ORF 363/COS 323	PROBLEM SET 3	Fall 2016, Princeton University
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Due at 1:30pm in class, on October 13th, 2016		

Problem 1: For each function below, tell us if it is "not convex", "convex but not strictly convex", or "strictly convex". Justify your answer.

(a) $f(x) = x^T Q x + b^T x + c$, where $x = (x_1, x_2)^T$, $Q = \begin{pmatrix} 2 & 2 \\ 2 & 3 \end{pmatrix}$, $b = (-1, -3)^T$ and $c = 5$,

(b) $f(x_1, x_2) = (2x_1 - 3x_2)^2 + 4x_1^3$,

(c) $f(x_1, x_2) = (5x_1 - x_2)^2$.

Problem 2: Prove or disprove the following claim:
The sum of two quasiconvex functions is quasiconvex.

Problem 3: Minimizers of convex problems.
Let $f : \mathbb{R}^n \to \mathbb{R}$ be a convex function and $\Omega \subseteq \mathbb{R}^n$ be a convex set. Show that the set of minimizers of f over Ω is convex (i.e., a convex set).

Problem 4: Consider a homogeneous polynomial $p(x) = p(x_1, \dots, x_n)$ of degree $d \geq 2$. Show that if p is convex, then it is nonnegative; i.e., $p(x) \geq 0$, for all $x \in \mathbb{R}^n$. (Note: A polynomial is homogeneous of degree d if all of its monomials have degree exactly d. The degree of a monomial $x_1^{\alpha_1} \dots x_n^{\alpha_n}$ is equal to $\alpha_1 + \cdots + \alpha_n$.)

Problem 5: Regression with different penalties using convex optimization
We are given 7 data points (x_i, y_i) in $\mathbb{R}^2$: $\{(0,0), (1,3), (2,7), (3,-1), (4,0), (5,5), (6,10)\}$. We would like to fit a cubic polynomial

$$p(x) = c_3 x^3 + c_2 x^2 + c_1 x + c_0$$

to our data to minimize the so-called L^2-error :

$$\underset{c_0,\dots,c_3}{\text{minimize}} \quad \sum_{i=1}^{7} (p(x_i) - y_i)^2, \tag{1}$$

1. (a) Confirm that problem (1) is a least squares problem by writing it as the problem of minimizing $||Ac - b||^2$ over $c = (c_0, \ldots, c_3)^T$, for some appropriate matrix A and vector b. Find the optimal solution and the optimal value using the closed-from solution we derived in class for the least squares problem.

   (b) Solve problem (1) in CVX and compare the answer with what you got in part (a).

2. Now consider two other types of error:

   - the L^1-error:

$$\underset{c_0,\ldots,c_3}{\text{minimize}} \quad \sum_{i=1}^{7} |p(x_i) - y_i|, \tag{2}$$

   - and the L^∞-error :

$$\underset{c_0,\ldots,c_3}{\text{minimize}} \quad \max_{i=1,\ldots,7} |p(x_i) - y_i|. \tag{3}$$

   Although these problems no longer admit a closed-form solution, we can just as easily solve them using convex optimization.

   (a) Show that problems (2) and (3) are convex optimization problems. Solve them using CVX and report their optimal values and optimal solutions.

   (b) Generate a single plot with the seven data points, as well as the three cubic polynomials that are solutions to problems (1), (2), and (3). Let the x axis run from -1 to 7 and label your plots clearly. You may want to use the `hold on` command to display multiple plots on the same figure.