Name:_____

PRINCETON UNIVERSITY

ORF 363/COS 323
Midterm Exam, Fall 2015

OCTOBER 22, 2015, FROM 1:30 PM TO 2:50 PM

Instructor: *AIs:*

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PLEASE DO NOT TURN THIS PAGE. START THE EXAM ONLY
WHEN YOU ARE INSTRUCTED TO DO SO.

1. You are allowed a single sheet of A4 paper, double-sided, hand-written or typed.

2. No electronic devices are allowed (e.g., cell phones, calculators, laptops).

3. All questions should be answered in the booklet. Please write out and sign the following
   pledge on the booklet "I pledge my honor that I have not violated the honor code during
   this examination."

4. Make sure you write your name on the first page of these questions and return the
   questions to us at the end of the exam. Please don't forget to write your name on the
   booklet as well.

5. You are allowed to cite results proved in lecture or lecture notes without proof.

Grading

Problem 1	20 *pts*	
Problem 2	20 *pts*	
Problem 3	20 *pts*	
Problem 4	20 *pts*	
Problem 5	20 *pts*	
TOTAL	100	

**Problem 1: Local minima and maxima.**

Find all the local minimizers and the local maximizers of

$$f(x_1, x_2) = \frac{1}{2}x_1^2 + x_1 x_2 - \frac{3}{2}x_2^2 + 2x_1 + 5x_2 + \frac{1}{3}x_2^3.$$

Show your reasoning.

Problem 2: Halloween stalking.

It's October 31st and you want to be close to the Halloween parade in Manhattan but also not too far from either of two bars in town where your crush is supposed to show up tonight. You don't know which one (s)he is going to go to yet, but you are checking his/her Facebook status non-stop.
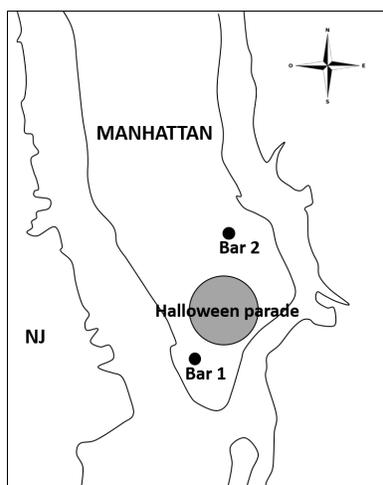
Write a convex optimization problem that finds an optimal location for you to wait at, which (i) minimizes the amount of walking you have to do in worst case to get to the bar, and (ii) allows you to be within 0.5 km walking-distance of the parade. Bar 1 is located at coordinates $(0.5, 5)$, bar 2 is in location $(-0.8, -3)$, and the parade is happening in a circle of radius 1 km centered at $(0, 0)$. Note that in Manhattan you can only walk West-East and North-South. You don't need to justify that your problem is convex, but if you write down a non-convex problem, you will not receive full credit. You also don't need to solve the problem.

Problem 3: Least squares with a Tikhonov regularizer
Consider the optimization problem

$$\min_{x \in \mathbb{R}^n} \ ||Ax - b||_2^2 + \lambda||x||_2^2,$$

where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, $\lambda \geq 0$ (a scalar) are given.

(a) Is this a convex optimization problem? Why?

(b) Give two different conditions on A, λ under which the solution is unique.

(c) Assuming that the solution to the problem is unique, find a closed form expression for it in terms of A, b and λ.

Problem 4: For each function below, tell us if it is "not convex", "convex but not strictly convex", or "strictly convex". Justify your answer.

(a) $f(x) = x^T Q x + b^T x + c$, where $x = (x_1, x_2)^T$, $Q = \begin{pmatrix} 2 & 2 \\ 2 & 3 \end{pmatrix}$, $b = (1, -3)^T$ and $c = 5$.

(b) $f(x_1, x_2) = (x_1 - x_2)^2 + x_1^3$

(c) $f(x_1, x_2) = (x_1 - x_2)^2$

Problem 5: Show by a counterexample that the sum of two quasiconvex functions is not always quasiconvex.