

Name: _____

PRINCETON UNIVERSITY

ORF 363/COS 323
Midterm Exam, Fall 2014

OCTOBER 23, 2014, FROM 1:30 PM TO 2:50 PM

Instructor:

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As:

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PLEASE DO NOT TURN THIS PAGE. START THE EXAM ONLY
WHEN YOU ARE INSTRUCTED TO DO SO.

1. You are allowed a single sheet of A4 paper, double sided, hand-written or typed.
2. No electronic devices are allowed (e.g., cell phones, calculators, laptops).
3. All questions should be answered in the booklet. Please write out and sign the following pledge on the booklet "I pledge my honor that I have not violated the honor code during this examination."
4. Make sure you write your name on the first page of these questions and return the questions to us at the end of the exam.

Grading

Problem 1	20 <i>pts</i>	
Problem 2	20 <i>pts</i>	
Problem 3	20 <i>pts</i>	
Problem 4	20 <i>pts</i>	
Problem 5	20 <i>pts</i>	
TOTAL	100	

Problem 1: Local minima and maxima.

Find all the local minimizers and the local maximizers of the function

$$f(x_1, x_2) = \frac{1}{2}x_1^2 + 4x_1x_2 + \frac{1}{2}x_2^2 - x_2^3.$$

Show your reasoning.

Problem 2: Convex hulls.

Consider four points $a = (0, 1)$, $b = (1, 3)$, $c = (2, 1)$, $d = (5, 2)$ in \mathbb{R}^2 . Write down a convex optimization problem that finds a point in the convex hull of $\{a, b, c\}$ that is closest (in Euclidean distance) to d . Clearly mark your decision variables, constraints, and objective function. You don't need to solve the optimization problem. You also don't need to prove that it is convex. However, if you write down a nonconvex problem, you won't receive full credit.

Problem 3: Minimizers of convex problems.

Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be a convex function and $\Omega \subseteq \mathbb{R}^n$ be a convex set. Show that the set of minimizers of f over Ω is convex (i.e., a convex set).

Problem 4: Estimating $\sqrt{20}$.

You are stuck in a place with no electronic devices allowed and asked to compute $\sqrt{20}$. How can you use Newton's method for root finding to approximate $\sqrt{20}$ using only the operations $+$, $-$, \times , \div ? Apply two iterations of Newton's method starting from $x_0 = 5$. Report your final estimate of $\sqrt{20}$ as a rational number.

Problem 5: True or False?

Either way, you need to fully and accurately justify your answer to receive credit.

- (a) The point $(2, -\frac{1}{2})^T$ is the unique global minimum of $f(x_1, x_2) = 2x_1^2 + x_2^2 - 8x_1 + x_2$ over the set $\{(x_1, x_2) \mid x_1^2 + x_2^2 \leq 5\}$.
- (b) Consider the iterations $x_{k+1} = x_k + \alpha_k d_k$ with constant stepsize (i.e., $\alpha_k = \alpha > 0, \forall k$) used to minimize a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$. If the directions d_k are descent directions for all k , and if the function f is convex, then the iterations satisfy $f(x_{k+1}) \leq f(x_k)$ for all k .