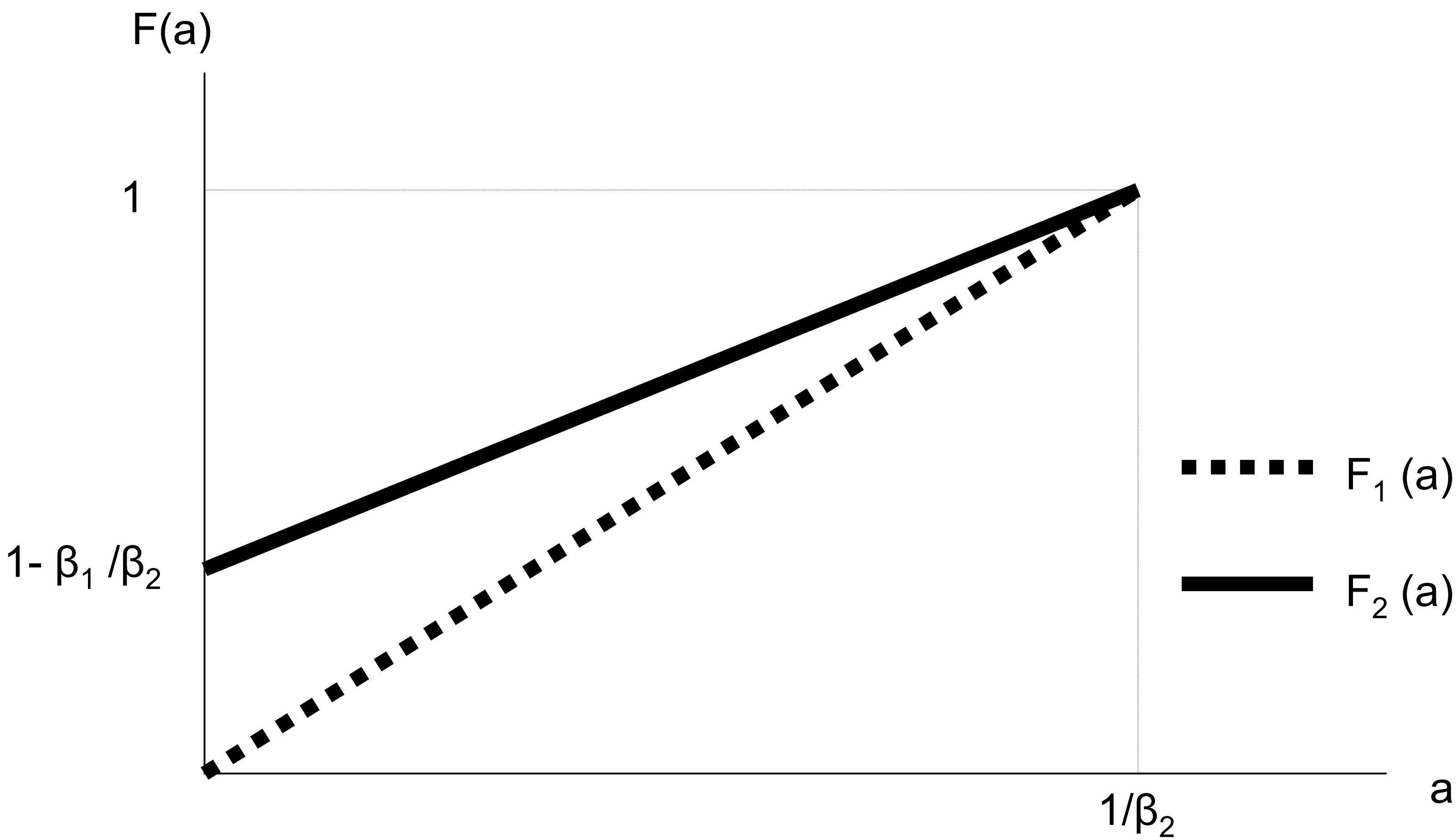

Fig 2.Equilibrium mixtures -asymmetric costs